Damage prediction of clamped circular plates subjected to contact underwater explosion

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Abstract

Contact explosion damage prediction of circular steel plates is of interest to the naval architects and warship designers. During contact explosion, a fraction of the explosive energy goes for plastic deformation and fracture of the plate. Experiments were conducted on air backed circular high strength low alloy (HSLA) steel plates to establish this fraction. There was a large amount of plastic deformation preceding fracture in the contact exploded plates. The deformation contour was found to be spherical showing maximum absorption of energy for the depth of bulge attained. The radius of penetration of the test plate was obtained by applying the terminal strain to fracture and the non-uniform strain distribution in the target plate. A case history is presented where the current predictions are compared with the existing empirical methodology and experimental data.

Keywords: Contact explosion; Shock energy; Deformation energy

1. Introduction

Research on contact explosion damage studies started way back in 1924 on the discarded Japanese battle ship Tosa \cite{1}. Imperial Japanese Navy carried out contact underwater explosion damage on Tosa’s hull with different charges at three widely separate parts to obtain necessary technical background to develop the underwater protection systems to their war ships. Tests carried out by Keil \cite{2} on scaled models ruled out the influence of depth of submergence on the damage potential of contact explosion. Keil described that there is a definite relation between the radius, \( R \), of the hole that is being made by an explosive of quantity, \( W \), during contact explosion.
Nomenclature

$e_1, e_2, e_3$ principal strains in three directions
$e_f$ fracture strain
$E$ energy (J)
$E_{\text{TNT}}$ energy of the TNT explosive (kcal/kg)
$E_{\text{EqTNT}}$ TNT equivalent of the explosive
$J$ energy conversion factor in work units (J/kcal)
$r$ instantaneous radius of the circular plate (m)
$R$ radius of the hole that is penetrated (m)
$t$ thickness of the plate (m)
$W$ charge weight (kg)
$W_{\text{crit}}$ critical charge weight
$\delta$ apex bulge depth (m)
$\eta$ coupling factor
$\sigma_y$ yield stress (MPa)

Suffix
in input
def deformation

with a plate of thickness $t$.

$$R = 0.0704 \sqrt{\frac{W}{t}}, \quad (1)$$

where $R$ and $t$ are in m and $W$ is in kg. This relation holds good only above certain charge quantity since a minimum quantity of explosive is required for making a hole in the plate of specified thickness. The critical charge weight $W_{\text{crit}}$ above which Eq. (1) holds good is given by

$$W_{\text{crit}} = 2.72t. \quad (2)$$

The basis on which the above prediction was obtained is not available in the literature.

Shock loading of circular plates subjected to non-contact underwater explosion loading was extensively studied by various authors [3–7]. Investigations on the circular plate subjected to uniformly distributed air blast impulse were reported in literature [8–11]. Response of plates subjected to high intensity of impulsive loading and undergoing large deformation is determined by membrane stretching resistance [12]. During membrane deformation, internal energy dissipation occurs predominantly through the action of membrane forces on middle surface strain, and flexural work could be neglected.

The empirical relations formulated by Keil [2] are nearly five decades old and there has been immense development in the field of materials and ship building technology. Therefore, it is natural to go for a new basis for the damage prediction of a plate subjected to contact explosion. This
situation has lead to the conduct of systematic contact explosions on the plane plates. The
deformation energy of the plate was correlated to the input shock energy. Remarkably, there was a
definite relation between the input shock energy and the deformation energy. This relation brings
out the available plastic energy as a function of explosive energy. The total energy of deformation is
obtained from this input fractional energy. Deformation of the plate was spherical in nature
suggesting maximum amount of energy absorption for the attained depth of bulge. Membrane theory
was applied since the ratio of the radius of the plate to the thickness of the plate was very large.

2. The methodology

The input shock energy, $E_{in}$, was obtained by using the energy content of the explosive and is
given by

$$E_{in} = WE_{TNT}E_{q_{TNT}}J,$$  

(3)

where $E_{in}$ is in J, $W$ is the charge weight in kg, $E_{TNT}$ is the energy content of TNT in kcal/kg
(1060 kcal/kg [2]), $J$ is the energy conversion factor in work units and $E_{q_{TNT}}$ is the TNT equivalent
of the explosive used.

Since large deformation is dominated by membrane stretching [12], the work done due to plastic
bending is neglected. The effect of local dimpling is also omitted as an approximation. It is assumed
that the plate undergoes spherical deformation. For a spherical deformation

$$(2\rho - \delta)\delta = R^2,$$  

(4)

where $\rho$ is the radius of the sphere, $\delta$ is the apex depth of bulge and $R$ is the radius of the plate as
shown in Fig. 1.

From Eq. (4),

$$\rho = \frac{R^2 \delta}{2^2} + \frac{\delta}{2}.$$  

(5)

It is assumed that the plate material deforms to a negligible amount until the yield point is reached
and subsequently flows plastically without further increase in stress. In other words, it is assumed
that no strain hardening occurs. For a circular plate stretching like a rigid plastic membrane, the
work done is given by [13]

$$W_{def} = \sigma_{y} t \Delta A,$$  

(6)

Fig. 1. A schematic representation of the spherical deformation of the circular plate.
where \( t \) is the thickness of the plate, \( \sigma_y \) is the static yield stress and \( \Delta A \) is the increase in area. The surface area of the sphere \( A_1 \) is given as

\[
A_1 = 2\pi \rho \delta
\]

as shown in Fig. 2. Subtracting the area of the circle from the area of the sphere,

\[
\Delta A = \pi \delta^2.
\]

Substituting Eq. (8) in Eq. (6), the deformation energy \( E_{\text{def}} \) is given as

\[
E_{\text{def}} = \pi \sigma_y t \delta^2.
\]

The deformation energy as a function of input shock energy is assumed as

\[
E_{\text{def}} = \eta E_{\text{in}},
\]

where \( \eta \) is the coupling factor which is the fraction of the explosive energy that has been converted into the deformation energy of the plate. The fraction of the energy that has gone in for tearing of the plate after sufficient amount of thinning has been neglected for the computation of the energy consumption of the plate.

During contact explosion, preceding fracture, thinning of the target plate takes place which absorbs energy in the form of plastic deformation. It can be approximately assumed that the hole that is being made by the explosive is circular in nature and significant energy goes only for the hole radius. Therefore, for the given quantity of charge, the radius of penetration of the explosive in the plate has to be worked out in accordance with these assumptions. Since the depth of bulge is dependent on the size of the hole being penetrated which in turn depends on the charge weight of the explosive, a method needs to be evolved relating the strain pattern across the radial line of the plate, the rupture strain and the radius of the hole.

von-Mises' proposed that yielding would occur when the second invariant of the stress deviator exceeds some critical value \([14]\). For a circular plate in plane stress condition, the principal stresses are equal and the stress distribution is like that of a soap film or a stretched membrane. The principal strains \( \varepsilon_1 \) and \( \varepsilon_2 \) at any distance \( r \) from the centre of the membrane plate deforming
with a radius $R$ and a depth of bulge $\delta$ are [15]

$$\varepsilon_1, \varepsilon_2 = \frac{\delta^2}{R^2} \left( 1 - \frac{r^2}{R^2} \right).$$ (11)

The strain is zero at the edge and equal to $\delta^2/R^2$ at the centre as shown in Fig. 3. The equivalent plastic strain $\varepsilon_{eq}$ is given by

$$\varepsilon_{eq} = \sqrt{2(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)/3},$$ (12)

where $\varepsilon_3$ is the principal strain in the thickness direction. By theory of plasticity,

$$\varepsilon_3 = -(\varepsilon_1 + \varepsilon_2).$$ (13)

Combining Eqs. (11)–(13),

$$\varepsilon_{eq} = 2\varepsilon_1 = 2\varepsilon_2 = -\varepsilon_3.$$ (14)

The test plate fractures when its equivalent strain at its centre is equal to the uniaxial fracture strain $\varepsilon_f$. Therefore,

$$\varepsilon_1 = \varepsilon_2 = \frac{\varepsilon_f}{2} = -\frac{\varepsilon_f}{2} = \frac{\varepsilon_{eq}}{2}. $$ (15)

The fracture strain, $\varepsilon_f$, is material specific and is assessed from the uniaxial tensile test. From Eq. (11), at the centre of the deforming plate

$$\frac{\delta^2}{R^2} = \frac{\varepsilon_f}{2}$$ (16)

from which

$$\delta^2 = R^2 \varepsilon_f \frac{2}{2}.$$ (17)

The deformation energy for a spherical geometry, which is given by Eq. (9) becomes

$$E_{def} = \frac{\pi \tau \sigma_y R^2 \varepsilon_f}{2}.$$ (18)
From Eq. (18) the radius of the hole is estimated as

\[ R^2 = \frac{2E_{\text{def}}}{\pi t \sigma_y \varepsilon_f}. \]  

Substitution of Eq. (10) in Eq. (19) results in

\[ R = \sqrt[2]{\frac{2\eta E_{\text{in}}}{\pi t \sigma_y \varepsilon_f}}. \]  

Substituting for \( E_{\text{in}} \) from Eq.(3), Eq. (20) can be rewritten as

\[ R = \sqrt[2]{\frac{2\eta W E_{\text{TNT}} E_{\text{TN}}}{J \pi t \sigma_y \varepsilon_f}}. \]  

The value of \( R \) thus obtained from Eq. (21) predicts the radius of the hole that is being made by an explosive of known quantity in plate having a thickness \( t \), a static yield stress \( \sigma_y \) and the uniaxial fracture strain \( \varepsilon_f \). During the actual deformation process, the yield stress of the plate increases due to the dynamic nature of the load but it is not uniform throughout the plate. The strain rate is maximum at the centre and almost zero at the edge [5]. The distribution of the strain rate across the radial line of the plate during contact underwater explosion is not available for consideration in this methodology.

### 3. Experimental

The target material was high strength low alloy (HSLA) steel whose chemical composition and mechanical properties are given in Table 1. A total of ten tensile tests were carried out at a strain rate of 0.01 s\(^{-1}\) (quasi-static regime of testing [14]), five in the rolling direction and five in the transverse direction as per ASTM E8M-89 [16]. The average of quasi-static yield stress is 400 MPa and percentage elongation is 28. The tensile test result of the HSLA steel for a strain rate of 0.01 s\(^{-1}\) is shown in Fig. 4.

A drum model target of \( \Phi 290 \times 4 \) mm as shown in Fig. 5 was used for the study of contact explosion damage. The plate was supported at its edge by a rigid unyielding baffle. Experiments were conducted with charge weights varying from 1 to 10 g.

<table>
<thead>
<tr>
<th>Element Content (%)</th>
<th>Chemical composition weight (%)</th>
<th>Mechanical properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C 0.12</td>
<td>Mn 0.7</td>
</tr>
<tr>
<td></td>
<td>Ultimate tensile strength (MPa) 560</td>
<td>Elongation (%) 28</td>
</tr>
</tbody>
</table>
Fig. 4. Stress–strain diagram of the HSLA steel; strain rate $0.01 \text{s}^{-1}$.

Fig. 5. A schematic representation of the drum model target ready for testing.
The explosive used was PEK-1 (1.17 TNT equivalent) along with Mk-79 electrical detonator. The Mk-79 detonator (which is equivalent to one gram of TNT) is 41.2 mm in length and 6.2 mm in diameter. The detonator was placed vertically on the centre of the plate for one gram explosive. For 2 g onwards, the PEK-1 explosive (with a density of 1700 kg/m$^3$) was shaped to a sphere and placed at the end of the detonator with steel grip tape. The explosive was fixed at the centre of the plate with the detonator perpendicular to it. The whole assembly was submerged in the underwater shock test tank to a depth of 2 m. After each explosion, plate with assembly was taken out of the tank, dismantled and observed for cracking. The depth of bulge at the apex of the plate was also measured. The contour of the deformed plate was measured with a Mututyo height gauge as shown in Fig. 6.

4. Results and discussion

The deformation contours of the plates subjected to contact underwater explosion are shown in Fig. 7. The contours closely adhere to the spherical geometry. Dimpling of the plate at its apex is seen in the deformed contours in Fig. 7. For 1 g charge only plastic deformation took place and no localised dimple was observed as shown in Fig. 8. For a charge weight of 2 g localised dent was observed at the centre of the plate along with spherical deformation as shown in Fig. 9. For the charge weight of 3 g and above there was localised cracking at the centre of the plate along with
Fig. 7. Contour of the deformed plate: (a) 1 g of Mk-79 detonator; (b) 2 g of explosive; (c) 3 g of explosive; (d) 4 g of explosive; (e) 5 g of explosive; (f) 6 g of explosive; (g) 10 g of explosive.

Fig. 8. A photographic view of the plastic deformation of the plate for Mk-79 detonator explosion: No dimple was observed.
plastic deformation. Fig. 10 shows the photographic view of the cracked plates for 3, 4, 5, 6 and 10 g of charge weight. There was considerable extension of crack for the explosion of 3–10 g of charge. After every firing, during dismantling of the plate, it was observed that the clamping bolts had got loosened.

The deformation to input energy ratio was 7.26% for 1 g detonator and it increased to 11.76% for 2 g and 16.07% for 3 g and thereafter it fluctuated around that value. This trend is attributed to the elongated geometry of the detonator placed perpendicular to the test plate for 1 g charge and the unsymmetric distribution of the explosive around the detonator for 2 g charge. With increase in charge weight, the symmetry of the charge improved and hence the energy ratio was maintained.

The underwater explosion instantly converts the explosive charge into a hot gas of high pressure of approximately 5000 MPa and high temperature of the order of 3000°C [15]. This hot gaseous ball propagates the shock in spherical direction in the water medium. Therefore the explosive energy which is not incident on the test plate (around 84–92%) propagates as spherical front in other directions. A detailed investigation on the energy balance of contact underwater explosion is out of the scope of this work.

Table 2 shows the summary of the depth of bulge, the deformation energy and the input energy for the exploded charges. The size of the dimple varied from 4 mm to 13.6 mm. The diameter of the charge varied from 6.2 mm to 21.6 mm. It can be observed from Table 2 that the size of the dimple was less than the diameter of the charge (the dimple diameter to explosive diameter ratio varying from 0 to 0.9). Also, the dimple diameter bears a ratio of 0–0.06 to the plate diameter. This justifies the assumption that the work done due to the localised dimpling and cracking may be omitted for analysis. It appears that the dimpling is due to spallation in the plate as a result of the high-velocity compression of the hot gas ball of the explosive which was in contact with the test plate. Fig. 11 shows the variation of deformation energy as a function of input shock energy. Least-squares fit to the above data yields \( \eta \) as 0.1236.
5. Case study

Applying the material parameters for the HSLA steel plate under investigation, and the TNT equivalent for PEK-1 charge:

\[ \eta = 0.1236, \quad Eq_{\text{TNT}} = 1.17, \quad E_{\text{TNT}} = 1060 \text{ kcal/kg}, \quad J = 4180 \text{ kcal/kg}, \quad \sigma_y = 400 \text{ MPa}, \]
Table 2
Summary of contact explosion experiment results

<table>
<thead>
<tr>
<th>Sl no.</th>
<th>Weight of PEK-1 (kg)</th>
<th>Charge diameter (mm)</th>
<th>Depth of bulge (mm)</th>
<th>Input energy</th>
<th>Deformation energy</th>
<th>Coupling factor (%)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>6.2</td>
<td>8.00</td>
<td>4430</td>
<td>322</td>
<td>7.26</td>
<td>NoD; NoC</td>
</tr>
<tr>
<td>2.</td>
<td>2</td>
<td>10.3</td>
<td>15.00</td>
<td>9613</td>
<td>1131</td>
<td>11.76</td>
<td>DF4 mm; NoC</td>
</tr>
<tr>
<td>3.</td>
<td>3</td>
<td>13.1</td>
<td>21.75</td>
<td>14796</td>
<td>2378</td>
<td>16.07</td>
<td>D:C; F7 mm</td>
</tr>
<tr>
<td>4.</td>
<td>4</td>
<td>15.0</td>
<td>25.50</td>
<td>19973</td>
<td>3269</td>
<td>16.37</td>
<td>D:C; F13.6 mm</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>16.5</td>
<td>26.25</td>
<td>25162</td>
<td>3464</td>
<td>13.78</td>
<td>D:C; F7.4 mm</td>
</tr>
<tr>
<td>6.</td>
<td>6</td>
<td>17.8</td>
<td>28.00</td>
<td>30346</td>
<td>3941</td>
<td>12.99</td>
<td>D:C; F10.4 mm</td>
</tr>
<tr>
<td>7.</td>
<td>10</td>
<td>21.6</td>
<td>35.50</td>
<td>51058</td>
<td>6334</td>
<td>12.41</td>
<td>D:C; F12.7 mm</td>
</tr>
</tbody>
</table>

*D: Dimpling; C: Cracking.

Fig. 11. Variation of plate deformation energy as a function of the explosive energy.

\[ \varepsilon_f = \ln(1 + 0.28) = 0.2469, \]

Eq. (21) reduces to

\[ R = 0.0643 \frac{W}{r}. \]  \hspace{1cm} (22)

Eq. (22) is in good comparison with Eq. (1). For a charge weight of 19 g of PEK-1 with 1 g Mk-79 detonator, on a 4 mm HSLA steel plate the radius of the hole that is being made is 0.143 m.
Fig. 12. Hole bored into the test plate at its centre by contact explosion (seen like lotus); Charge quantity = 19 g of PEK-1 + 1 g of Mk-79 detonator; Hole radius = 0.120 m; Prediction by present methodology = 0.143 m; Prediction by Keil's method = 0.157 m.

Application of Keil's relation gives $R$ as 0.157 m. Experiment was conducted for the above conditions on a f 290 circular plate and it ruptured at its centre like a lotus as shown in Fig. 12. The radius of the hole (taken as half of its average diameter) in the plate was 0.120 m. The present approach thus is 84% in agreement with the experimental result and it is 91% in agreement with Keil's approach. In addition, the new approach takes into account the salient material properties like the yield stress and the terminal strain to fracture.

6. Conclusions

Underwater contact explosion damage phenomenon was empirically modelled by Keil relating the charge quantity to the thickness of the plate. The model lacks explanation for the variation in the structural material properties. Also, the basis of the empirical model is not known.

A new damage prediction model has been proposed in this investigation relating the input shock energy, the contour of deformation, the material properties and the thickness of the structural material. This model assumes rigid plastic membrane stretching and neglects the energy dissipated in localised dimpling and cracking during the dynamic deformation process.

A case study has been illustrated correlating Keil's empirical model and the present approach. The results are in good agreement with each other and the experimental observation.

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