Robust Control Design for Nonlinear Magnetic Levitation System using Quantitative Feedback Theory (QFT)

P. S. V. Nataraj and Mukesh D. Patil

Systems and Control Engineering
Indian Institute of Technology Bombay, Mumbai India.
nataraj@sc.iitb.ac.in, mdpatil@sc.iitb.ac.in

Abstract—There is a growing demand for magnetic suspension systems, especially in the field of high accuracy multi-dimensional positioning. Magnetic Levitation (Maglev) systems offer many advantages such as frictionless, low noise, the ability to operate in high vacuum environments and so on. Reliable and robust controller synthesis of this system is of great practical interest. Robustness is a key issue in designing a control system for a Magnetic Levitation as the models are never 100 percent accurate and the uncertainties in the model must be accounted. In the present work, a new approach using Quantitative Feedback Theory (QFT) is presented for design of a robust two degree of freedom controller for Magnetic Levitation. Experimental tests are conducted to check the reliability and robustness of the designed control system by subjecting it to disturbances. The experimental test results show the control system design to be reliable and robust.

Index Terms—Magnetic Levitation system, reliable control, Robust Control, QFT.

I. INTRODUCTION

Magnetic Levitation (Maglev) is becoming an attractive technology for application areas such as high-speed trains, vibration isolation systems, magnetic bearings and photolithography steppers (see [5]). Recently, its application to the launching of space missions has received attention since the use of Magnetic Levitation setup technology as the zeroth stage of the launch is viewed as a safe, reliable and inexpensive launch assist for launching payloads into orbit (see [4]). Magnetic Levitation setup systems offer many advantages such as frictionless, low noise, the ability to operate in high vacuum environments and so on. Various applications of micro-robotic technology suggest the use of new actuator systems which allow motions to be realized with micrometer accuracy.

There are several robust control methodologies and the debate on superiority of one method over the other would perhaps be everlasting (see [7]. However, Horowitz’s approach of quantitative feedback theory (QFT) (see [2],[6]) to robust control has been gaining popularity in the control literature for design of robust feedback systems. The specific characteristics of QFT are:

- Design trade-offs at each frequency are highly transparent between stability, performance, plant uncertainty, disturbance level, controller complexity and controller bandwidth.
- The method extends classical frequency domain loop shaping concepts to cope with simultaneous specification and plants with uncertainties.
- There is one design for full envelope.
- The QFT approach can handle single-input single-output (SISO) and multi-input multi-output (MIMO), linear and non-linear, time-varying and time-invariant, and lumped and distributed parameter systems.

To the best of our knowledge, QFT has not been applied in literature to Magnetic Levitation systems. We have applied this technique for designing a controller of magnetic levitation, and obtained simple, low order controller. The controller is successfully implemented on an experimental Magnetic Levitation system in our laboratory.

The paper is organized as follows: The brief background about QFT is given in section 2. Mathematical modeling and control system of Magnetic levitation system is given in section 3. Control design for magnetic levitation is given in section 4. The experimental procedure is discussed in section 5. The conclusions of the work are drawn in section 6.

II. SOME PRELIMINARIES

A. Quantitative Feedback Theory

Consider a two degree of freedom feedback system configuration (see Fig 1), where \( P(s), G(s) \) and \( F(s) \) are uncertain linear time-invariant plant, the controller and pre-filter to be designed respectively.

Fig. 1. The Two Degree-of-Freedom Structure in QFT

The open loop transmission function is defined as

\[
L(s) = G(s)P(s)
\] (1)
and the nominal open loop transmission function is

$$L_0(s) = G(s)P_0(s)$$  \hspace{1cm} (2)$$

The objective in QFT is to synthesize $G(s)$ and $F(s)$ such that the various stability and performance specifications are met for all $P(s) \in \mathcal{P}$. In general following specifications are considered in QFT. (see [2]):

- Robust stability margin
  $$\left| \frac{L(j\omega)}{1 + L(j\omega)} \right| \leq W_s$$

- Robust tracking performance
  $$|T_L(j\omega)| \leq \left| \frac{F(j\omega)L(j\omega)}{1 + L(j\omega)} \right| \leq |T_U(j\omega)|$$

- Robust input disturbance rejection performance
  $$\left| \frac{G(j\omega)}{1 + L(j\omega)} \right| \leq W_{d_i}(\omega)$$

- Robust output disturbance rejection performance
  $$\left| \frac{1}{1 + L(j\omega)} \right| \leq W_{d_o}(\omega)$$

In practice, the objective is to satisfy the given specifications over a finite design frequency set $\Omega$. The design procedure which is to be followed for applying QFT robust design technique is as follows (see [3]):

- Synthesize the desired tracking model
- Specify the plant models that define the region of plant parameter uncertainty
- Obtain the plant templates at specified frequencies that pictorially describe the region of plant parameter uncertainty on the Nichols Chart.
- Select the nominal plant transfer function $P_0(s)$.
- Determine the stability contour on the Nichols Chart.
- Determine tracking and optimal bounds on the Nichols Chart.
- Synthesize the nominal loop transmission function $L_0(s) = G(s)P_0(s)$ that satisfies all the bounds and stability contour.
- Synthesize the pre-filter $F(s)$.

III. CONTROL SYSTEM FOR MAGNETIC LEVITATION SYSTEM

A. Magnetic Levitation

The magnetic levitation test bed supplied by ECP model 730 (see [1]) which is used for our experiment is shown in figure 2.

The plant shown if figure 3, consists of upper and lower coils that produce a magnetic field in response to a DC current. One or two magnets travel along a precision ground Pyrex glass guide rod. By energizing the lower coil, a single magnet is levitated through a repulsive magnetic force. As the current in the coil increases, the field strength increases and the levitated magnet height is increased. For the upper coil, the levitating force is attractive. Two magnets may be controlled simultaneously by stacking them on the glass rod, the magnets are of an ultra-high field strength rare earth (Nd:FeB) type and are designed to provide large levitated displacements to clearly demonstrate the principle of levitation and motion control.

Two laser-based sensors measure the magnet positions. The lower sensor is typically used to measure a given magnet’s position in proximity to the lower coil, and the upper one for proximity to the upper coil. This proprietary ECP sensor design utilizes light amplitude measurement and includes special circuitry to desensitize the signal to stray ambient light and thermal fluctuations.

The Magnetic Levitation setup apparatus dramatically demonstrates closed loop levitation of permanent and ferromagnetic elements. The apparatus includes laser feedback and high flux magnetics to affect large displacements and provide visually stimulating tracking and regulation demonstrations. The system is quickly set up in the open loop stable and unstable (repulsive and attractive fields) configurations as shown in figure. By adding a second magnet, two SIMO plants may be created, and by driving both actuators with both magnets, MIMO control is studied. The field interaction between the two magnets causes strong cross coupling and thus produces a true multi-variable system. The inherent magnetic field nonlinearities may be inverted via provided real-time algorithms for linear control implementation or the
full system dynamics may be studied.

The complete experimental setup is comprised of the three subsystems as shown in figure 4.

1) The first subsystem is the Magnetic Levitation system itself (described above) which consists of the electromagnetic coils, magnets, high resolution encoders.

2) The next subsystem is the real-time controller unit that contains the digital signal processor (DSP) based real-time controller, servo/actuator interfaces, servo amplifiers, and auxiliary power supplies. The DSP is capable of executing control laws at high sampling rates allowing the implementation to be modeled as continuous or discrete time systems. The controller also interprets trajectory commands and supports such functions as data acquisition, trajectory generation, and system health and safety checks.

3) The third subsystem is the executive program which runs on a PC under the Windows operating system. This menu-driven program is the user’s interface to the system and supports controller specification, trajectory definition, data acquisition, plotting, system execution commands, and more. Controllers may assume a broad range of selectable block diagram topologies and dynamic order. The interface supports an assortment of features which provide a friendly yet powerful experimental environment. Real-time implementation of the controllers is also possible using the Real Time Windows Target (RTWT).

B. Mathematical Model:

Consider the free body diagram of Magnetic Levitation setup ECP Model 730 is shown in figure 5.

![Free Body Diagram](image)

Fig. 5. Force Balance Diagram, Lower Coil (Coil Number 1) And A Magnet Forming A Siso Open Loop Stable System.

The balancing forces for the lower magnets are given by following eq. 3,

$$m\ddot{y}_1 + c_1 \dot{y}_1 + mg = F_{u11}$$  \hspace{1cm} (3)

Where, the repulsive force from coil 1 to magnet 1 is given by

$$f_{u11} = \frac{i_1}{a(y_1 + b)^N}$$

For the ECP model 730, the friction forces are also typically small. Often \( N = 4 \) has been shown empirically to yield a close approximation of the force/distance relationships. The following simplified model is therefore valid for control design and analysis purposes.

$$m\ddot{y}_1 + mg = f_{u11}$$  \hspace{1cm} (4)

where,

$$f_{u11} = \frac{u_1}{a(y_1 + b)^4}$$

The coil current \( i_1 \) is the control effort \( u_1 \). The constants \( a, b \) are found so that smooth actuator characteristics are obtained. For small motions, the systems may be modeled as being linear. The linearized equations of motion are found by customary method of solving for the zeroth and first order terms of the Taylor’s Series expansion of the respective equations about the operating point. For the purposes of control design, we choose the operating point to be at an equilibrium so that,

$$(mg - f_{u11})|_{y_{10}, u_{10}} = 0$$  \hspace{1cm} (5)

Linearizing eq. 4 about the equilibrium point, the linearized equation becomes,

$$m\ddot{y}_1 + \left(\frac{4u_{10}}{a(y_{10} + b)^4}\right)(y_1 - y_{10}) = \frac{1}{a(y_{10} + b)^4}(u_1 - u_{10})$$  \hspace{1cm} (6)

which can be rewritten as:

$$m\ddot{y}_1^* + k_1^* y_1^* = k u_1^* u_1^*$$  \hspace{1cm} (7)

where,

$$y_1^* = y_1 - y_{10}$$

$$u_1^* = u_1 - u_{10}$$

$$k_1^* = \frac{4u_{10}}{a(y_{10} + b)^5}$$

$$ku_1^* = \frac{1}{a(y_{10} + b)^4}$$

From eq. 5, we may solve for the equilibrium conditions where control effort value is,

$$u_{10} = a(y_{10} + b)^4 mg$$

A state space realization of this system is,

$$\dot{x} = Ax + Bu \hspace{1cm} y = Cx$$  \hspace{1cm} (8)

where,

$$x = \begin{bmatrix} y_1^* \\ \dot{y}_1^* \end{bmatrix}, \hspace{0.5cm} A = \begin{bmatrix} 0 & 1 \\ -\frac{k_1^*}{m} & 0 \end{bmatrix}$$

$$B = C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
C. Sensor Nonlinearity

The sensor output characteristics is nonlinear. The following equation is motivated by examining the optical design and geometry of the laser/detector/magnet system.

\[ y_{1\text{cal}} = \frac{e_{1}}{y_{1\text{raw}}} + \frac{f_{1}}{\sqrt{y_{1\text{raw}}}} + g_{1} + h_{1}y_{1\text{raw}} \quad (9) \]

where \( y_{1\text{cal}} \) is the linearized/calibrated output of the sensor and \( y_{1\text{raw}} \) is the raw sensor output. The curve fitting is done by iteration starting with the inverse term with the inverse square root term equal to zero and adjusting the bias and slope for a best fit. The inverse square term may be strengthened and again the bias and slope adjusted. The coefficients \( e, f, g, h \) are found by regression based curve fitting method. The values of the constants (in terms of counts) are as follows:

\[
\begin{align*}
  e &= -115720000 \\
  f &= 7208826 \\
  g &= -30540 \\
  h &= -0.2411
\end{align*}
\]

A comparison of the test and the corrected sensor is shown in figure 6.

D. Actuator Nonlinearity

The actuator is also nonlinear in nature. The relation between force developed by electromagnetic and the input control effort is given by,

\[ f_{u11} = \frac{u_{1}}{a(y_{1} + b)^{4}} \quad (12) \]

where the actuator correction can be found by manual iteration. The coefficients \( a, b \) are found so that smooth actuator characteristics are obtained. Again the curve fitting for the actuator correction can be found by manual iteration. The values of the constants are as follows,

\[
\begin{align*}
  a &= 1.05/10000 \\
  b &= 6.2
\end{align*}
\]

The raw control effort data and the compensated control effort curve are shown in figure 7. The closeness of the fitted curve appears to satisfy the use of fourth power approximation of the magnetic force term in eq. 12.

IV. CONTROL DESIGN

The transfer function of the plant with nonlinear actuator and linearized sensor linearized at the equilibrium point of 2 cm is obtained by converting the state space model in eq. 8 and using eq. 4 through 10 is given by:

\[ P(s) = \frac{-935}{s^{2} + 425.63} \quad (13) \]

The equivalent block diagram of closed loop system including nonlinear and gravity compensation elements is shown in figure 8.

IV. CONTROL DESIGN

Consider the identified transfer system of the Magnetic Levitation system for lower magnet in eq. 13. Input is current to the lower coil and output is displacement of magnet in cm.
For the design, the uncertainty of $10 \text{ gm}$ is added to check the robustness of the controller, Plant:

$$P(s) = \frac{k}{s^2 + a}$$

The uncertainties are introduced in the parameters $k$ and $a$ as follows,

$$k \in [-863, -936]; \quad a \in [418, 434]$$

Performance specifications:

- Stability margin specification: Gain margin $\geq 4.5 \text{ dB}$, phase margin $\geq 45^\circ$.
- Tracking specifications: $0.4 \leq \text{rise time} \leq 0.6 \text{ sec}$, no offset and no overshoot.

The nominal loop transmission function $L_0(s) = G(s)P_0(s)$ that satisfies all the bounds and stability contour is synthesized using "lpshape" environment of MATLAB. (see figure 9)

Corresponding controller transfer function obtained for magnetic levitation system for lower magnet is as follows:

$$G(s) = \frac{3.524(s^2 + 1)(s^2 + 1)}{s(s^2 + 1)(s^2 + 1)}$$

To satisfy the tracking specifications, a pre-filter is designed using "pfshape" environment of the MATLAB such that the closed loop response of the system satisfies the given tracking specifications. The corresponding pre-filter is as follows:

$$F(s) = \frac{1}{s^2 + \frac{a}{5.598} + \frac{b}{4.661} + 1}$$

V. EXPERIMENTAL TESTING

The steps followed to carry out the experiment are as follows:

- Linearization of the sensor using eq. 9. The comparison plot of raw sensor count and calibrated sensor is shown in figure 6. The constants $e, f, g, h$ are fitted such that calibrated sensor output is close approximation of magnet position.
- Nonlinear compensation of the actuator using eq. 12. The figure 7 shows the uncompensated and compensated actuator. The constants $a$ and $b$ are fitted such that the actuator response becomes smooth.
- Construct the design control system in Simulink environment as shown in figure 10 along with reference command signal.
- Build and execute the real time model using Real Time Workshop Target(RTWT), to convert the control algorithm in C++ code. Download this code onto the DSP via RTWT.
- Start the real-time implementation from within RTWT environment for desired length of time.
- After the experiment is over, make the appropriate conversions and plot the data.

The above designed control system is now implemented real time and experimentally tested for its performance. Initially, the magnet is brought to equilibrium position of 2 cm as plant is linearized to 2 cm. Now the reference signal is applied. The response of the closed loop system for a given reference command signal is shown in figure 11.

The rise time of the closed loop system is 0.42 sec with no overshoot which is within the range of tracking specifications. The required control effort is low, with a peak of 2.8 volts. (see 12).

VI. CONCLUSION

This paper presents a methodology for design of robust control system of nonlinear magnetic levitation system. The
Fig. 11. Step response of closed loop system.

Fig. 12. Control Effort Required To Levitate The Magnet.

design problem with quantitative bounds on the plant parameters and quantitative tolerances on the acceptable closed loop system response can easily be ascertained using the QFT feedback design. The designed robust controller is tested on ECP Model 730 Magnetic Levitation setup through Real Time Workshop Target (RTWT). It has been successfully applied to experimental Magnetic Levitation setup and desired performance specifications are also achieved.

REFERENCES