A Neural Solution for Signal Detection In Non-Gaussian Noise

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Abstract—In this paper, we suggest a neural network signal detector using radial basis function network for detecting a known signal in presence of Gaussian and non-Gaussian noise. We employ this RBF Neural detector to detect the presence or absence of a known signal corrupted by different Gaussian and non-Gaussian noise components. In case of non-Gaussian noise, computer simulation results show that RBF network signal detector has significant improvement in performance characteristics. Detection capability is better than to those obtained with multilayer perceptrons and optimum matched filter detector.

Index Terms—Radial basis function neural network, non-Gaussian noise, signal detection.

I. INTRODUCTION

In radar, sonar and communication applications, ideal signals are usually contaminated with non-Gaussian noise. Detection of known signals from noisy observations is an important area of statistical signal processing with direct applications in communications fields. Optimum linear detectors, under the assumption of additive Gaussian noise are suggested in [1]. A class of locally optimum detectors are used in [2] under the assumptions of vanishingly small signal strength, large sample size and independent observation. Recently, neural networks have been extensively studied and suggested for applications in many areas of signal processing. Signal detection using neural network is a recent trend [3] - [6]. In [3] Watterson generalizes an optimum multilayer perceptron neural receiver for signal detection. To improve performance of the matched filter in the presence of impulsive noise, Lippmann and Beckman [4] employed a neural network as a preprocessor to reduce the influence of impulsive noise components. Michalopoulou et al [5] trained a multilayer neural network to identify one of $M$ orthogonal signals embedded in additive Gaussian noise. They showed that, for $M = 1$, operating characteristics of the neural detector were quite close to those obtained by using the optimum matched filter detector. Gandhi and Ramamurti [6], [7] has shown that the neural detector trained using BP algorithm gives near optimum performance. The performance of the neural detector using BP algorithm is better than the Matched Filter (MF) detector, used for detection of Gaussian and non-Gaussian noise. In our previous work [10], we suggest the signal detector for two non-Gaussian cases such as Double exponential and Contaminated Gaussian. In this work, we explore it further and propose a neural network detector using radial basis function network and we employ this neural detector to detect the presence or absence of a known signal corrupted by Gaussian and non-Gaussian noise components. For many non-Gaussian noise distributions such as double exponential, Contaminated Gaussian, Cauchy noise etc. We found that RBF network signal detector performance is very close to that of MF and BP detector for Gaussian noise. While, we observed that in non-Gaussian noise environments the RBF network signal detector show better performance characteristics and good detection capability compared to neural detector using BP.

II. PRELIMINARIES

Probability of detection $P_d$ and the probability of false alarm $P_{fa}$ are the two commonly used measures to assess performance of a signal detector [1] That is, $P_d$ is defined as the probability of choosing $H_1$ given that $H_1$ is true, and $P_{fa}$ is defined as the probability of choosing $H_1$ given that $H_0$ is true.

$$P_d = P(\Lambda(X(t)) > \eta / H_1)$$

(1)

and

$$P_{fa} = P(\Lambda(X(t)) > \eta / H_0)$$

(2)

Consider a data vector $X(t) = [x_1(t), x_2(t), ..., x_N(t)]^T$ as an input to the detector in Figure 1. Using the additive observational model, we have

$$X(t) = S(t) + C(t)$$

(3)

for the hypothesis that the target signal is present (denoted by $H_1$) and

$$X(t) = C(t)$$

(4)

for the hypothesis that the signal is absent (denoted by $H_0$), where $S(t) = [s_1(t), s_2(t), ..., s_N(t)]^T$ is the target signal vector and $C(t) = [c_1(t), c_2(t), ..., c_N(t)]^T$ is the noise vector. The likelihood ratio is defined by

$$\Lambda(X(t)) = \frac{P(X(t)/H_1)}{P(X(t)/H_0)}$$

(5)

where $P(X(t)/H_1)$ and $P(X(t)/H_0)$ are the jointly conditional probability density functions of $X(t)$ under $H_1$ and $H_0$, respectively. Denoting the decision threshold by $\eta$, we
network is shown in Figure 2 [9]. This neural network signal
two possible states. The two bias nodes are included as part
of neurons with a linear function. One hidden layer of
neurons with nonlinear transfer functions such as the Gaussian
network detector test statistic $T_{NN}(x)$ may now be expressed
as,

$$T_{NN}(x) = \sum_{i=1}^{K} w_i \varphi_i(x) + u^2$$

where $\{\varphi_i(x), i = 1, 2, 3, \ldots, K\}$ is a set of basis functions.
The $w_i$ constitutes a set of connection weights for the output
layer. When using RBF the basis is

$$\varphi_i(x) = G(||x - \Sigma t_i||^2) + u^2_i, i = 1, 2, 3, \ldots, K$$

where $t_i = [t_{i1}, t_{i2}, \ldots, t_{in}]^T$ with $t_i$ as unknown centers
to be determined. $\Sigma$ is a symmetric positive definite weighting
matrix of size $N \times N$. $G(.)$ represents a multivariate Gaussian
distribution with mean vector $t_i$ and covariance matrix $\Sigma$. By
using above equations we redefine $T_{NN}(x)$ as

$$T_{NN}(x) = \sum_{i=1}^{K} w_i \varphi_i(x, t_i) = \sum_{i=1}^{K} w_i G(||x - t_i||).$$

(8)

We determine the set of weights $w = [w_1, w_2, \ldots, w_K]^T$ and
the set $t$ of vectors $t_i$ of centers such that the cost functional,

$$\xi(W, \tau) = \frac{1}{M} \sum_{i=1}^{M} (d_i - \sum_{j=1}^{K} w_j G(||x_i - t_j||))^2$$

(9)

where $\{\varphi_i(x), i = 1, 2, 3, \ldots, M\}$ is a new set of basis
functions. The first term on the right hand side of the
equation may be expressed as the squared Euclidean norm

$$||d - GW||^2,$$

where $d = [d_1, d_2, d_3, \ldots, d_M]^T$ and $W = [w_1, w_2, w_3, \ldots, w_K]$. [8].

The first step in the learning procedure is to define the
instantaneous value of the the cost function,

$$\xi = 1/2 \sum_{i=1}^{K} e_i^2$$

(10)

where $K$ is the size of the training sample used to do the
learning, and $e_i$ is the error signal defined by

$$e_i = d_i - F(X_i)$$

(11)

We assume $\Sigma = diag[\sigma_1, \sigma_2, \ldots, \sigma_N]$. $\Sigma$ is to be
optimized with respect to the parameters $w_i$, $t_i$, and $\sigma_i^{-1}$. The
cost function $\xi$ is convex with respect to the linear parameters
$w_i$, but non convex with respect to the centers $t_i$ and matrix
$\sigma_i^{-1}$. The search for the optimum values of $t_i$ and $\sigma_i^{-1}$
may get stuck at a local minimum in parameter space. The
different learning-parameters assigned updated value.
$t_i$ and $\sigma_i^{-1}$. RBF networks with supervised learning were able to exceed substantially the performance of multilayer perceptrons [8]. After updating at the end of an epoch, the training is continued for the next epoch and it continues until the maximum error among all K training patterns is reduced to a prespecified level.

IV. EXPERIMENTAL RESULTS AND PERFORMANCE EVALUATIONS

Neural weights are obtained by training the network at 10-dB SNR using $\theta = \sqrt{10}$ and $E[N_i^2] = 1$. During simulation, the threshold $r_{NN}$ is set to 0.5, and the bias weight $w_b$ value that gives a $P_{fa}$ value in the range $0.001 - 1$. For each $w_b$ value that gives a $P_{fa}$ value in the above range, the corresponding $P_d$ value are also simulated. These $P_d$ values are plotted against the corresponding $P_{fa}$ values to obtain the receiver operating characteristics. Of course, for a given $P_{fa}$ value, larger $P_d$ value implies a better signal detection at that $P_{fa}$. The 10-dB-SNR-trained neural network is tested in the 5-dB and 10-dB SNR environment. This latter experiment is carried out to study the neural detector’s sensitivity to the training SNR. To achieve 5-dB SNR environment, we keep $\theta$ at $\sqrt{10}$ and sufficiently increase the noise variance $E[N_i^2]$.

A. Performance in Gaussian Noise (Constant Signal, 10 dB)

Performance characteristics of neural detectors using RBF, MLP and MF detectors are presented in Figure 3 for Gaussian noise. The RBF and MLP neural detectors are trained using the constant signal and ramp signal with SNR = 10 dB. And then both neural detectors and match filter detector are tested with 10-dB SNR inputs.

B. Performance in Gaussian Noise (Ramp Signal, 10 dB)

For gaussian noise, the receiver operating characteristics of neural detectors as well as matched filter detectors are presented in Figure 4. In this case, RBF and MLP neural detectors are trained using the ramp signal with SNR = 10 dB. All detectors are then tested with 10-dB SNR inputs. In both Constant and Ramp Signal cases, the RBF and MLP neural detectors performance is very close to that of the MF detector.

C. Testing of Signal Detector in Non-Gaussian Noise

In this work, we consider the classical problem of detecting known signals in non-Gaussian noise. Performance characteristics of RBF and MLP neural detectors are presented at small false alarm probabilities (in the range $10^{-3}$ to $10^{0}$) that are of typical practical interest.

D. Performance in Double Exponential Noise (Ramp Signal)

Here we illustrate performance comparisons of the LR, MF, LO, and neural detectors using RBF and MLP for ramp signal embedded in additive double exponential noise. Figure 5-a and 5-b show the comparison for a 10-dB-SNR-trained neural detectors operating in the 10-dB and 5-dB SNR environment.

In this testing, the signal detector using RBF network continues to provide performance improvement, compare to MLP neural, MF and LO signal detectors.

E. Performance in Contaminated Gaussian Noise (Ramp Signal)

The same experiment is repeated for the ramp signal embedded in contaminated Gaussian noise with parameters $\epsilon = 0.2$, $\sigma_0^2 = 0.25$ and $\sigma_1^2 = 4$. Figure 6-a and 6-b show the comparison for a 10-dB-SNR-trained neural detectors operated in the 10-dB-SNR and 5-dB-SNR environment respectively. In all cases, we see that both MF and LO detectors perform similarly and that the neural detector using RBF...
clearly provides the best detection performance compared to MLP neural detector.

F. Performance in Cauchy Noise (Constant and Triangular Signal)

In this case, we are not consider SNR as the random variable is not finite in Cauchy noise. Here, we consider the signal energy $E$ and $\sigma^2$ of the Cauchy pdf. Performance of detector are illustrated in Figure 7 and 8. We observe that the neural detector using RBF outperforms compared to other detectors. But for relatively high $P_{fa}$ values its performance decreases compared to the matched filter and locally optimum detectors.

G. Detection Performance as a Function of SNR (Ramp Signal)

Here we try to study the behavior of MF, LO and RBF, MLP neural detector’s for fixed $E(N^2)$ and varying $\theta$ values. The noise variance is set to unity during training and testing. Here, we consider the case of contaminated Gaussian noise distribution with $\epsilon = 0.2$, $\sigma^2 = 0.25$ and $\sigma^2 = 4$, as before. The neural detectors are trained using the ramp signal at 0, 10 and 15-dB SNR. During testing, we adjust the bias weight in both the neural detector’s to ensure that the neural detector’s operation at $P_{fa} = 0.001$. We set $E[N^2]$ to unity and vary $\theta$ for SNR values between 0-15 dB. These probability of detection values are plotted in Figure 9-a,9-b and 9-c as a function of SNR. The neural detector using RBF network clearly yields superior performance characteristics in all three cases.

V. Conclusion

In this paper radial basis function network is proposed for known signal detection in non-Gaussian noise. Neural detector using radial basis function network show better performance characteristics for many non-Gaussian noise distributions such as double exponential, contaminated Gaussian and Cauchy noise. We observed that in non-Gaussian noise environments the RBF neural network signal detector show good detection capability compared to neural detector using multilayer perceptron (BP) and conventional signal detectors.

REFERENCES