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Optimal two-level choice designs for estimating main effects and specified two-factor interactions

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Abstract

Over two decades, optimal choice designs have been obtained for estimating the main effects and the main plus two-factor interaction effects under both the multinomial logit model and the linear paired comparison model. However, there are no general results on the optimal choice designs for estimating main plus *some* two-factor interaction effects. We consider a model involving the main plus two-factor interaction effects with our interest lying in the estimation of the main effects and a specified set of two-factor interaction effects. The specified set of the two-factor interaction effects include the interactions where only one of the factors possibly interact with the other factors. We first characterize the information matrix and then construct universally optimal choice designs for choice set sizes 3 and 4.

1 Introduction

Discrete choice experiments are widely used in various areas including marketing, transport, environmental resource economics and public welfare analysis. A choice experiment consists of N choice sets with each set containing m options such that there are no repeated options in a choice set. A respondent is shown each of the choice sets in turn and is asked for the preferred option as per his perceived utility. Each option in a choice set is described by a set of n factors, where each factor has two or more levels. In this paper we consider each factor to be at two levels, 0 and 1 (say). Thus there are a total of 2^n options. It is ensured that respondents choose one of the options in each choice set (termed forced choice in the literature). A choice design is a collection of choice sets employed in a choice experiment and may contain repeated choice sets.

We denote αth choice set by $\mathcal{T}_\alpha = (T_{\alpha 1}, T_{\alpha 2}, \ldots, T_{\alpha m})$, where $T_{\alpha i}$ is the ith option in the αth choice set, $\alpha = 1, 2, \ldots, N$ and $i = 1, 2, \ldots, m$. Since an option in the choice set is a representation of n factors, $T_{\alpha i}$ can be written as $(i_1 i_2 \cdots i_n)_\alpha$ where i_q represents the level of the qth factor f_q in the ith option. The collection of all such choice sets $\mathcal{T}_\alpha$, $\alpha = 1, 2, \ldots, N$ is called a choice design, say d, with parameters N, n and m.

Most of the work on optimal choice designs is based on the multinomial logit model approach of Huber and Zwerina (1996), or the multinomial logit model approach of Street and Burgess (2007). It follows from Singh et al. (2015) that, under the main effects model, the two approaches are equivalent for the purpose of finding optimal paired choice designs (designs with $m = 2$). This result can be easily extended to the models involving main effects and some or all two-factor interaction effects. Street and Burgess (2007) and Street and Burgess (2012) present a comprehensive exposition of designs for choice experiments under multinomial logit model. The multinomial logit model specifies the probability of an individual choosing one of the m options from a choice set. Simultaneously, Graßhoff et al. (2003) and Graßhoff et al. (2004) studied linear paired comparison designs which are analyzed under the linear paired comparison model. In the linear paired comparison model, rather than just choosing the preferred option, the respondents have to indicate on a scale how strong their preference is. More precisely, the response is described by the model, $y = P\theta + \epsilon$, where P is the effects coded difference matrix corresponding to the parameters of interest θ, and ϵ is the random error vector. The matrix $P'P$ is recognized as the information matrix under the linear paired comparison model. Großmann and Schwabe (2015) showed that for choice set size m, the average information matrix under the multinomial logit model is

$$M = \frac{4}{m^2} \sum_{1 \leq i < j \leq m} M_{ij} \tag{1}$$

where M_{ij} is the average information matrix of a paired choice design corresponding to options i and j. This also means that the contribution of the choice set $\mathcal{T}_\alpha$ to the information matrix is equal to $4/m^2$ times the sum of the individual contributions of the $m(m-1)/2$ different *component pairs* $(T_{\alpha i}, T_{\alpha j})$ that $\mathcal{T}_\alpha$ contains.

Optimal designs under the linear paired comparison model have been described in Graßhoff et al. (2003) and Graßhoff et al. (2004) and that under the equal choice probability multinomial logit model in Street and Burgess (2007), Demirkale et al. (2013), Bush (2014) and Singh et al. (2015).

Most of the work on designs for choice experiments is based on an a priori assumption that either only the main effects of the factors or the main effects and all two-factor interaction effects are to be estimated. In practice however, an experimenter may be interested in estimating all main effects and only a specified set of two-factor interaction effects when interaction effects involving three or more factors are negligible. The model then includes all main and two-factor interaction effects and that experimenter's interest lies in only *some* but not all two-factor interaction effects. In the traditional factorial design setup, the issue of estimability and optimality in situations of this kind has been addressed by Hedayat and Pesotan (1992), Wu and Chen (1992), Hedayat and Pesotan (1997), Chiu and John (1998), Dey and Mukerjee (1999) and Dey and Suen (2002).

Street and Burgess (2012) and Großmann and Schwabe (2015) observed that there are no general results on the optimal choice designs for estimating main plus *some* two-factor interaction effects, though Street and Burgess (2007) highlighted the problem giving few examples.

Let F_h represent the main effects corresponding to the factor f_q, $h = q = 1, \ldots, n$ and F_h represent the two-factor interaction effects corresponding to the factors f_q and $f_{q'}$, $h = \sum_{i=0}^{q-1}(n-i) + (q'-q)$, $1 \leq q < q' \leq n$. Note that for the two-factor interaction effects F_h corresponding to the factors f_q and $f_{q'}$, $1 \leq q < q' \leq n$, the value of h ranges from $n+1$ to $n + n(n-1)/2$. Under the hierarchical model, with all interaction effects involving three or more factors absent, we have only one of the factors that can possibly interact with the others. Without loss of generality, the factor f_1 interacts with each of the other $n-1$ factors and thus our interest lies in the n main effects $F_h, h = 1, \ldots, n$ and $n-1$ specified two-factor interaction effects $F_h, h = n+1, \ldots, 2n-1$. Interest on such specified two-factor interaction effects arise in situations when one factor like price of a product interacts individually with the other $n-1$ factors of the product.

In this paper, we construct universally optimal choice designs for estimating main effects and the specified set of two-factor interaction effects when $m = 3$ and $m = 4$, under the assumption that all three or higher order interaction effects are absent.

2 The information matrix

Using the effects coded matrix as in Großmann and Schwabe (2015), for the ith option $(i_1 i_2 \cdots i_n)_\alpha$ in αth choice set, the corresponding effects coded vector $P_{\alpha i} = ((P_{\alpha i})_1, \ldots, (P_{\alpha i})_n)$ is a $1 \times n$ vector with $(P_{\alpha i})_q = 1$ if $i_q = 0$ and -1 otherwise, $q = 1, \ldots, n$. Also, $Q_{\alpha i} = ((Q_{\alpha i})_{12}, \ldots, (Q_{\alpha i})_{(k-1)k})$ where $(Q_{\alpha i})_{qq'} = (P_{\alpha i})_q (P_{\alpha i})_{q'}, 1 \leq q < q' \leq n$. Thus, for the ith option, an $N \times n$ matrix $P_i = (P'_{1i}, \ldots, P'_{Ni})'$ and an $N \times \binom{n}{2}$ matrix $Q_i = (Q'_{1i}, \ldots, Q'_{Ni})'$. Furthermore, for any two options $i < j$, $X_{ij} = P_i - P_j$ and $Y_{ij} = Q_i - Q_j$. Following Großmann and Schwabe (2015), the information matrix corresponding to options i and j, for estimating the main plus two-factor interaction effects, is $M_{ij} = \frac{1}{N}\begin{bmatrix} X'_{ij}X_{ij} & X'_{ij}Y_{ij} \\ Y'_{ij}X_{ij} & Y'_{ij}Y_{ij} \end{bmatrix}$.

For F_h, $h = 1, \ldots, n$, we define the hth positional value corresponding to the option $T_{\alpha i}$ as i_h. Also, for $h = n+1, \ldots, n+n(n-1)/2$, the hth positional value corresponding to the option $T_{\alpha i}$ is defined as $i_q + i_{q'} \pmod 2$ $(= i_h^*$, say). For the option $T_{\alpha i}$, the hth and kth positional value is $(i_h i_k)_{hk}$; $(i_h i_k^*)_{hk}$; and $(i_h^* i_k^*)_{hk}$ respectively for $h \neq k, (h,k) \in \{1, \ldots, n\}$; $h \in \{1, \ldots, n\}, k \in \{n+1, \ldots, n + n(n-1)/2\}$; and $h \neq k, (h,k) \in \{n+1, \ldots, n+n(n-1)/2\}$. Similarly for the *component pair* $(T_{\alpha i}, T_{\alpha j})$, the hth and kth positional value for the above three cases respectively are $(i_h i_k, j_h j_k)_{hk}$; $(i_h i_k^*, j_h j_k^*)_{hk}$; and $(i_h^* i_k^*, j_h^* j_k^*)_{hk}$.

Since our interest lies in the n main effects $F_h, h = 1, \ldots, n$ and the $n-1$ specified two-factor interaction effects $F_h, h = n+1, \ldots, 2n-1$, let $Y_{ij} = (Y_{(1)ij}\ Y_{(2)ij})$, where $Y_{(1)ij}$ is a $N \times (n-1)$ matrix corresponding to the selected two-factor interaction effects and $Y_{(2)ij}$ is a $N \times (n-1)(n-2)/2$ matrix of the remaining two-factor interaction effects. Also, let $X'X = \sum_{1 \leq i < j \leq m} X'_{ij} X_{ij}$, and for $t = 1, 2$, $X'Y_{(t)} = \sum_{1 \leq i < j \leq m} X'_{ij} Y_{(t)ij}$. Finally for $s = 1, 2$ and $t = 1, 2$, $Y'_{(s)} Y_{(t)} = \sum_{1 \leq i < j \leq m} Y'_{(s)ij} Y_{(t)ij}$.

Then the information matrix for estimating the main effects and the specified two-factor interaction effects is,

$$\tilde{M} = \frac{1}{N} \begin{bmatrix} X'X & X'Y_{(1)} \\ Y'_{(1)}X & Y'_{(1)}Y_{(1)} \end{bmatrix} - \begin{bmatrix} Y'_{(2)}X & Y'_{(2)}Y_{(1)} \end{bmatrix} [Y'_{(2)}Y_{(2)}]^{-} \begin{bmatrix} X'Y_{(2)} \\ Y'_{(1)}Y_{(2)} \end{bmatrix} \qquad (2)$$

.

A choice design for estimating the main effects and the specified two-factor interactions is said to be *connected* if $rank(\tilde{M}) = 2n - 1$. In what follows, the class of all connected choice designs involving n two-level factors and N choice sets each of size m is denoted by $\mathcal{D}_{N,k,m}$.

For F_h and F_k, $h \neq k$, let N_{hk}^{+} and N_{hk}^{-} be the total number of *component pairs* of the positional value type $(00, 11)_{hk}$ and $(01, 10)_{hk}$ respectively, across all $m(m-1)/2$ possible pairs of a choice set of size m and among all such N sets in the choice design.

Theorem 2.1. *The off-diagonal elements of $\tilde{M}$ are zero if,*
(i) $N_{hk}^{+} = N_{hk}^{-}$, for $h \neq k, (h,k) \in \{1, \ldots, 2n-1\}$, and
(ii) $N_{hk}^{+} = N_{hk}^{-}$, for $h \in \{1, \ldots, 2n-1\}$ and $k \in \{2n, \ldots, n + n(n-1)/2\}$.

Proof. It is easy to see that for $(h,k) \in \{1, \ldots, n + n(n-1)/2\}$, the exhaustive cases leading to possible values of (h,k)th entries of $X'_{ij}X_{ij}$, $X'_{ij}Y_{ij}$, $Y'_{ij}Y_{ij}$ and its associated *component pairs* $(T_{\alpha i}, T_{\alpha j})$, are

(i) Case 1: For $h \neq k, (h,k) \in \{1, \ldots, n\}$, (h,k)th entry in $X'_{ij}X_{ij}$ is -4 if $(i_h i_k, j_h j_k)_{hk} \equiv (01, 10)_{hk}$, is 4 if $(i_h i_k, j_h j_k)_{hk} \equiv (00, 11)_{hk}$ and is 0 otherwise.

(ii) Case 2: For $h \in \{1, \ldots, n\}, k \in \{n+1, \ldots, n+n(n-1)/2\}$, (h,k)th entry in $X'_{ij}Y_{ij}$ is 4 if $(i_h i_k^*, j_h j_k^*)_{hk} \equiv (01, 10)_{hk}$, is -4 if $(i_h i_k^*, i_h j_k^*)_{hk} \equiv (00, 11)_{hk}$ and is 0 otherwise.

(iii) Case 3: For $h \neq k, (h,k) \in \{n+1, \ldots, n+n(n-1)/2\}$, (h,k)th entry in $Y_{ij}'Y_{ij}$ is -4 if $(i_h^* i_k^*, j_h^* j_k^*)_{hk} \equiv (01, 10)_{hk}$, is 4 if $(i_h^* i_k^*, j_h^* j_k^*)_{hk} \equiv (00, 11)_{hk}$ and is 0 otherwise.

Applying the above three cases, proof follows from (2) and the definition of N_{hk}^+ and N_{hk}^-. $\square$

In a choice set $\mathcal{T}_\alpha$, let $n_{h_\alpha} \in \{0, 1, 2, \ldots, m\}$ represents the number of options such that the hth positional value is 0. The following Theorem gives upper bound to $trace(\tilde{M})$.

Theorem 2.2. *For a choice design d, with N choice sets of size m, an upper bound of $trace(\tilde{M})$ is*

$$trace(\tilde{M}) \leq \begin{cases} 4(2n-1) & \text{for } m \text{ even} \\ 4(2n-1)(m^2-1)/m^2 & \text{for } m \text{ odd} \end{cases},$$

with equality attaining when the following two conditions are satisfied:

(i) $n_{h_\alpha} = m/2$ for m even and $n_{h_\alpha} = (m-1)/2$ or $(m+1)/2$ for m odd for every $h = 1, \ldots, 2n-1$ and for every choice set $\mathcal{T}_\alpha$ and
(ii) $N_{hk}^+ = N_{hk}^-$, for $h \in \{1, \ldots, 2n-1\}, k \in \{2n, \ldots, n+n(n-1)/2\}$.

Proof. Note that from (2), $trace(\tilde{M}) \leq 4\{trace(X'X/m^2N) + trace(Y_{(1)}'Y_{(1)}/m^2N)\}$ since the matrix $\left[Y_{(2)}'X \quad Y_{(2)}'Y_{(1)}\right] [Y_{(2)}'Y_{(2)}]^- \left[Y_{(2)}'X \quad Y_{(2)}'Y_{(1)}\right]'$ is non-negative definite. Now we find the maximum possible trace of $X'X$ and $Y_{(1)}'Y_{(1)}$.

For $h \in \{1, \ldots, n\}$, hth diagonal entry in $X_{ij}'X_{ij}$ is 4 if $i_h - j_h = \pm 1$ and is 0 otherwise. For $h \in \{n+1, \ldots, n+n(n-1)/2\}$, hth diagonal entry in $Y_{ij}'Y_{ij}$ is 4 if $i_h^* - j_h^* = \pm 1$ and is 0 otherwise. This implies that the value of hth diagonal entry of $X'X$ and $Y_{(1)}'Y_{(1)}$ is non-zero when hth factor differs among two options and this happens $n_{h_\alpha}(m - n_{h_\alpha})$ times. Therefore, every choice set $\mathcal{T}_\alpha$ adds a value $4n_{h_\alpha}(m - n_{h_\alpha})$ to the hth diagonal entry of $X'X$ and $Y_{(1)}'Y_{(1)}$. Clearly, $4n_{h_\alpha}(m - n_{h_\alpha})$ is maximum when $n_{h_\alpha} = m/2$ for m even, and $n_{h_\alpha} = (m-1)/2$ or $(m+1)/2$ for m odd. By simple addition of $(1/m^2N)\max(4n_{h_\alpha}(m - n_{h_\alpha}))$ over all choice sets $\alpha = 1, \ldots, N$, we get

$$trace(X'X/m^2N) \leq \begin{cases} n & \text{for } m \text{ even} \\ n(m^2-1)/m^2 & \text{for } m \text{ odd} \end{cases}$$

and

$$trace(Y_{(1)}'Y_{(1)}/m^2N) \leq \begin{cases} (n-1) & \text{for } m \text{ even} \\ (n-1)(m^2-1)/m^2 & \text{for } m \text{ odd} \end{cases}.$$

$\square$

Remark 1. *For $m = 2$, it is noted that one cannot simultaneously maximize each diagonal element of $\tilde{M}$. For given N and n, with respect to maximum trace$(\tilde{M})$, (i) all designs with m even are equivalent and (ii) a design with m odd is always inferior to a design with m even.*

3 Construction of universally optimal designs

The criteria of *universal optimality* was introduced by Kiefer (1975) and is a strong family of optimality criteria which includes $A-$, $D-$, and $E-$ criteria as particular cases. Kiefer (1975) also obtained the following sufficient condition for universal optimality. *Suppose $d^* \in \mathcal{D}$ and $\tilde{M}_{d^*}$ satisfies (i) $\tilde{M}_{d^*}$ is scalar multiple of I_p and, (ii) trace$(\tilde{M}_{d^*}) = max_{d \in \mathcal{D}}$ trace$(\tilde{M}_d)$. Then d^* is universally optimal in $\mathcal{D}$.*

We now provide a simple method for constructing universally optimal two-level choice designs with choice set size $m = 3$ and $m = 4$.

Theorem 3.1. *Let $n = 4t - j$, where t is a positive integer and $j = 0, 1, 2, 3$. Also, given a Hadamard matrix H of order $4t$ in normal form, let H_1 be the Hadamard matrix derived from H by multiplying the first column of H by -1. Let $Z_1 = H, Z_2 = -H, Z_3 = H_1, Z_4 = -H_1$. For $w = 1, 2, 3, 4$, let A_w be respective matrices obtained by replacing every entry i ($i = 1, -1$) of Z_w by $(1-i)/2$, and then deleting rightmost j columns from Z_w, where $j = 4t - n, j \in \{0, 1, 2, 3\}$. Consider rows of A_w as options. Then, $d_1 = \big[(A_1, A_2, A_3, A_4)\big]$ and $d_2 = \begin{bmatrix} (A_1, & A_2, & A_3) \\ (A_1, & A_2, & A_4) \end{bmatrix}$ are universally optimal two-level choice design in $\mathcal{D}_{4t,n,4}$ and in $\mathcal{D}_{8t,n,3}$, respectively.*

Proof. To prove that d_1 and d_2 are universally optimal choice designs, we show that the information matrix $\tilde{M}$ for the designs d_1 and d_2 are of the form βI_n for some scalar β and that d_1 and d_2 maximizes *trace*$(\tilde{M})$ in the respective classes of designs $\mathcal{D}$. First we show that for every $h \neq k$, $(h, k) \in \{1, \ldots, n + n(n-1)/2\}$, the (h, k)th element of the $\tilde{M}$ is zero. Note that the design d_1 consists of the *component pair* designs $\{(A_\delta, A_{\delta'}), 1 \leq \delta < \delta' \leq 4\}$. Denoting the *component pair* designs of d_1 by $d_1^{\delta\delta'}, 1 \leq \delta < \delta' \leq 4$, we now calculate N^+_{hk} and N^-_{hk} for the design d_1.

Since H is a Hadamard matrix of order $4t$, for all $h \neq k$, $(h, k) \in \{1, \ldots, n + n(n-1)/2\}$, the combinations from the set $\{(00)_{hk}, (11)_{hk}\}$ and from the set $\{(10)_{hk}, (01)_{hk}\}$ occurs equally often for each of the *component pair* designs (A_1, A_2), (A_1, A_3), (A_2, A_4), (A_3, A_4), i.e., $N^+_{(\delta\delta')hk} = N^-_{(\delta\delta')hk} = 0$ or $2t$ for $(\delta, \delta') = (1, 2)$, $(1, 3)$, $(2, 4)$, $(3, 4)$, where $N^+_{(\delta\delta')hk}$ is the total number of pairs of the type $(00, 11)_{hk}$ corresponding to hth and kth positional values in $d_1^{\delta\delta'}$, and $N^-_{(\delta\delta')hk}$ is the total number of pairs of the type $(01, 10)_{hk}$ corresponding to hth and kth positional values in $d_1^{\delta\delta'}$. Furthermore, for $(\delta, \delta') = (1, 4), (2, 3)$, the respective *component*

pair designs (A_1, A_4) and (A_2, A_3) have $N^+_{(\delta\delta')hk} = N^-_{(\delta\delta')hk} = 0$ or $2t$ for all $h \neq k$, $(h, k) \in \{1, \ldots, n+n(n-1)/2\}$, except $(h, k) = (h, n+h-1)$, $h = 2, 3, \ldots, n$, i.e., (h, k) corresponding to the main effects involving f_h and the two-factor interaction effects involving f_1 and f_h, $h = 2, \ldots, n$. For such (h, k)'s, $N^+_{(14)hk} = N^-_{(23)hk} = 4t$, and $N^-_{(14)hk} = N^+_{(23)hk} = 0$. Therefore, using the result of Theorem 2.1 it follows that $\tilde{M}$ for the design d_1 has off-diagonal elements zero.
The design d_1 also ensures that $n_{h_\alpha} = 2$, for $h \in \{1, 2, \ldots, 2n-1\}$ and for every choice set. Therefore using Theorem 2.2, it follows that each of the diagonal elements of $\tilde{M}$ equals 4 and $trace(\tilde{M})$ is maximum for the design d_1. Thus d_1 is universally optimal in $\mathcal{D}_{4t,n,4}$.
To establish that the design d_2 is universally optimal in $\mathcal{D}_{8t,n,3}$, one can see that the *component pairs* of the design are similar to the ones corresponding to d_1 and thus $\tilde{M}$ for the design d_2 has off-diagonal elements zero. Regarding the diagonal elements of the $\tilde{M}$, the design d_2 ensures that $n_{h_\alpha} = 1$ or 2, for $h \in \{1, 2, \ldots, 2n-1\}$ and for every choice set. Therefore, from Theorem 2.2, each of the diagonal elements of $\tilde{M}$ equals 32/9 and $trace(\tilde{M})$ is maximum for d_2. Thus, the design d_2 is universally optimal in $\mathcal{D}_{8t,n,3}$. □

Remark 2. *As an alternative to d_2, if situation demands, one may consider a choice design* $d_{2'} = \begin{bmatrix} (A_1, & A_2, & A_3) \\ (A_1^*, & A_2^*, & A_4^*) \end{bmatrix}$ *with distinct options, which is also universally optimal in $\mathcal{D}_{8t,n,3}$. Here, for $w = 1, 2, 4$, A_w^* is obtained from A_w by adding 1 to the elements of the 2nd column of A_w, reduced mod 2.*

Example 3.1. *Consider a 2^{8-j} choice experiment ($j = 0, 1, 2, 3$) conducted through 8 choice sets of size 4 each. The 2^8 ($j = 0$) choice design d_1 (as below), is universally optimal in $\mathcal{D}_{8,8,4}$.*

$$d_1 = \begin{bmatrix} (00000000, & 11111111, & 10000000, & 01111111) \\ (01010101, & 10101010, & 11010101, & 00101010) \\ (00110011, & 11001100, & 10110011, & 01001100) \\ (01100110, & 10011001, & 11100110, & 00011001) \\ (00001111, & 11110000, & 10001111, & 01110000) \\ (01011010, & 10100101, & 11011010, & 00100101) \\ (00111100, & 11000011, & 10111100, & 01000011) \\ (01101001, & 10010110, & 11101001, & 00010110) \end{bmatrix}.$$

Deleting the last j factors we get the corresponding universally optimal design in $\mathcal{D}_{8,8-j,4}$, $j = 1, 2, 3$. Now consider the design d_2.

$$d_2 = \begin{bmatrix} (00000000, & 11111111, & 10000000) \\ (01010101, & 10101010, & 11010101) \\ (00110011, & 11001100, & 10110011) \\ (01100110, & 10011001, & 11100110) \\ (00001111, & 11110000, & 10001111) \\ (01011010, & 10100101, & 11011010) \\ (00111100, & 11000011, & 10111100) \\ (01101001, & 10010110, & 11101001) \\ (00000000, & 11111111, & 01111111) \\ (01010101, & 10101010, & 00101010) \\ (00110011, & 11001100, & 01001100) \\ (01100110, & 10011001, & 00011001) \\ (00001111, & 11110000, & 01110000) \\ (01011010, & 10100101, & 00100101) \\ (00111100, & 11000011, & 01000011) \\ (01101001, & 10010110, & 00010110) \end{bmatrix}.$$

Deleting the last j factors ($j = 0, 1, 2, 3$) of the design d_2 we get the corresponding universally optimal design in $\mathcal{D}_{16,8-j,3}$, $j = 0, 1, 2, 3$.

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