Linear elastic shock response of plane plates subjected to underwater explosion

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Abstract

Linear elastic shock response of plane plates subjected to underwater explosion is of importance to warship designers. Underwater explosion experiments were carried out with small quantity of explosive charges on air-backed 4 mm high strength low alloy (HSLA) steel circular plates of 290 mm diameter and rectangular plates of $300 \times 250$ mm with varying stand-off. Strain gauges were fixed at regular intervals from the centre to the edge of the plates and dynamic strains were recorded. Strain distribution function was formed from the experimental data using which semi-analytical models were derived for predicting the elastic strain. The models showed good agreement with the experiments.

Keywords: Underwater explosion; Elastic shock response; HSLA steels; Strain analysis

1. Introduction

The effect of underwater explosion against a vessel will depend both on the proximity of the explosion and the way in which the hull is constructed [1]. The quantity of charge required to hole a single-hulled vessel during contact explosion is relatively small when compared to that of the non-contact explosion [2]. If the effect of the explosive power is small enough or the stand-off is large enough during non-contact explosion, all effects of hull structure are within its elastic range and there is no permanent damage. Estimating the elastic stresses developed during non-catastrophic attack of mines, depth charges and torpedoes is of interest to warship designers. Underwater explosion experiments were carried out on air backed circular and rectangular plates with
strain gauges fixed at equal intervals from the centre to the edge of the plates for various stand offs and dynamic strains were recorded. Strain distribution functions were found from the experimental data. These functions were used for deriving the semi-analytical models for predicting the elastic strains. A comparison of experimental data with the models showed good agreement.

2. Experimental

The high strength low alloy (HSLA) steel plates used for this investigation were 4mm in thickness having an Young’s modulus of 211 GPa and Poisson’s ratio of 0.29. The circular plate chosen for investigation was 550 mm in diameter with an exposed area of 290 mm in diameter. Rectangular plates were 550 × 450 mm with an exposed area of 300 × 250 mm. These dimensions meet the condition of infinite plane plate since the shock pulse generated during the experiments had a maximum length of 35 μs which is much less when compared to the diffraction time of the plate (96.6 μs for the circular plate and 83 μs for the rectangular plate) [3]. A pipe was inserted to the target assembly and strain gauge cables from the test plate were run through it to the data acquisition system. A rubber gasket was provided between the test plate and the flange to prevent leakage into the assembly.
A general-purpose Micro Measurements gauge CEA-06-125UW-120 and four CEA-06-125UT-120 T-rosettes were fixed on the circular plate as shown schematically in Fig. 1. The test plate was assembled into the air-backed drum model target. Micro measurements CEA-06-125UR-120 rectangular rosettes were fixed from the centre at intervals of 25 mm on the rectangular plate as shown schematically in Fig. 2. The rectangular plate was assembled into the box model target.

PEK-1 explosive (1.17 TNT equivalent) with Mk79 detonator was used for all the tests reported herein. The explosive was weighed, inserted in a plastic container and positioned at required stand
Fig. 3. Schematic of the experimental set-up used for measuring the dynamic strains on the circular plate.

off such that its centre coincided with the centre of the plate. A firing cable was led from the detonator to the firing circuit situated in the control room. The whole set up was submerged in an underwater shock tank (15 × 12 × 10 m) during the experiments. Dynamic strain was recorded on 42-channel SE7000D tape recorder through Micro Measurements 2310 amplifier and analysed using Iwatsu SM 2100B signal analyser. All strains were recorded and replayed at 720 mm/min. A schematic of the instrumentation set-up used for the experiments is shown in Fig. 3.

3. Results and discussion

3.1. Strain analysis

3.1.1. Circular plates

The results of the strain analysis experiments for circular plates are summarised in Table 1. A typical strain time history of the plate at its centre for the stand off of 1.2 m is shown in Fig. 4. The second peak of the dynamic strain [2] is greater than the first peak due to the addition of the gas bubble pulse on to the plate which has already begun to spring back from elastic deflection. The variation of strain data due to the primary pulse for the stand off varying from 2 m to 60 cm as a function of radial distance of the plate is shown in Fig. 5. Theoretically, the radial and tangential strains must be equal due to equi-biaxial membrane stretching [3]. The experimental results closely follow the theoretical trend.

3.1.2. Rectangular plates

From the strain transformation equations of two-dimensional state of stress[4], it can be deduced for rectangular plates that

\[ \varepsilon_{xx} = \varepsilon_B, \]  
\[ \varepsilon_{yy} = \varepsilon_A - \varepsilon_B + \varepsilon_c, \]  
\[ \gamma_{xy} = \varepsilon_c - \varepsilon_A, \]
Table 1
Summary of results of dynamic elastic strain experiments on the circular plate; charge quantity = 0.01 kg of PEK-1

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<th>S2B (με)</th>
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*A: tangential; B: radial.

Fig. 4. A typical strain time history recorded on the circular plate. Charge weight = 10 g of PEK-1; stand off = 1.2 m; Location: centre of the plate.

where $\varepsilon_{xx}$, $\varepsilon_{yy}$ and $\gamma_{xy}$ have their usual meanings. The principal strains are deduced from the formula

$$\varepsilon_{1,2} = \frac{1}{2} (\varepsilon_{xx} + \varepsilon_{yy}) \pm \frac{1}{2} \left[ (\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2 \right]^{1/2}$$

where $\varepsilon_{1,2}$ are the principal strains. The results of the strain analysis experiments for rectangular plates are summarised in Table 2. The variation of the strain data due to the primary pulse for the stand off varying from 2 m to 50 cm as a function of the longitudinal distance from the centre of the plate is shown in Fig. 6.

3.2. Semi-analytical models

3.2.1. Circular plates

The underwater shock energy transmitted per unit area for an air-backed plate is [5,6]

$$E_d = \frac{2mP_n^2}{\rho^2c^2} x^{2/(1-x)}$$

where $m$ is the mass of the charge, $P_n$ is the peak pressure of the primary pulse, $\rho$ is the density of the water, and $c$ is the speed of sound in water.
Fig. 5. Variation of tangential and radial strains along the radius of the circular plate. S is stand off.
Table 2
Summary of results of dynamic elastic strain experiments on the rectangular plate; charge quantity = 0.005 kg of PEK-1

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$\varepsilon_{A} = -45^\circ$; $\varepsilon_{B} = 0^\circ$; $\varepsilon_{C} = 45^\circ$.

where $E_d$ is the energy density, $m$ is the mass per unit area of the plate, $P_m$ is the peak pressure, $\rho$ is the density of water and $c$ is the velocity of sound in the water medium and $\chi$ is the inverse weight number which is equal to $\rho c \theta / m$ where $\theta$ is the time constant of the shock pulse. Since the mass per unit area can be given as the thickness times the mass density of the plate, Eq. (5) can be written as

$$E_d = \frac{2t \rho_p P_m^2}{\rho^2 c^2 x^{2/(1-x)}},$$

where $\rho_p$ is the mass density and $t$ is the thickness of the plate.

The total energy transmitted to a circular plate of radius $R$ is

$$E_t = \frac{2t \rho_p P_m^2 \pi R^2}{\rho^2 c^2 x^{2/(1-x)}}.$$
Fig. 6. Variation of strain along the longitudinal axis of the plate; $S =$ stand off; A: $-45^\circ$; B: $0^\circ$; C: $45^\circ$.
where $\varepsilon$ is the principal strain at any point on the plate, $\varepsilon_a$ is the apex principal strain and $r$ is the instantaneous radius. The elastic deformation energy for an incremental radius $dr$ of the plate is

$$U = \int_0^R U_d 2\pi r \, dr.$$  \hspace{1cm} (10)

Substituting for $U_d$ from Eq. (8) for equibiaxial membrane stretching and simplifying,

$$U = \frac{Et_0^2 \pi R^2}{3(1 - \nu)}.$$  \hspace{1cm} (11)
where \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio, respectively. It is assumed that all the kinetic energy of the plate is converted to strain energy. Equating the kinetic energy of the plate in Eq. (7) to the strain energy in Eq. (11), the apex principal strain is given as

\[
\varepsilon_a = \sqrt{\frac{6 \rho_p P_m^2 \lambda^{1/2} x (1 - \nu)}{E \rho^2 c^2}}.
\] (12)

For plane stress, the von Mises’ stress \([8]\) is given by

\[
\sigma^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2,
\] (13)

where \( \sigma \) is the von Mises’ stress of the material. Eq. (13) reduces to

\[
\sigma = \frac{E \varepsilon}{(1 - \nu)}.
\] (14)

since \( \sigma_1 \) and \( \sigma_2 \) are equal due to membrane stretching.

A comparison is made between the von Mises’ semi-analytical stress with the experimental stress in Fig. 7. There is good agreement between the model and the experimental results.

### 3.2.2. Rectangular plates

The energy transmitted to the air-backed rectangular plate is

\[
E_d = \frac{2 m P_m^2}{\rho^2 c^2} x^{2/(1 - \nu)} 4a b, \]

where \( a \) is half the length and \( b \) is half the breadth of the rectangular plate. The mass per unit area of the test plate is given by the thickness \( t \) multiplied by the mass density \( \rho_p \). Eq. (15) is modified as

\[
E_d = \frac{2 t \rho_p P_m^2}{\rho^2 c^2} x^{2/(1 - \nu)} 4a b. \] (16)

It is assumed that the rectangular plate deforms in membrane mode and all the effects of bending are neglected \([7]\). The strain energy density \( U_d \) of the elastically deforming material in plane stress is given by Eq. (8) \([8]\). Substituting for \( \sigma_1 \) and \( \sigma_2 \)

\[
U_d = \frac{E}{2(1 - \nu^2)} (\varepsilon_1^2 + \varepsilon_2^2 + 2\nu \varepsilon_1 \varepsilon_2). \] (17)

Introducing the term ‘equivalent or effective strain’ in the elastic region analogous to the equivalent plastic strain of a biaxially loaded plate, Eq. (17) modifies to

\[
U_d = \frac{1}{2} E \varepsilon_e^2, \] (18)

where

\[
\varepsilon_e^2 = \frac{1}{(1 - \nu^2)} (\varepsilon_1^2 + \varepsilon_2^2 + 2\nu \varepsilon_1 \varepsilon_2). \] (19)

From the experimental data, it is observed that the strains vary in parabolic fashion along the longitudinal axis of the plate. A similar trend is expected for the transverse direction of the plate also.
Fig. 7. Variation of von-Mises' strain along the radius of the circular plate. S is stand-off.
Therefore, it is justifiably assumed that the equivalent strain varies in parabolic fashion both in longitudinal and transverse directions of the plate. Therefore, the strain energy is computed as

$$U = 4 \int_0^a \int_0^b U_a \left(1 - \frac{x^2}{a^2}\right)^2 \left(1 - \frac{y^2}{b^2}\right)^2 t \, dy \, dx$$

(20)
which simplifies to

\[ U = \frac{64}{225} \left( \frac{E_{bc}^2}{2} \right) t^{4ab}. \]  

(21)
It is assumed that all the kinetic energy of the rectangular plate is converted to elastic strain energy. Comparing the elastic strain energy in Eq. (21) with the kinetic energy of the plate in Eq. (16),

\[
\varepsilon_e = \sqrt{\frac{14\rho_p P_m^2 x^2(1-x)}{E\rho^2 c^2}}.
\]

(22)

A comparison is made between the semi-analytical effective strain with the experimental effective strain through Fig. 8. There is an excellent agreement between the semi-analytical model and the experimental results.

4. Conclusions

Underwater explosion experiments were carried out on a thin circular HSLA steel plate and dynamic radial and tangential strains were recorded to derive a strain energy function for the variation of the apex thickness strain as a function of radial distance. A semi-analytical model was developed for predicting the strain on the circular plate by equating the kinetic energy to the strain energy of the plate. This model takes into account the shock wave parameters, the plate elastic properties, material density and thickness and the acoustic properties of the water medium. A comparison is made between the semi-analytical prediction and the experimental result which show excellent agreement.

For rectangular plates, a term called equivalent strain has been introduced analogous to the equivalent plastic strain for developing a semi-analytical model. A strain energy distribution was proposed from the experimental data for the variation of the apex strain as a function of the longitudinal and the transverse distances. A semi-analytical model was developed for predicting the strain on the rectangular plate by equating the kinetic energy to the strain energy of the plate. A comparison is made between the semi-analytical prediction and the experimental result which show excellent correlation.

References