Response of piping system on friction support to bi-directional excitation

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Abstract

Installation of friction devices between a piping system and its supporting medium is an effective way of energy dissipation in the piping systems. In this paper, seismic effectiveness of friction type support for a piping system subjected to two horizontal components of earthquake motion is investigated. The interaction between the mobilized restoring forces of the friction support is duly considered. The non-linear behavior of the restoring forces of the support is modeled as an elastic-perfectly plastic system with a very high value of initial stiffness. Such an idealization avoids keeping track of transitional rules (as required in conventional modeling of friction systems) under arbitrary dynamic loading. The frictional forces mobilized at the friction support are assumed to be dependent on the sliding velocity and instantaneous normal force acting on the support. A detailed systematic procedure for analysis of piping systems supported on friction support considering the effects of bi-directional interaction of the frictional forces is presented. The proposed procedure is validated by comparing the analytical seismic responses of a spatial piping system supported on a friction support with the corresponding experimental results. The responses of the piping system and the frictional forces of the support are observed to be in close agreement with the experimental results validating the proposed analysis procedure. It was also observed that the friction supports are very effective in reducing the seismic response of piping systems. In order to investigate the effects of bi-directional interaction of the frictional forces, the seismic responses of the piping system are compared by considering and ignoring the interaction under few narrow-band and broad-band (real earthquake) ground motions. The bi-directional interaction of the frictional forces has significant effects on the response of piping system and should be included in the analysis of piping systems supported on friction supports. Further, it was also observed that the velocity dependence of the friction coefficient does not have noticeable effects on the peak responses of the piping system.

1. Introduction

The technology of \textit{passive control} of structures has now initialized its applications in the lifeline structures like piping systems in industrial installations and utilities like nuclear power plants (Kunieda et al., 1987). Owing to its advantages over its sister branches i.e. \textit{active control} or \textit{semi-active control} technologies, this technology finds a wide range of applications varying from its use in upcoming piping installations to the retrofitting of existing facilities without significantly affecting the production/operation and the structural characteristics. The primary objective of the control technology is to satisfy the motion related design requirements wherein strength is viewed as a constraint and not as a primary requirement (like assumed in conventional designs). The passive control technology is now having a reputation of over five decades since its emergence and had been successfully used in structures like buildings, bridges and water tanks in the form of supplemental devices. Several passive control devices are reported in the literature namely friction, viscous or visco-elastic dampers, yielding elements or auxiliary oscillators like tuned mass dampers and multiple tuned mass dampers (Mahmoodi, 1972; Hanson and Soong, 2001; Oliveto, 2002). These passive control devices act as energy sinks or fuses and control the motion of the system by significantly increasing the damping due to energy dissipation in the structural system. These devices can be easily replaced after an earthquake if required due to occurred damage.

There had been an attempt to prove seismic effectiveness of supplemental devices for piping systems in the past (Kunieda et al.,1987). However, the proposed devices were very complex in design, fabrication and implementation, as a result, there was an urgent need to develop simple control devices that are capable to dissipate the earthquake forces and at the same time allow free expansion of the piping systems for thermal load-
ings. Using friction in the supports was the best choice as it satisfies the above-mentioned requirements without altering the piping characteristics. Anderson and Singh (1976) were the first to propose the modeling of friction at the piping support. Kobayashi et al. (1989) modeled friction between the piping system and its gap support to establish its seismic effectiveness. Later on, Suzuki et al. (1992), Chiba et al. (1992) and Shimizu et al. (1996) performed extensive shake table tests on a large-scale piping system resting on a friction support in the form of Teflon, which were further studied by Yokoi et al. (1993). Based on the results obtained from above experimental studies (Suzuki et al., 1992; Chiba et al., 1992; Shimizu et al., 1996; Yokoi et al., 1993), Reddy et al. (1999) presented the designer’s methods to analyze piping systems resting on friction supports. Recently, Bakre et al. (2004) have presented the comparison of time-history analysis of the piping system on friction support, with the methods presented by Reddy et al. (1999) and the corresponding experimental results, by modeling the frictional forces independently in the two orthogonal directions (two-dimensional planar) and ignoring the effects of bi-directional interaction. Few observations from all of the above-mentioned studies to be noted are: (i) the non-linear friction support is referred as equivalent linear, (ii) the frictional forces are assumed to be acting independently in two orthogonal directions without any effects of bi-directional interaction, (iii) the limiting friction force of the friction support is assumed to remain constant throughout the analysis, and (iv) the effects of the velocity dependence of friction coefficient on the controlled piping system were not considered. Mokha et al. (1993) reported the interaction effects of restoring forces for the friction type isolation system on the seismic responses of buildings, which are further confirmed by Jangid (1996). The seismic behavior of the piping systems is far different than the building structures, as a result, there is a need to conduct separate study on the behavior of piping systems with friction supports under bi-directional excitation.

In this paper, seismic response of a spatial piping system with friction support is investigated for two horizontal components of four narrow-band (frequency range of 3–9 Hz) and four real earthquake ground motions. The specific objectives of the present study are summarized as: (i) to present a method which incorporates the interaction effects of the frictional forces for dynamic analysis of piping systems with friction support to bi-directional ground excitations; (ii) to validate the proposed analytical procedure by comparing the analytical and experimental results; (iii) to study the seismic response of piping system under variation of the friction coefficient; (iv) to study the effects of bi-directional interaction of the frictional forces mobilized at the friction support; and (v) to investigate the effects of velocity dependence of the friction coefficient at friction support on the seismic response of piping system.

2. Structural model

Fig. 1(a) shows a schematic layout of the piping system considered in the present study which was tested by Shimizu et al. (1996) and further studied by Reddy et al. (1999). The piping system is made of pipe elements of carbon steel having Young’s modulus \((E)\) of 192.2 GN/m\(^2\), poisons ratio = 0.214. All the bends in the piping system are 90° with bend radius
of 225 mm. The piping system is supported on a friction support and on rod restraints with its ends rigidly anchored to the shake table, as shown in Fig. 1(a). The damping in the piping system without friction support obtained from the experimental test model is equal to 1.3% (Reddy et al., 1999). Fig. 1(b) shows an enlarged view of the friction support illustrating the Teflon coated mild steel plate that was used to dissipate the input dynamic energy. The plate was placed below a steel slider that was welded to the piping system. The coefficient of friction between the two surfaces (Teflon and steel) is adopted as $\mu_{\text{max}} = 0.15$ (at large velocity). The energy gets dissipated when relative sliding between the two interfaces occurs during shaking of the piping system.

The experimental results of the selected piping system model with friction support are available for comparison to validate the proposed analysis procedure and to establish the need of considering bi-directional interaction of the frictional forces. These experimental tests were performed at the lab facilities of National research Institute for Earth Science and Disaster prevention (NIED), Tsukuba, Japan, which houses a shake table of area $15 \, \text{m} \times 15 \, \text{m}$ with 500 T capacity. From the series of tests performed on the full-scale piping system, experimental results of the piping system without internal water pressure and subjected to only narrow-band random ground motions are considered for comparison between analytical and experimental response. To study the effect of broad-band random motions analytically, few components of real earthquake ground motions are also considered. It is to be noted that the piping system was placed diagonally (in plan) for performing the tests to produce the effects in the two orthogonal directions of the piping system. The piping system was subjected to few components of narrow-band random ground motions, which were again recorded on the shake table. The recorded ground motions are having peak ground acceleration (PGA) of 0.2, 0.27, 0.35 and 0.53 g, and 0.22, 0.29, 0.35 and 0.56 g, along X- and Y-directions, respectively. In addition to this, the piping responses and the frictional forces mobilized at the friction support in the two orthogonal directions were also recorded.

3. Finite element model of piping system

Fig. 1(c) shows finite element (FE) model of the piping system with a friction support. Following assumptions are made for dynamic analysis of piping systems supported on friction support subjected to bi-directional excitation.

1. The straight members in the piping system are modeled as three dimensional (3D) Beam elements and the bends are modeled as 3D curved beams (considering the flexibility factors) having 6 degrees-of-freedom at each node. Before step-by-step integration, the degrees-of-freedom with zero mass are statically condensed out.
2. The mass of each member is assumed to be distributed between its two nodes as a point mass. In addition to the mass of piping system the externally lumped masses are assumed to be effective in the three translational degrees-of-freedom.
3. Only the friction support behaves non-linearly and the piping system is assumed to behave linearly throughout the dynamic analysis.
4. The non-linear force-deformation relationship at the friction support is considered similar to an elastic-perfectly plastic system (with a very high value of initial stiffness), based on the non-linear model proposed by Park et al. (1986), in which there is no need to keep track of transitional rules for sliding and non-sliding phases.

Solving above FE model for its eigenvalues and eigenvectors yields the fundamental natural frequency of the piping system equal to 4.38 Hz, which is in close agreement with the corresponding experimental frequency of 4.74 Hz (Reddy et al., 1999).

4. Modeling of the friction support

Conventionally, the non-linear friction support is represented by its transitions between the stick and slip conditions, where, it is required to keep track of the transitional rules. In this paper, instead of using conventional method, a non-linear model proposed by Park et al. (1986) is used to represent the frictional forces of the support. The characteristic of this type of modeling is that the friction support behaves like a fictitious spring with very high stiffness during non-slip mode and zero stiffness during slip mode. The following relation expresses the restoring forces at the friction support in the X and Y-directions

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} Z_x \\ Z_y \end{bmatrix}$$

where $F_x$ and $F_y$ are the restoring or frictional forces in X- and Y-directions of the friction support, respectively; $Z_x$ and $Z_y$ are the non-dimensional hysteretic components (value varies from $-1$ to $+1$); and $F_s$ is the limiting friction force at the friction support.

The hysteretic components, $Z_x$ and $Z_y$, satisfies the following coupled non-linear first order differential equations presented in (Park et al., 1986)

$$q \begin{bmatrix} Z_x \\ Z_y \end{bmatrix} = [G] \begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix}$$

where

$$G = \begin{bmatrix} A - \beta \text{sgn}(\dot{x}_p)|Z_x|Z_x - \tau Z_y^2 & -\beta \text{sgn}(\dot{y}_p)|Z_y|Z_y - \tau Z_x Z_y \\
-\beta \text{sgn}(\dot{x}_p)|Z_x|Z_x - \tau Z_y Z_y & A - \beta \text{sgn}(\dot{y}_p)|Z_y|Z_y - \tau Z_x^2 \end{bmatrix}$$

where $\beta, \tau$ and $A$ are the parameters, which controls the shape and size of the hysteretic loop of the friction support; $q$ is the yield displacement; $\text{sgn}$ denotes the signum function; and $\dot{x}_p$ and $\dot{y}_p$ represent the relative velocity of piping system in X- and Y-directions of the friction support, respectively. The force-deformation behavior of friction support can be modeled by properly selecting the parameters $\beta, \tau$ and $A$. However, as the behavior of friction support is modeled as elastic-perfectly...
plastic, the yield displacement should be taken equal to zero, however, to avoid computational error a very small value has to be adopted.

The off-diagonal terms of the matrix \([G]\) expressed by Eq. (3) provide the coupling or interaction between the restoring forces at the friction support. If these terms are ignored (i.e. assumed equal to zero), the frictional forces behave independently in the two orthogonal directions. It will be interesting to investigate the effects of this interaction (i.e. ignoring and considering the off-diagonal terms of matrix \([G]\)) on the seismic response of piping system. Two typical hysteresis loops of the selected are:

The off-diagonal terms of matrix \([G]\) expressed by Eq. (3), by zero. The difference between the frictional forces at the friction support starts sliding at the lower value of sliding force i.e. as soon as the resultant of the frictional forces attains the limiting frictional force (i.e. \(F_0^2 + F_1^2 = F_2^2\)).

The limiting friction force at the friction support is generally considered to be invariant (i.e. assumed as a constant) during the excitation by earthquake ground motions. However, it is well-known that the normal reaction at the friction support will be varying with time, consequently, the limiting friction force will also be affected. This change in the limiting friction force is included in the present study and is expressed by

\[
F_s(t) = \mu F_s(t)
\]

where \(F_s(t)\) is the time dependent vertical support reaction at the friction support; and \(\mu\) is the coefficient of friction.

Constantinou et al. (1990) performed several tests on Teflon-steel interfaces and observed that the coefficient of friction is dependent on the relative velocity between the contact interfaces and provided an expression to track this effect. Using the expression, the coefficient of sliding friction, \(\mu\), at a resultant sliding velocity \(\dot{u} = \sqrt{x^2 + y^2}\) is approximated as

\[
\mu = \mu_{\text{max}} - (\Delta \mu) \exp(-\alpha|\dot{u}|)
\]

where \(\mu_{\text{max}}\) is the coefficient of friction at large velocity of sliding (after leveling off); \(\Delta \mu\), is the difference between the friction coefficient at large and zero velocity of the system; and \(\alpha\) is the constant dependent on the bearing pressure and condition of interface. The value of the constant \(\alpha\) is taken as 0.2 s/cm (Constantinou et al., 1990).

5. Governing equations of motion

The equations of motion for the piping system model as shown in Fig. 1(a) under the two horizontal components of earthquake ground motions are expressed in the following matrix form

\[
[M][\ddot{u}] + [C][\dot{u}] + [K][u] + [D][F] = -[M][r][\ddot{u}_g]
\]

\[
[u] = [x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3, x_i, y_i, z_i \ldots]^T
\]

\[
[\ddot{u}_g] = \{\ddot{x}_g \ddot{y}_g \ldots \}
\]

where \([M]\), \([C]\) and \([K]\) represents the mass, damping and stiffness matrix, respectively of the piping system; \([\ddot{u}]\), \([\dot{u}]\) and \([u]\) represent structural acceleration, velocity and displacement vectors, respectively; \([D]\) is the location matrix for the frictional forces at the friction support; \([F]\) is the vector containing the frictional forces governed by Eq. (1); \([r]\) is the influence coefficient matrix; \([\ddot{u}_g]\) is the earthquake ground acceleration vector expressed by Eq. (8); \(\ddot{x}_g\) and \(\ddot{y}_g\) represents the earthquake ground accelerations in the X- and Y-directions of the piping system, respectively; and \(x_i, y_i, z_i\) are the displacements of the node of the piping system in X-, Y- and Z-directions, respectively. The mass matrix has a diagonal form with the lumped mass at each node. The stiffness matrix of the piping system with friction support is constructed separately and then static condensation is carried out to eliminate the rotational degrees-of-freedom. The damping matrix of the system is constructed using Rayleigh’s method by considering the first two natural frequencies of the piping system and the damping ratio obtained from the experimental test results.

6. Incremental solution technique

Since the force-deformation behaviour of the friction support is non-linear, the governing equations of motion are to be solved in the incremental form using Newmark’s time-stepping method assuming linear variation of acceleration over small time interval \(\Delta t\). The equations of motion in the incremental form are expressed as

\[
[M][\Delta \ddot{u}] + [C][\Delta \dot{u}] + [K][\Delta u] + [D][\Delta F] = -[M][r][\Delta \ddot{u}_g]
\]
Table 1
Peak ground acceleration of various ground motions

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Recording station</th>
<th>Applied in X-direction</th>
<th>Applied in Y-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imperial Valley, 1940</td>
<td>El-Centro</td>
<td>N90E 0.214</td>
<td>N00E 0.348</td>
</tr>
<tr>
<td>Northridge, 1994</td>
<td>Sylmar Converter Station</td>
<td>N90E 0.605</td>
<td>N00E 0.843</td>
</tr>
<tr>
<td>Loma Prieta, 1989</td>
<td>Los Gatos Presentation Center</td>
<td>N90E 0.608</td>
<td>N00E 0.57</td>
</tr>
<tr>
<td>Kobe, 1995</td>
<td>JMA</td>
<td>N90E 0.629</td>
<td>N00E 0.834</td>
</tr>
</tbody>
</table>

where \( \{ \Delta F \} \) is the incremental restoring force vector of the friction support.

Following the assumption of linear variation of acceleration over the small time interval, \( \Delta t \), \( \{ \Delta \dot{u} \} \) and \( \{ \Delta u \} \) are given as

\[
\{ \Delta \dot{u} \} = a_0 \{ \Delta u \} + a_1 \{ \dot{u}' \} + a_2 \{ \ddot{u}' \}
\]

(10)

\[
\{ \Delta u \} = b_0 \{ \Delta u \} + b_1 \{ \dot{u}' \} + b_2 \{ \ddot{u}' \}
\]

(11)

where \( a_0 = (6/\Delta t^2) \); \( a_1 = (-6/\Delta t) \); \( a_2 = (-3) \); \( b_0 = (3/\Delta t) \); \( b_1 = -3 \); and \( b_2 = -(\Delta t/2) \) and the superscript denotes the time.

Substituting Eqs. (10) and (11) in Eq. (9),

\[
[\hat{K}] \{ \Delta u \} = \{ \Delta \hat{P} \} - [D] \{ \Delta F \}
\]

(12)

where

\[
[\hat{K}] = a_0 [M] + b_0 [C] + [K]
\]

(13)

\[
[\Delta \hat{P}] = -[M][r] \{ \Delta \dot{u}_g \} - [M] \{ a_1 \{ \dot{u}' \}
\]

\[
+ a_2 \{ \ddot{u}' \} \} - [C] \{ (b_1 \{ \dot{u}' \} + b_2 \{ \ddot{u}' \} ) \}
\]

(14)

After solving for incremental displacement vector from Eq. (12), the incremental acceleration and velocity vectors are obtained from Eqs. (10) and (11), respectively. Finally, the displacement and velocity vectors are obtained using Eqs. (12) and (13), respectively, as given below.

\[
\{ u^{t+\Delta t} \} = \{ u^t \} + \{ \Delta u \}
\]

(15)

\[
\{ \dot{u}^{t+\Delta t} \} = \{ \dot{u}^t \} + \{ \Delta \dot{u} \}
\]

(16)

To obtain the acceleration vector, the direct equilibrium equation expressed by Eq. (6) is adopted. The incremental Eq. (9) can be solved by iterative technique along with Runge-Kutta method, required to solve the coupled Eqs. (2) and (3), to obtain the

Fig. 3. Comparison of the analytical and experimental hysteresis loops of the friction support in X-direction of the piping system under narrow-band random ground motions (\( \mu_{\text{max}} = 0.15 \) and \( \Delta \mu = 0 \).
response of the system at time, $t + \Delta t$, where $\Delta t$ is expressed by

$$\Delta t = \frac{dr}{N}$$  \hfill (17)

where $dr$ is the time interval at which the ground motions are recorded and $N$ are the number of divisions adopted for convergence of the structural responses due to the non-linearity of frictional forces.

7. Numerical study

The seismic response of piping system with friction support is investigated under four narrow-band random and four real earthquake ground motions. Details of the narrow-band random ground motions are mentioned in earlier sections. However, the specific components of the real earthquake ground motions applied in the X- and Y-directions of the piping system are shown in Table 1. The response quantities of interest for the piping sys-

Fig. 5. Comparison of the hysteresis loops for uni- and bi-directional idealization of the friction support in X-direction of the piping system under 1995 Kobe earthquake ($\mu_{\text{max}} = 0.15$ and $\Delta\mu = 0$).
tem under consideration are the relative sliding displacements ($x_s$ or $y_s$) at the friction support, absolute accelerations ($\ddot{x}_p$ or $\ddot{y}_p$) at node $B$ of the piping system and the support reactions ($R_x$ or $R_y$) as indicated in Fig. 1(c). The absolute accelerations and reactions at the support are directly proportional to the forces exerted on the piping system. On the other hand, the relative sliding displacements are crucial from the design point of view of friction support. The $x$ and $y$ in the response quantities refer to the corresponding responses in the X- and Y-directions of the piping system, respectively. The responses are obtained for (i) uncontrolled system (i.e. piping system without friction support), (ii) controlled system (i.e. piping system with friction support but ignoring the bi-directional interaction of the frictional forces) and (iii) controlled system (i.e. piping system with friction support and considering the bi-directional interaction of the frictional forces).

### 7.1. Validation of the proposed analysis procedure

To validate the proposed seismic analysis procedure for the piping system, analytical force-displacement hysteresis loops are compared by considering and ignoring the variation of the limiting friction force with time along with the experimental hysteresis loops are shown in Fig. 3. The hysteresis loops of the friction support are shown in the X-direction under various narrow-band random ground motions and duly considering the effects of the bi-directional interaction. It is observed from the Fig. 3 that there is a better comparison of the analytical and experimental hysteresis loop when the effect of variation of limiting friction force with time due to change in the normal reaction is considered. A comparison of the corresponding FFT spectra of the frictional force of the friction support obtained by analytically and experimentally is made in Fig. 4. As observed earlier, there is a better comparison of the analytical and experimental FFT spectra of the frictional forces when the time-dependent normal reaction at the friction support is considered. Thus, there is a close agreement of the proposed procedure for the analytical seismic response of the piping with the corresponding experimental results. The limiting frictional force of the friction support must be modified due to change in the normal reaction during the earthquake excitation of the piping system.

The comparison of the hysteresis loops of the friction support in X-direction under 1995 Kobe earthquake is shown in Fig. 5 by ignoring and considering the interaction effects and the time dependency of the limiting friction force. As expected, there are significant effects of the bi-directional interaction of the frictional forces and time dependent limiting frictional force of the support under real earthquake excitation. Thus, these effects must be considered for seismic analysis of piping system supported with friction device.

The peak resultant responses of the piping system (i.e. $u_s = \sqrt{x_s^2 + y_s^2}$, $\ddot{u}_p = \sqrt{\ddot{x}_p^2 + \ddot{y}_p^2}$ and $R_R = \sqrt{R_x^2 + R_y^2}$) are plotted against $N$ (expressed by Eq. (17)) in Fig. 6 under 1994 Northridge earthquake to study the robustness of the proposed analytical procedure and its sensitivity to time interval of integration. It is observed from the figure that the minimum value of $N$ for numerically stable results is 20 and 40 for no-interaction and interaction cases, respectively. For reasonable accurate response of the piping system the minimum value of $N$ is found to be 40 for both cases. Thus, the nonlinear response of the piping system with friction support is quite sensitive to the time interval used for the integrations of the equations of the motion. The accuracy of the results shall be assured by taking the minimum time interval. In addition, the minimum time interval required for integration of equations of motions is also found to be sensitive to typical earthquake ground motions having different frequency contents and PGA.
7.2. Performance of the friction support

The time variation of various response quantities in X- and Y-directions of the piping system under a narrow-band random ground motion of PGA = 0.22 g and 1994 Northridge earthquake motion are shown in Figs. 7 and 8, respectively. The responses are plotted for the controlled piping system by considering and ignoring the effects of the bi-directional interaction along with the responses of the uncontrolled piping system. There is significant reduction in the response quantities of the controlled piping system.
system in comparison to uncontrolled system implying that the friction support is very effective in reducing the seismic response of piping systems. A comparison of the controlled response by considering and ignoring the bi-directional interaction indicates that the response quantities are increased when the interaction effects are considered into account. This happens because of the fact that the system with interaction effects starts sliding when the resultant value of the frictional force (i.e. \( F_x^2 + F_y^2 = F_r^2 \)) exceeds the limiting friction force, which results in smaller limiting friction force value compared to system without interaction, resulting in higher displacements. Thus, the friction support is quite effective for seismic response control of piping system and interaction of frictional forces should be duly considered.

Figs. 9 and 10 show the plot of peak resultant response quantities of the piping system against the friction coefficient (\( \mu_{\text{max}} \)) under narrow-band random ground motions and real earthquake ground motions, respectively. It is observed that under all ground motions the sliding displacements are relatively more by considering the interaction effects in comparison to that without interaction effects implying that there is significant under estimation of the sliding displacements. Moreover, it is also observed that the effects of the bi-directional interaction are dependent on the friction coefficient and are relatively less for low values of the friction coefficient of the friction support. This is due to the fact that for lower values of friction coefficient, the system remains most of the time in the sliding phase for both cases of the excita-

### Table 2

Peak response quantities of piping system on friction support under narrow-band random waves (\( \mu_{\text{max}} = 0.15 \) and \( \Delta \mu = 0 \))

<table>
<thead>
<tr>
<th>PGA</th>
<th>Direction</th>
<th>Uncontrolled</th>
<th>Controlled (no interaction)</th>
<th>Controlled (interaction)</th>
<th>Controlled (experimental)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( x_p ) or ( y_p ) (mm)</td>
<td>( x_p ) or ( y_p ) (g)</td>
<td>( R_x ) or ( R_y ) (kN)</td>
<td>( x_p ) or ( y_p ) (mm)</td>
</tr>
<tr>
<td>-----</td>
<td>-----------</td>
<td>--------------</td>
<td>-----------------------------</td>
<td>--------------------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>0.22 g</td>
<td>X</td>
<td>6.714</td>
<td>0.682</td>
<td>1.1</td>
<td>2.474</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>6.11</td>
<td>1.971</td>
<td>2.982</td>
<td>1.372</td>
</tr>
<tr>
<td>0.29 g</td>
<td>X</td>
<td>9.031</td>
<td>0.932</td>
<td>1.46</td>
<td>3.795</td>
</tr>
<tr>
<td></td>
<td>Y</td>
<td>7.989</td>
<td>2.513</td>
<td>3.866</td>
<td>3.234</td>
</tr>
<tr>
<td>0.35 g</td>
<td>X</td>
<td>11.58</td>
<td>1.174</td>
<td>1.882</td>
<td>5.331</td>
</tr>
<tr>
<td>0.56 g</td>
<td>X</td>
<td>17.85</td>
<td>1.787</td>
<td>2.89</td>
<td>9.404</td>
</tr>
</tbody>
</table>
Fig. 10. Effects of bi-directional interaction of frictional forces on peak resultant responses of piping system under real earthquake ground motions.

Fig. 11. Effects of the velocity dependence of friction coefficient on peak resultant responses of piping system under narrow-band random ground motions.
tion (i.e. with and without interaction). As a result, the difference in the sliding displacements for the two cases is relatively less. Thus, for effective design of the friction supports for the piping systems the bi-directional interaction effects of frictional forces must be considered.

In Tables 2 and 3, the peak response quantities of the piping system are compared by considering and ignoring the interaction effect, respectively, under narrow-band random and real earthquake ground motions. In Table 2, the responses in the two horizontal directions are compared with corresponding responses of the piping system without friction support along with the experimental results. Displacements of the piping system under narrow-band random ground motion of PGA = 0.56 g, by ignoring and considering the interaction effects, are observed to be equal to 10.092 and 13.297 mm, respectively, and the corresponding experimental value is 12.867 mm. Thus, the response values obtained by considering the interaction effects are in close agreement with the experimental results implying that the proposed analytical procedure yields analytical results those are in close agreement with the experimental results. From Table 3, similar observations are also noted for the piping system under real-earthquake ground motions. The displacements of the piping system are required for designing the piping system and supplemental devices, the nodal accelerations are direct representation of the body forces developed in the elements and the support reactions are required to design the supporting system.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Direction</th>
<th>Without friction support</th>
<th>With friction support (no interaction)</th>
<th>With friction support (interaction)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>Imperial Valley, 1940</td>
<td>x</td>
<td>9.366 0.686 1.334 2.341 3.341 4.31</td>
<td>10.092 0.743 1.334 2.091 3.42</td>
<td>12.867 0.743 1.334 2.091 3.42</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>3.636 0.84 1.677 2.091 1.272</td>
<td>4.231 0.343 0.685 1.077 0.242</td>
<td>5.021 0.427 0.757 2.001 0.625</td>
</tr>
<tr>
<td>Northridge, 1994</td>
<td>x</td>
<td>12.982 0.874 2.091 1.077 0.242</td>
<td>12.867 0.743 1.334 2.091 3.42</td>
<td>12.867 0.743 1.334 2.091 3.42</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>4.821 1.348 2.676 1.077 0.242</td>
<td>4.231 0.343 0.685 1.077 0.242</td>
<td>4.352 1.092 2.433 1.092 2.433</td>
</tr>
<tr>
<td>Loma Prieta, 1989</td>
<td>x</td>
<td>15.103 1.283 2.121 1.077 0.242</td>
<td>12.405 1.167 1.911 1.911 3.97</td>
<td>12.367 1.167 1.911 1.911 3.97</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>4.462 1.213 2.094 1.077 0.242</td>
<td>3.172 0.905 1.106 1.106 3.172</td>
<td>3.529 1.307 1.357 1.357 3.529</td>
</tr>
<tr>
<td>Kobe, 1995</td>
<td>x</td>
<td>22 1.391 3.132 3.132 6.56</td>
<td>12.902 0.656 2.097 2.097 6.56</td>
<td>15.024 0.845 2.166 2.166 6.56</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>4.51 1.069 2.426 2.426 6.56</td>
<td>2.834 0.913 1.701 1.701 3.37</td>
<td>3.372 0.891 1.929 1.929 3.37</td>
</tr>
</tbody>
</table>
These are the key variables in the design of piping systems. Piping systems, designed by considering the underestimated response quantities can be unsafe in case of industrial installations and nuclear power plants.

7.3. Effects of velocity dependent friction coefficient

The friction coefficient of various sliding devices is typically dependent on the relative velocity at the sliding interface. The velocity dependence of the friction coefficient is modeled by Eq. (5). It will be interesting to study this effect on the peak responses of the piping system with sliding friction support.

The peak resultant responses of the piping system are plotted in Figs. 11 and 12 under narrow-band random and real earthquake ground motions, respectively. The response is shown for three values of $\Delta \mu$, (i.e. $\Delta \mu = 0, 0.15$ and $0.3$). Note that $\Delta \mu = 0$ denotes that friction coefficient of the friction support is independent of the velocity (i.e. Coulomb-friction idealization). It is observed from Figs. 11 and 12 that dependence of friction coefficient on the relative sliding velocity does not have noticeable effects on the peak response of piping systems with friction support which were confirmed earlier by Fan et al. (1990) and Bhasker Rao and Jangid (2001) for buildings resting on sliding isolators. Thus, the effects of dependence of friction coefficient on sliding velocity may be ignored for finding out the peak responses of piping systems with friction supports.

8. Conclusions

The response of a piping system with friction support under two horizontal components of narrow-band and real earthquake ground motions is investigated. The interaction between the frictional forces at the friction support in two orthogonal directions is duly considered. The dependence of limiting friction force on the time dependent normal reaction and relative velocity at sliding interface are also considered. The proposed analytical procedure for seismic response of the piping system supported on a friction support is validated by comparing the analytical seismic responses with the corresponding experimental results. From the trends of the results of the present study, the following conclusions may be drawn

(1) The friction support is very effective in reducing the dynamic response of the piping system under ground motions with different frequency contents.
(2) The proposed analytical procedure for seismic response of the piping system supported on a friction support is found to be sufficiently robust and the results obtained are in close agreement with the experimental results.
(3) The analytical responses of the piping system obtained by considering bi-directional interaction are in close agreement with the experimental results than those obtained by ignoring the interaction.
(4) The sliding displacements of friction support isolating the piping system are underestimated if the interaction of frictional forces in two orthogonal directions is ignored. This can be crucial from design point of view. Similar effects of bi-directional interaction are also observed for nodal accelerations and support reactions of the piping system.
(5) The variation of the limiting frictional force of the friction support due to change in the normal reaction had significant effects on the response of the piping system supported on the friction support.
(6) The dependence of friction coefficient on the relative sliding velocity at the friction support does not have noticeable effects on the peak responses of the piping system. Therefore, these effects may be ignored for finding out the peak responses of the piping system.

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