TABLE II
COMPARISON BETWEEN BURG'S, RER AND SPAC METHODS; m = 105, n+ = 2000.

| Method | $\beta(\cdot, \cdot)$ | $\rho(\cdot, \cdot)$ | $R(\cdot, t)$ | a(\cdot) | $\sigma^2 10^2$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SA. bias</td>
<td>0.124</td>
<td>0.118</td>
<td>0.040</td>
<td>0.523</td>
<td>4.098</td>
</tr>
<tr>
<td>SA. $\sqrt{MSE}$</td>
<td>0.208</td>
<td>0.235</td>
<td>5.288</td>
<td>0.738</td>
<td>6.097</td>
</tr>
<tr>
<td>Bo. bias</td>
<td>0.125</td>
<td>0.118</td>
<td>0.040</td>
<td>0.531</td>
<td>4.153</td>
</tr>
<tr>
<td>Bo. $\sqrt{MSE}$</td>
<td>0.208</td>
<td>0.235</td>
<td>5.288</td>
<td>0.747</td>
<td>6.164</td>
</tr>
<tr>
<td>RER bias</td>
<td>0.066</td>
<td>0.119</td>
<td>0.039</td>
<td>0.017</td>
<td>0.499</td>
</tr>
<tr>
<td>RER $\sqrt{MSE}$</td>
<td>0.156</td>
<td>0.238</td>
<td>5.333</td>
<td>0.251</td>
<td>0.799</td>
</tr>
<tr>
<td>SPAC bias</td>
<td>0.065</td>
<td>0.118</td>
<td>0.040</td>
<td>0.017</td>
<td>0.475</td>
</tr>
<tr>
<td>SPAC $\sqrt{MSE}$</td>
<td>0.154</td>
<td>0.235</td>
<td>5.288</td>
<td>0.256</td>
<td>0.799</td>
</tr>
</tbody>
</table>

$\text{PAR}(6,6,6)$ model with

$\beta(1, \cdot) : 7.60, -0.80, -0.30, 0.85, -0.95, 0.80, 0.25,$

$\beta(2, \cdot) : 2.00, 0.50, 0.90, -0.90, 0.70, -0.60, 0.45,$

$\beta(3, \cdot) : 4.50, 0.00, -0.90, 0.30, -0.80, 0.55, -0.90.$

a very good approximation of the SPAC one. Table II indicates that the four methods are equivalent in $\rho(\cdot, \cdot)$ estimation. Otherwise, the Burg-type extensions give very bad results for the other parameters. This seems to be a consequence of the presence of constraints in the recursive filters construction. This clearly appears in the $\sigma^2$ case, where the bias corresponds to overestimation of residual variances of each period. It can be expected that this failure has an increased effect when the model order is large.

VII. CONCLUSION

The SPAC and RER methods are extended to the periodically correlated processes case. They are compared with the Yule-Walker method and two extensions of Burg’s method. Simulation results show that they eliminate some shortcomings of the other methods. On the other hand, we also consider the relationship between these different approaches and those related to the stationary multivariate process. The advantage of scalar approaches is to avoid use of matrices and to allow estimation of autoregressive models of any order.

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Nonuniform $M$-Band Wavepackets for Transient Signal Detection
Seema Kulkarni, V. M. Gadre, and Sudhindra V. Bellary

Abstract—In this paper, we present a scheme to detect significantly overlapping transients buried in white Gaussian noise. A nonuniform $M$-band wavepacket decomposition algorithm using $M$-band, translation-invariant wavelet transform (NMTI) is developed, and its application to transient signal detection is discussed. The robustness of the NMTI-based detector is illustrated.

Index Terms—$M$-band, receiver operating characteristics, transient, wavepackets.

I. INTRODUCTION

The transient signal detection problem has been studied using various techniques with different assumptions regarding a priori information about the signal like time of arrival, duration, time-bandwidth product and relative bandwidth, signal model, etc. [1]–[3]. In case of no prior knowledge, wavepackets and their variations have shown better performance due to their property to capture the transient signal in a

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fewer number of coefficients [4], [5]. The problem of matched subspace detection has also been addressed [10] for detection of one signal. In the case of multiple transients, however, the problem becomes more difficult, and it becomes necessary to develop a transform that is sufficiently generic.

In this paper, we have extended the transient signal detection algorithm based on UMTI and UETI [4], [5] for multiple transients overlapping in either of the time or frequency domains. No a priori information is assumed. We also have developed a new transform, a nonuniform M-band wavepacket decomposition algorithm using the translation invariant wavelet transform (NMTI), which is an extension of UMTI, and discussed its application to the detection of single as well as multiple transients. Detectors based on UMTI and NMTI are compared. The latter is seen to be more robust.

II. NONUNIFORM M-BAND TRANSLATION-INvariant WAVELET TRANSFORM (NMTI)

In M-band wavepacket decomposition, each node of the tree is decomposed using the same set of filters into M bands [4]. Here, we decompose a node in either P or Q bands, depending on which decomposition minimizes the cost. The cost function used in the algorithm, as well as elsewhere, is an additive and shift-invariant cost function

\[ C(x) = -\sum_{i} |x_i|^2 \log |x_i|^2 \]  

which, when minimized, minimizes the Shannon entropy function [7]

\[ E(x) = -\sum_{i} \frac{|x_i|^2}{\|x_i\|^2} \log \frac{|x_i|^2}{\|x_i\|^2}. \]

Consider for any \( M \in Z \), \( M \) QMF's \( a_i(n) \) for \( i = 0, 1, \ldots, M - 1 \) satisfying the following equations:

\[ \sum_{n} a_i(n + kM) a_j(n + lM) = M \delta_{i,j} \delta_{k,l}. \]  

The filter \( a_0(n) \) represents the lowpass filter, \( a_{M-1}(n) \) represents the highpass filter, and the remaining filters represent the bandpass filters. Let the operators \( F_{i,p} \) be defined for \( i = 0, 1, \ldots, P - 1 \) and \( F_{i,q} \) be defined for \( i = 0, 1, \ldots, Q - 1 \) on a signal \( x \), where \( P = (P)^N \), \( Q = (Q)^N \) for \( P, Q, N > 0 \). The operators correspond to filtering the input signal with the highpass and lowpass filters, respectively. Let the operators \( F_{i,p} \) be defined for \( i = 0, 1, \ldots, P - 1 \) and \( F_{i,q} \) be defined for \( i = 0, 1, \ldots, Q - 1 \) on a signal \( x \), where \( P = (P)^N \), \( Q = (Q)^N \) for \( P, Q, N > 0 \). The operators correspond to filtering the input signal with the highpass and lowpass filters, respectively.

The detection problem consists of choosing between the signal absent hypothesis \( H_0 \) and signal present hypotheses \( H_1 \cdots H_k \), where \( k \) is the number of transients. The idea is to design the best transformer by treating the overall waveform as comprised being of transient signal plus remainder signal and noise. At every step, the previously detected transient is zeroed out, and a new transient is identified. This could be expressed by the following steps:

\[ H_0: x(m) = n(m). \]
\[ H_1: x(m) = s_1(m) + s_2(m) + n(m). \]
\[ H_2: x(m) = s_1(m) + s_2(m) + s_3(m) + n(m) \]
\[ \vdots \]
\[ H_k: x(m) = \sum_{i=1}^{k} s_i(m) + n(m). \]  

The detector follows the steps described as follows.

1) Full wavepacket decomposition is performed using UETI, UMTI, or NMTI.
2) Best basis is selected by minimizing the cost function as in (1) over bases [8].
3) For J-level deep decomposition, a template at level L, block B, and index I consists of coefficients from \( I + (J - L) \) to \( I + (J - L) \). The best basis is searched for the template having the maximum energy. The maximum energy serves as the detection statistic.
4) The detection statistic is compared with a threshold to choose between hypotheses.
5) The maximum energy template is then forced to zero.
6) Steps 3)-5) are repeated \( k \) times, where \( k \) is the maximum possible number of transients in a signal.

A. Performance Measures

ROC and tiling pictures are used to evaluate the performance of the detectors. ROC is a plot of probability of detection versus the proba-
the magnitude of the coefficient, where more darkness corresponds to lower frequency bands below and higher frequency bands toward the top. The coefficients are indexed from left to right in order. Gray-scale represents probability of false alarm for various values of the threshold. Tiling picture is a time-scale representation of a signal. It is plotted with low frequency bands below and higher frequency bands toward the top. The coefficients are indexed from left to right in order. Gray-scale represents the magnitude of the coefficient, where more darkness corresponds to higher magnitude.

IV. SIMULATION RESULTS

The received signal was simulated as a superposition of exponentially damped sinusoids buried in white Gaussian noise

\[ x(m) = \sum_{i=1}^{k} \exp^{-\lambda_i}[\sin(B_i)]C_i + n(m) \]  

\[ x(m) = s(m) + n(m), \] where

\[ A_i = \lambda_i(m - \tau_0), \] 
\[ B_i = \sin(\omega_i(m - \tau_0) + \phi_i), \] 
\[ C_i = U(m - \tau_0) + n(m) \; \text{unit step function}; \] 
\[ \tau_0 \; \text{arrival time of the } i\text{th transient}; \] 
\[ k \; \text{number of transients}; \] 
\[ n(m) \; \text{added Gaussian noise}. \]  

A large set of signals with wide ranges of \( \lambda, \omega, \) and \( \phi \) was generated, with \( k \) ranging from one to five. The SNR is defined as the average signal energy to mean-square noise ratio. All the simulations were carried out using NMTI \((P = 2, Q = 3)\), UMTI \((M = 2 \text{ or } 3)\), or UETI \((M = 2 \text{ or } 3)\). Standard Daubechies and Coiflet filters were used for the two-band filter bank. For the three-band filter bank, the coefficients were derived using the scheme given in [6] and are given in the Appendix. ROC’s, tiling pictures, and locations of detected transients were found.

1) Fig. 1 shows ROC’s for the detectors based on different transforms in comparison with the matched filter. The signal with \( \lambda = 1/16, \omega = \pi/2, \) and \( k = 1 \) with noise of variance of 0.1 was used. NMTI is clearly seen to perform better than the rest of the transforms. The ROC corresponding to NMTI is closer to that for the matched filter, as compared with other transform.

2) The second simulation illustrates the robustness of NMTI to resolve the transients overlapping in time domain. The signal parameters with length \( = 216 \) and \( k = 3 \) in (8) are \( \tau_0_1 = \tau_0_2 = \tau_0_3 = 50, \lambda_1 = \lambda_2 = \lambda_3 = 3/64, \phi_1 = \phi_2 = \phi_3 = \pi, \omega_1 = \pi/6, \omega_2 = \pi/2, \) and \( \omega_3 = 5\pi/6 \). The maximum number of levels is \( J = 3 \) for all the transforms. Tiling pictures for UMTI and UETI are similar and are shown in Fig. 2. The tiling picture for the NMTI is shown in Fig. 3. For this particular example, only NMTI could resolve all the transients.

3) Finally, we have compared different transforms based on the correctness of the time-frequency location of the detected transient. The signal parameters with \( k = 5 \) are given in Table 1. SNR = –1.23 dB. Figs. 4 and 5, respectively, show original and noisy

- Fig. 1. ROC’s for detectors based on different transforms showing superiority of NMTI over rest of the transforms. Upper-most ROC corresponds to the detector based on matched filter. Below that one is ROC for NMTI. The detectors based on other transforms, namely, two-band MTI, two-band ETI, three-band MTI, and three-band ETI have more or less similar performance in this case.
- Fig. 2. Tiling picture for the signal whose parameters are given in Section IV using two-band UETI and three-band UETI.
- Fig. 3. Tiling picture for the signal whose parameters are given in Section IV using NMTI with \( P = 2, Q = 3 \). Here, labels A, B, and C correspond to different transients.
signals. Tables II–IV give the mean time of arrival, mean frequency, and energy statistic for the two-band UETI, three-band UETI, and NMTI, respectively. Results for UETI and UMTI were more or less similar. It is seen that NMTI is able to resolve and detect all the transients. The results obtained using NMTI are comparable with those obtained using UETI.

### V. Conclusion

In this correspondence, we have extended the scheme of single transient detection based on [4] for multiple transients. NMTI is developed, and its application to transient signal detection is discussed. NMTI is seen to outperform UMTI over a wide range of signals. It provides more flexibility and, hence, adaptability to the nature of the signal.

### APPENDIX

#### Three-Band Filter Bank

The full wavelet matrix $A$, with $M = 3$, number of vanishing moments $N = 2$, and the rank-3 DCT for its characteristic Haar matrix [6] is given as:

\[
\begin{bmatrix}
0.58 & 0.91 & 1.25 & 0.41 & 0.08 & -0.25 \\
-0.17 & -0.27 & -0.37 & 1.39 & 0.27 & -0.85 \\
0.70 & -1.14 & 0.70 & 0.00 & 0.00 & -0.00 
\end{bmatrix}
\]

### REFERENCES


An Aperiodic Phenomenon of the Extended Kalman Filter in Filtering Noisy Chaotic Signals

Henry Leung, Zhiwen Zhu, and Zhen Ding

Abstract—In this correspondence, we report an interesting behavior of the extended Kalman filter (EKF) when it is used to filter a chaotic system. We show both theoretically and experimentally that the gain of the EKF may not converge or diverge but oscillates aperiodically. More precisely, when a nonlinear system is periodic, the Kalman gain and error covariance of the EKF converge to zero. However, when the system is chaotic, they either converge to a fixed point with magnitude larger than zero or oscillate aperiodically. Our theoretical analyses are verified using Monte Carlo simulations based on some popular chaotic systems.

Index Terms—Chaos, extended Kalman filter, Lyapunov exponent, stability.

I. INTRODUCTION

Recently, there has been a great deal of interest in applying chaos to engineering problems such as control [1], signal processing [2], [3] and communications [4]. Not only have many real systems and signals been shown to be chaotic, but simulated chaotic signals can also be used in various applications such as image encryption [5], secure communication [6], and multiple access [7]. Since chaotic signals are neither entirely deterministic nor stochastic, new signal processing techniques are under development to process them efficiently.

One problem in chaotic signal processing that has drawn a lot of attention is noise filtering. In a recent workshop on complexity and communications is therefore limited. A natural choice to filter a chaotic system on-line is through a nonlinear filter developed using statistical estimation theory [11], [12]. Although a chaotic system is not stochastic, it fits into the stochastic state space representation used in the filtering theory with a zero system driven noise. In fact, the most widely used nonlinear filter, namely, the extended Kalman filter (EKF), has recently been applied to chaotic signal processing [13], chaotic control [14], and chaotic communications [15], [16]. It is hoped that the EKF will have an optimal or near optimal performance in processing noisy chaotic signals. In [15], we apply the EKF to demodulate a chaotic communication system. This chaotic modulation scheme has been shown to have great potential in spread spectrum communication and multiple access because it has a high capacity and does not require any code synchronization. While the concept of this novel chaotic communication scheme has been validated in a noise-free environment, the main problem for putting this system into practice is the lack of an efficient demodulator when noise exists. We therefore propose using an adaptive filter such as the least mean square (LMS) algorithm and the EKF in the receiver to demodulate the noisy signal. Although the EKF demodulator is based on a correct state-space representation and the LMS method only uses a simple approximated model that treats the measurement noise as the system driven noise, the performance of the EKF is found to be even worse than that of the LMS demodulator in some cases.

This unusual behavior and the potential of chaotic communication have motivated us to investigate why the EKF performs unexpectedly poorly on the simple logistic map used in chaotic modulation system. Here, we use the one-dimensional (1-D) discrete time dynamical systems to investigate the EKF performance in filtering chaotic systems because they almost include all of the behaviors and characteristics in the multidimensional systems, and they are most widely used for chaotic communication and signal processing. We show in this correspondence that when the EKF is applied to certain types of chaotic maps, the Kalman gain does not necessarily converge to zero but oscillates aperiodically. To the best of our knowledge, this strange oscillatory behavior has never been reported. Since the Kalman gain does not converge to zero, the performance of the EKF in noise filtering and chaotic demodulation is therefore rather poor.

A natural choice to filter a chaotic system on-line is through a nonlinear filter developed using statistical estimation theory [11], [12]. Although a chaotic system is not stochastic, it fits into the stochastic state space representation used in the filtering theory with a zero system driven noise. In fact, the most widely used nonlinear filter, namely, the extended Kalman filter (EKF), has recently been applied to chaotic signal processing [13], chaotic control [14], and chaotic communications [15], [16]. It is hoped that the EKF will have an optimal or near optimal performance in processing noisy chaotic signals. In [15], we apply the EKF to demodulate a chaotic communication system. This chaotic modulation scheme has been shown to have great potential in spread spectrum communication and multiple access because it has a high capacity and does not require any code synchronization. While the concept of this novel chaotic communication scheme has been validated in a noise-free environment, the main problem for putting this system into practice is the lack of an efficient demodulator when noise exists. We therefore propose using an adaptive filter such as the least mean square (LMS) algorithm and the EKF in the receiver to demodulate the noisy signal. Although the EKF demodulator is based on a correct state-space representation and the LMS method only uses a simple approximated model that treats the measurement noise as the system driven noise, the performance of the EKF is found to be even worse than that of the LMS demodulator in some cases.

This unusual behavior and the potential of chaotic communication have motivated us to investigate why the EKF performs unexpectedly poorly on the simple logistic map used in chaotic modulation system. Here, we use the one-dimensional (1-D) discrete time dynamical systems to investigate the EKF performance in filtering chaotic systems because they almost include all of the behaviors and characteristics in the multidimensional systems, and they are most widely used for chaotic communication and signal processing. We show in this correspondence that when the EKF is applied to certain types of chaotic maps, the Kalman gain does not necessarily converge to zero but oscillates aperiodically. To the best of our knowledge, this strange oscillatory behavior has never been reported. Since the Kalman gain does not converge to zero, the performance of the EKF in noise filtering and chaotic demodulation is therefore rather poor.

II. THEORETICAL ANALYSIS

Consider a nonlinear dynamical system

$$x_n = f(x_{n-1})$$  \hspace{1cm} (1)

where $f : \mathbb{R}^1 \rightarrow \mathbb{R}^1$ is a smooth function, and the measurement equation is given by

$$y_n = x_n + w_n$$  \hspace{1cm} (2)

where $w_n$ is a zero mean white Gaussian noise process with variance $\sigma_w^2$. When the global Lyapunov exponent of (1) is larger than zero, we consider (1) to be chaotic. Otherwise, the system is nonchaotic.

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