Because of the increase in the nucleon-nucleon interaction energies (30–65 MeV) have recently been reported [1]. Included are new measurements for 19 neutron-rich nuclei in this region. The initial RIB experiments [2,3] measured the total reaction cross sections of neutron-rich nuclei in the vicinity of 28 closed shells, on a 28Si target at intermediate projectile energies (30–65 MeV). To supplement the zero-range analysis at high energies, a finite-range Glauber model (FR-GM) analysis has been carried out for recently measured [1] reaction cross sections in the FR-GM is expressed as

\[ \sigma_R = 2\pi \int_{2m/k}^{\infty} b' \left( 1 - \frac{\eta}{kb'} \right) \left[ 1 - T(b') \right] db', \]

(1)

where \( \eta \) is the usual Sommerfeld parameter, \( k \) is the wave number of the projectile, and the Coulomb-modified impact parameter \( b' \) can be written as

\[ b' = \frac{1}{k} \left[ \eta + (\eta^2 + k^2 b^2)^{1/2} \right]. \]

(2)

The transparency function \( T(b) \), within the finite-range approximation, is written as [9,10]

\[ T(b) = \exp \left\{ -\int_p \int_T \sum_{ij} \left[ \Gamma_{ij}(b_{\text{eff}}) \tilde{p}_i(t) \tilde{p}_j(s) \right] d\tilde{s} d\tilde{t} \right\}, \]

(3)

where the summation indices \( i \) and \( j \) run over neutrons and protons for both the target and the projectile. The subscript \( T \) (\( P \)) refers to target (projectile) and \( \tilde{\rho}(s) \) is the \( z \)-directional-integrated nucleon density distribution expressed as

\[ \tilde{\rho}(s) = \int_{-\infty}^{+\infty} dz \rho(\sqrt{s^2 + z^2}), \]

(4)

with \( s^2 = (x^2 + y^2) \).

The profile function \( \Gamma_{ij} \) is given by [9,10]

\[ \Gamma_{ij}(b_{\text{eff}}) = \frac{1}{2\pi \beta^2} \sigma_{ij} \exp \left( -\frac{b_{\text{eff}}^2}{2\beta^2} \right), \]

(5)

with the requirement that its norm should be \( \sigma_{ij} \). In these expressions,

\[ b_{\text{eff}} = \left| \vec{b} - \vec{s} + \vec{t} \right|. \]

(6)

where \( \vec{b} \) is the impact parameter \( \vec{s} \) and \( \vec{t} \) are dummy variables for integration over the \( z \)-integrated target and projectile densities. The range parameter \( \beta \) reads [10]

\[ \beta = 0.996 \times \exp \left( -\frac{E}{106.679} \right) + 0.089, \]

(7)

The present analysis proceeds in two steps:

(i) First, the well-tested and very reliable relativistic mean field (RMF) theory [6,7] is used to calculate the ground-state properties [such as binding energies, deformations, sizes (radii), and densities].

(ii) The resulting RMF neutron and proton density distributions of the relevant projectiles and the targets required as input are then used in a finite-range Glauber model to calculate \( \sigma_R \).

The reaction cross sections for several neutron-rich nuclei with \( 7 \leq Z \leq 18 \) on a 28Si target at intermediate energies (30–65 MeV) are calculated and are compared with the corresponding recently reported new measurements. A finite-range Glauber model along with a Coulomb modification is used. The required nucleon density distributions of the relevant projectiles and the targets are obtained in the relativistic mean field framework. The calculations reproduce the experiment well. A simple phenomenological modification of the zero-range Glauber model is proposed to incorporate the finite-range effects. This one-parameter expression is found to reproduce the experimental reaction cross sections quite well.

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where $E$ is the projectile energy in units of $A$ MeV. This range parameter has been obtained phenomenologically by fitting the observed $^{12}$C on $^{12}$C cross sections from 30A MeV to 1A GeV energies [10]. The nucleon-nucleon cross section $\sigma_{ij}$ is taken either from the experiment or from some empirical fit to the experimental nucleon-nucleon cross sections (see, for example, Ref. [11]). In this work, we have taken the $\sigma_{ij}$ value from Ref. [11].

Replacing the nucleon profile function [4] by the $\delta$ function times the nucleon-nucleon cross section $\sigma_{ij}$ (zero-range limit) reduces the transparency function $T(b)$ to

$$T(b) = \exp \left[ - \sum_{i,j} \sigma_{ij} \int \rho_i(s) \rho_j(|\vec{b} - \vec{s}|) \right] d\vec{s}. \quad (8)$$

The expression for the total reaction cross section ($\sigma_R$) in the zero-range limit is identical to Eq. (1), with $T(b)$ given by Eq. (8). In the limit of high projectile energies, the expression

FIG. 1. The calculated reaction cross sections along with the corresponding experimental values [1]. The corresponding results obtained by using the phenomenological single-parameter expression [Eq. (10)] are denoted by “Fit.”
The RMF calculations reproduce the experiment rather well as expected. The experimental binding energies [15] are reproduced within ~0.25%. The deformation parameters $\beta$ agree with the experiment (where data are available) and in general match closely with the corresponding Möller and Nix (MN) values [16]. Transitions from oblate (prolate) to prolate (oblate) shapes are noticed at several places. At some places the RMF $\beta$ has opposite sign to that of MN. However, at these very places, there appears another solution at a small (≤0.5 MeV) excitation, with $\beta$ having the same sign as and value close to that of MN. The calculated charge radii differ from their corresponding experimental values (where available) [17] only at the second decimal place of a fermi. The neutron skin ($r_n - r_p$) for a given $Z$ increases almost linearly with the addition of neutrons. These observations are now standard and well established (see, for example, Refs. [18,19]).

for $\sigma_R$ [Eq. (1)] reduces to

$$\sigma_R = 2\pi \int_0^\infty b [1 - T(b)] \, db, \tag{9}$$

where $T(b)$ is the transparency function [Eq. (8)] at impact parameter $b$.

First, the RMF calculations have been carried out for the relevant nuclei (projectiles and target) appearing in the measured $\sigma_R$ data analyzed here. The RMF equations are then solved in the axially symmetric deformed oscillator basis. The pairing correlations are incorporated by using the frozen gap approximation. These calculations require the parameters appearing in the RMF Lagrangian and the pairing gap parameters $\Delta$ for the calculation of occupancies. In the present work, we use one of the most successful Lagrangian parameter sets, NL3 [12]. The required neutron and proton pairing gaps are adjusted so as to reproduce the corresponding pairing energies obtained by solving the relativistic Hartee-Bogolyubov equations in the spherical oscillator basis, with the Gogny D1S interaction [13,14] in the pairing channel.

FIG. 2. The calculated reaction cross sections along with the corresponding experimental values [1]. The results obtained by using the phenomenological single-parameter expression [Eq. (10)] are denoted by “Fit.”
The Glauber calculations for the cross sections require both the target and the projectile density distributions. All the projectile densities are obtained in the RMF framework. In the case of the deformed densities, the $L = 0$ (spherical) component is projected out and then renormalized, separately for protons and neutrons. The resulting nucleon density distributions closely agree with the experiment values (where available).

The Glauber model calculations also require an additional phenomenological parameter: the ratio of the experimental free-nucleon-nucleon cross section to the in-medium nucleon-nucleon cross section. In almost all the Glauber model analysis, this parameter is assumed to have a fixed value, 0.8 \cite{2}.

The calculated cross sections (DEF) along with the experimental values (Expt.) and the corresponding values (Fit) obtained by using a single-parameter phenomenological expression [Eq. (10)] are displayed in Figs. 1 and 2. The experiment has been performed \cite{1} at two different settings of magnetic rigidity. The experimental results corresponding to each rigidity are shown separately in the figures. It is amply clear from both figures that on average the calculation reproduces the experiment rather well. Appreciable deviations are noticed particularly at places where the experimental $\sigma_R$ has large uncertainties. Close inspection further reveals that there are certain apparent inconsistencies in the experimental data. For example, it is expected that, at least in the domain of projectile energies with which we are concerned here, the reaction cross section should increase as the projectile energy decreases. At some places the experimental trend is inconsistent with this expectation. Further, within a given set of isotopes, a few unusual trends are noticed (e.g., the results of $^{34,36}$Mg projectiles at different rigidities). Therefore, we think that more precise measurements of $\sigma_R$ are required.

It should be mentioned that the range parameter solely determined from the fit to the $^{12}$C-$^{12}$C data used in the calculation may have a different value for heavier targets (e.g., $^{28}$Si) and relatively heavy projectiles. We do not intend to introduce additional parameter(s). Instead, based on the inspection and study of the variation of the calculated $\sigma_R$ with energy and different projectile-target combinations, we propose the following one-parameter expression to incorporate the finite-range effects in a simple, phenomenological manner, by starting from the zero-range ($\sigma_{ZR}$) values of the reaction cross sections:

$$
\sigma_R = \sigma_{ZR} \left[ 1 + \frac{\alpha}{A_T^{3/2} + A_P^{3/2}} \exp (-E/200) \right],
$$

(10)

Here, $A_P (A_T)$ is the mass number of projectile (target) and $E$ is the projectile energy, expressed in units of A MeV. This ansatz ensures that at high projectile energies, the factor multiplying $\sigma_{ZR}$ is close to unity, as desired. The parameter $\alpha$ has been determined by a chi-square fit to reproduce experimental reaction cross sections for a set of target-projectile combinations spanning a wide energy range and has the value 2.1738. The $\chi^2$ obtained for FR-GM (“Fit”) reaction cross sections turns out to be 3.64 (2.97) for this data set. It is clearly seen from the figures that the proposed single-parameter expression is quite accurate. Therefore, it is possible to get accurate $\sigma_R$ values without carrying out explicit finite-range calculations, that is, merely by using the corresponding zero-range values of reaction cross sections.

In summary, a finite-range Glauber model is used to calculate the recently measured reaction cross sections for neutron-rich nuclei with $7 \leq Z \leq 18$ on a $^{28}$Si target at intermediate energies (30–65 A MeV). The calculation proceeds in two steps. First, the ground-state properties of the relevant nuclei are calculated by using the RMF formulation. As expected, the RMF calculations give an excellent account of the ground-state properties (e.g., binding energies, deformations, radii, and densities) of the relevant nuclei. The calculated (RMF) point target/projectile densities of both neutrons and protons are then used in the Glauber model to compute the cross sections. Overall, the calculations reproduce the experiment well. An accurate, simple expression involving a single parameter is proposed to incorporate the finite-range effects in the zero-range Glauber model calculations.

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