A New Approach to Model Nonquasi-Static (NQS) Effects for MOSFETs—Part II: Small-Signal Analysis

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Abstract—We present a new approach to model nonquasi-static (NQS) effects in a MOSFET in a small-signal situation. The model derived here is based on the large-signal NQS model proposed in [1]. The derivation of the small-signal model is presented. The small-signal parameters obtained with this model prove to be accurate up to very high frequencies. An excellent match between the new model and device simulation results has been observed even when the frequency is many times larger than the cutoff frequency.

Index Terms—MOSFET compact model, nonquasi-static (NQS) effect, small-signal MOS transistor model, $Y'$ parameters.

I. INTRODUCTION

LOW-FREQUENCY MOSFET models can be derived from dc analysis using the quasi-static (QS) approach. In principle, it means that the time dependent behavior is treated as a succession of steady-state situations. However, at higher frequencies, the QS model loses its validity, since the behavior of the MOSFET cannot be treated as a succession of steady-state situations, and the channel charge becomes an explicit function of time. This behavior is called nonquasi-static (NQS) behavior. A first order estimate of the time scale at which NQS behavior becomes noticeable is given by the average transit time of the carriers. For modeling this NQS behavior, the time dependence in the basic semiconductor equations has to be taken into account explicitly.

Apart from theoretical interest, the study of NQS effects and models becomes increasingly important because of practical reasons. The operating frequency of most high-frequency circuits is in a range where NQS effects are almost negligible ($f \ll f_T$) due to the capacitive loads of the active device. However, when an inductive load tunes out the capacitance at some nodes, NQS effects might be a limiting factor [2]. The PMOS biasing current sources of RF circuits might also suffer from these NQS effects because of low mobilities of the holes. NQS effects may become important for high-frequency circuits which sometimes require relatively long channel devices. It may also be possible to profitably make use of the NQS effects by treating the MOS transistor as a distributed element [3]. For the above reasons, a good understanding and modeling of the NQS behavior, even beyond the cutoff frequency is essential.

Although the small-signal NQS regime of the intrinsic MOS transistor has already been investigated in several papers [3]–[8], these descriptions are often mathematically complicated, and they give little insight to circuit designers. Most of the analytical solutions of the problem are given in terms of Bessel functions of fractional orders and of complex arguments. Such functions are not available in most programming languages, and their numerical evaluation tends to be slow with poor convergence. One of the alternatives is to truncate the series after some terms. But this can drastically change the behavior of various $Y'$ parameters at high frequency [3]. Moreover, these solutions cannot be implemented over a conventional QS model; instead, one has to rewrite the whole small-signal model. Recently [2], an approximate NQS $Y'$ parameter model was presented, based on asymptotic behavior of Bessel functions and relative independence of those functions over normalized inversion charge density ratio. This model removes the above-mentioned difficulties. After using suitable approximations, their final form contains only sine and cosine hyperbolic terms and can easily be implemented in a circuit simulator.

In this paper, we present a completely new approach to model the NQS effect for small-signal analysis. We separate out the $Y'$ parameters of the intrinsic transistor in two parts: $Y = Y'^{qs} + Y'^{tr}$. The $Y'^{qs}$ represent the $Y'$ parameters predicted by QS model and $Y'^{tr}$ arise due to NQS effect. Our model can therefore be implemented over a conventional QS model. $Y'^{tr}$ has a very simple representation; it is a parallel combination of single-pole transfer function, where every pole carries a physical meaning. This kind of representation may be a great help for circuit designers and may be more desirable than [2] where the poles are not represented explicitly and frequency-dependent sine and cosine hyperbolic terms are involved.

The paper is organized as follows. The derivation of the model is given in Section II. In Section III, results from the new model are compared with both an exact solution and device simulation. In Section IV, some implementation issues of the model are discussed, and conclusions drawn. In the Appendix, the exact expressions for the $Y'^{tr}$ parameters are derived.
II. MODEL FORMULATION

The small-signal model presented here is essentially an extension of the large-signal model described in [1]. We begin with the assumption that the n-MOSFET channel is in steady state. A small step change is made in one or more terminal voltages at \( t = 0 \). We want to investigate how the channel reaches its new steady state, i.e., how \( X_n(t) \) changes with time. In this special case, for \( t > 0 \), (16) in [1] reduces to

\[
\sum_{n=1}^{n} p_{mn} X_m = \sum_{m=1}^{n} \alpha_{mn} X_m + \sum_{m=1}^{n} \sum_{l=1}^{n} \beta_{mnl} X_m X_l.
\]  

(1)

There is no \( \lambda \) term, because under our assumption, \( dV_i/dt = 0 \) for \( t > 0 \).

Now, under the small-signal assumption, since the change in the voltage is small, \( X_n \) will also be small. We can therefore neglect the second order \( X_m X_l \) terms. When this is done, we get a linear system of differential equations of the following form:

\[
\dot{X}_n = \sum_{m=1}^{n} \alpha_{mn} X_m.
\]  

(2)

Now, a column vector \([X]\) is introduced whose rows are \( X_n(t) \). The system of equations can be written as

\[
[X(t)] = [\alpha][X]
\]  

(3)

where \([\alpha]\) is a square matrix and \( \alpha_{mn} = \alpha_{nm} \). The solution of this equation is of the form \([X] = \{v\} \exp(\gamma t)\); therefore, \( \gamma \) are the eigenvalues of the matrix \([\alpha]\), and \([v]\) is the eigenvector of the matrix \([\alpha]\). The general solution of the system is given by

\[
[X(t)] = \sum_{m=1}^{n} C_m [v)_m \exp(\gamma_m t)
\]  

(4)

where \( \gamma_m \) is the \( m \)th eigenvalue and \( [v]_m \) is the \( m \)th eigenvector.

In order to determine the \( C_m \)'s, we need to find the initial condition. From the definition of \( Q_{\text{NQS}} \) [1], we have

\[
Q_{\text{NQS}}(x; 0) = \Delta Q_{\text{QS}}(x)
\]  

(5)

where \( \Delta Q_{\text{QS}}(x) \) denotes the change in the QS charge distribution between \( t < 0 \) and \( t > 0 \). We have

\[
\Delta Q_{\text{QS}}(x) = \sum_{i=D, S, B, G} \frac{\partial Q_{\text{QS}}}{\partial V_i} \Delta V_i.
\]  

(6)

From (6), we obtain

\[
X_n(0) = \sum_{i=D, S, B, G} \lambda_{ni} \Delta V_i
\]  

(7)

where

\[
\lambda_{ni} = \int_0^L \frac{\partial Q_{\text{QS}}}{\partial V_i} \varphi_n dx = \frac{\partial}{\partial V_i} \left( \int_0^L Q_{\text{QS}} \sin \left( \frac{\pi x}{L} \right) dx \right).
\]  

(8)

From (4), we get

\[
X(0) = \sum_{m=1}^{n} C_m [v]_m.
\]  

(9)

Now, a matrix \([\tilde{V}_0]\) is defined such that the columns of the matrix \([\tilde{V}_0]\) are the eigenvectors \([v]_m\), and \([C]\) is a vector whose rows are \( C_m \). From (4), we get

\[
C_m = \sum_{k=1}^{n} [V_0^{-1}]_{mk} X_k(0).
\]  

(10)

Substituting (9) and (10) into (4), we get the final expression for \( X_n(t) \):

\[
X_n(t) = \sum_{n=1}^{n} \sum_{m=1}^{n} \sum_{l=1}^{n} [V_0^{-1}]_{mk} X_m(0) \lambda_{kl} \tilde{V}_0 \exp(\gamma_m t) \Delta V_i.
\]  

(11)

In order to find the terminal NQS current, we need to find the terminal partition of NQS charges. The terminal charge for any terminal \( j \) is given by

\[
\begin{align*}
Q_{\text{NQS},j}(t) &= WL \left( \sum_{i=D, S, B, G} \sum_{m=1}^{n} \sum_{l=1}^{n} \sum_{i=D, S, G, B} [V_0^{-1}]_{mk} \lambda_{kl} [V_0]_{im} \right. \\
& \quad \left. \times \exp(\gamma_m t) \Delta V_i P_{ij} \right) \end{align*}
\]  

(12)

\( P_{ij} \) denotes the contribution of \( i \)th harmonic to the terminal \( j \). For example, with our choice of basis functions as \( \varphi_n(x) = \sin(n\pi x/L) \) [1] for the drain terminal, \( P_{ij} = ((-1)^{n+1})/(\pi n) \).

If \( \Delta V_i \) is very small (i.e., it does not disturb the bias condition which is true under small signal assumption), we can think of the system as a linear one and use superposition theorem to calculate the various NQS charges. In \( s \) domain

\[
Q_{\text{NQS},j}(s) = WL \left( \sum_{i=D, S, B, G} \sum_{m=1}^{n} \sum_{l=1}^{n} \sum_{i=D, S, G, B} [V_0^{-1}]_{mk} \right. \\
& \quad \left. \lambda_{kl} [V_0]_{im} \frac{1}{s - \gamma_m} \Delta V_i P_{ij} \right). \]  

(13)

The different NQS terminal currents are given by the time derivatives of the terminal NQS charges. In \( s \) domain, the relation becomes a multiplication by \( s \). The NQS component of the \( j \)th terminal current is given by

\[
\begin{align*}
i_{\text{NQS},j}(s) &= sQ_{\text{NQS},j} \\
&= WL \left( \sum_{i=D, S, B, G} \sum_{m=1}^{n} \sum_{l=1}^{n} \sum_{i=D, S, G, B} [V_0^{-1}]_{mk} \right. \\
& \quad \left. \lambda_{kl} [V_0]_{im} \frac{s}{s - \gamma_m} \Delta V_i P_{ij} \right).
\]  

(14)

Now, changing the order of the summation, we can rewrite the above expression as

\[
\begin{align*}
i_{\text{NQS},j} &= \sum_{i=D, S, G, B} \sum_{n=1}^{n} \sum_{m=1}^{n} \sum_{l=1}^{n} [V_0^{-1}]_{mk} \lambda_{kl} [V_0]_{im} \\
& \quad \frac{s}{s - \gamma_m} P_{ij} \Delta V_i.
\]  

(15)
To find the various \( Y \) parameters in \( s \) domain, we can write

\[
\hat{Y}_{ji}^{\text{PF}} = \sum_{i=1,3} \sum_{k=1}^{N} \lambda_{ki} \left( V_0^{-1} \right)_{rk} V_i(s)
\]

where \( V_0^{-1} = \left( (i_{\text{PF}} - s) V_i(s) \right) \) when all other \( V_j(s) \) are 0. From this definition, we can find \( Y_{ji}^{\text{PF}}(s) \). In our case, the input to the system is a step change of strength \( \Delta V_j \) so \( V_j(s) = (\Delta V_j/s) \), and we obtain

\[
Y_{ji}^{\text{PF}} = WL \left( \sum_{i=1}^{N} \sum_{k=1}^{N} \left( V_0^{-1} \right)_{rk} \lambda_{ki} \left( V_0 \right)_{rn} P_{ni} s^{-2} \gamma_m \right).
\]

We now rearrange the terms and notice that the \( Y^{\text{PF}} \) parameters can be written as a parallel combination of single pole functions.

\[
Y_{ji}^{\text{PF}} = WL \left( \sum_{m=1}^{N} C_m s^{-2} \gamma_m \right)
\]

where \( C_m = \left( \sum_{i=1}^{N} \left( V_0^{-1} \right)_{mk} \lambda_{ki} \left( \sum_{i=1}^{N} \left( V_0 \right)_{rn} P_{ni} \right) \right) \).

We will now illustrate the steps involved in the model with a simple example and discuss some guidelines to choose the number of harmonics. Let us consider a case when the charge profile is nearly constant along the channel. In this special case, \( \alpha_{mn} \) becomes \( \alpha_{mn} = \frac{-\left( \mu Q_{0m} \right)}{\left( \mu C_{ox} E_2 \right) \left( n^2 \pi^2 \alpha_{mn} \right)} \). The term \( \left( \mu C_{ox} E_2 \right) \left( \mu Q_{0m} \right) \) is of the order the transit time of the MOSFET. We define this term by \( \tau \). As \( \left[ c \right] \) is a diagonal matrix, the eigenvalues are \( \gamma_n = \left( -n^2 \pi^2 / \tau \right) \), and both \( \left[ V_0 \right] \) and \( \left[ V_0^{-1} \right] \) will be unity matrices. Next, we find \( \lambda_{ni} \) and \( P_{ni} \). In our simple example, \( \lambda_{ni} = \left( \partial \tilde{Q} / \partial \tilde{V} \right) \left( 1 - (-1)^n \right) / \left( \pi \gamma_n \right) \) and \( P_{ni} = \left( \left( 1 \right) 1 / \pi \gamma_n \right) \). Therefore, \( C_m = \left( \partial \tilde{Q} / \partial \tilde{V} \right) \left( 1 - (-1)^n \pi^2 / n^2 \right) \). We notice that \( C_m \) vanishes when \( n \) is even and it is monotonically decreasing. Now, the poles \( \gamma_n \) are located at \( \left( n^2 \pi^2 / \tau \right) \). In reality, the charge profile will be modulated by \( \left( 1 / \pi \gamma_n \right) \). As a result, the pole will move about this value but the order of the pole location will not change. This observation sets a guideline to choose the number of harmonics. If the maximum frequency of interest is \( f \), then we need to ensure that poles outside this frequency range will not begin to affect the phase characteristics of the system. If the \( n \) numbers of harmonics is chosen to model the behavior up to \( f \), then the order of \( n \) will be given by \( \left( n^2 \pi^2 / \tau \right) \). We have verified that, up to a frequency of \( 10 f \), three harmonics are adequate to describe the \( Y \) parameters accurately.

III. SIMULATION RESULTS

In order to validate our model, we have compared it with both an analytical solution and device simulation; the analytical solution has been derived in Appendix I. We have also made a comparison with BSIM3v3 NQS model [9], [10]. For this purpose, we have derived a new analytical solution of the problem in Appendix I. To compare it with device simulation, a MOSFET was designed using process simulator ISE-DIOS with \( N_{sub} = 1.0 \times 10^{16} \text{ cm}^{-3}, \epsilon_{ox} = 18 \text{ nm}, \ V_{FB} = -0.8 \text{ V}, \mu_0 = 540 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}, W = 1 \mu\text{m} \). A device with a relatively large gate length \( (L = 6 \mu\text{m}) \) was simulated for this study because we wanted to extend the process window well above the cut off frequency of the device. If we had chosen a device with a smaller channel length, then the device behavior would have been completely dominated by the external parasitics at high frequencies, thus masking the NQS effects. Device simulation is done using ISE-DESSIS [10]. The QS component of the \( Y \) parameters of the device were extracted from low-frequency simulation results. The NQS component of the \( Y \) parameters was obtained by subtracting the QS \( Y \) parameters from the total \( Y \) parameters. The BSIM3v3 parameters were extracted using ISE-EXTRACT [10], and NQS components of the various \( Y \) parameters were obtained in a similar manner. To compare our new model with analytic solution and device simulation, we will show the plots of normalized admittance versus normalized frequency. The admittance is normalized with respect to \( Y_0 \) (as defined in Appendix I), and the frequency is normalized with respect to \( \omega_0 \) which corresponds to the cutoff frequency of the device (see Appendix I). We have gone up to 100 times the cutoff frequency for saturation and 1000 times for linear region. We have used 6 harmonic terms to model the NQS charge profile.

A. \( Y \) Parameters in Saturation Region

Fig. 1 shows the real and imaginary part of \( Y_{dg}^{\text{PF}} \) in saturation \((V_S = 0 \text{ V}, V_B = 0 \text{ V}, V_D = 2 \text{ V}, V_G = 1 \text{ V}) \). As frequency increases, the inversion layer charge does not have enough time to respond fully, and thus \( |Y_{dg}| \) which models this response will be small. Thus, both real and imaginary parts of \( Y_{dg} \) will tend to zero. But the QS model predicts a constant value for the real part of \( Y_{dg} \) and a constant capacitance \( C_{dg} \), hence, a monotonically increasing imaginary part of \( Y_{dg} \). In order to compensate that,
must show a constant negative real part and a monotonically increasing imaginary part. Our model correctly predicts that behavior. It shows a real part of normalized NQS admittance equal to $-1$ and a monotonically increasing imaginary part. The BSIM3v3 NQS model, on the other hand, gives a wrong prediction. Instead of going through a minimum and then stabilizing at $-1$, $Y_{\text{NQS}}^{\text{QP}}$ monotonically decreases and saturates at a completely different value.

Fig. 2 shows the real imaginary part of $Y_{\text{NQS}}^{\text{QP}}$. As frequency increases, the small-signal charge distribution in the channel moves toward the source. So, the source partition of the charge does not suffer as drastically as the drain partition. The part of $Y_{\text{NQS}}$ arising due to charge redistribution is proportional to the product of frequency and source partition of the small-signal charge. We therefore expect that, as the frequency increases, both real and imaginary parts of $Y_{\text{NQS}}^r$ will also increase. The QS model predicts a constant real part of $Y_{\text{NQS}}$. So, the real part of $Y_{\text{NQS}}^{\text{QP}}$ has to increase with frequency. From the results, we find that the imaginary part of $Y_{\text{NQS}}^{\text{QP}}$ does increase with frequency. Similarly, Fig. 3 shows the real and imaginary part of $Y_{\text{NQS}}^{\text{QP}}$.

### B. Y Parameters in Linear Region

Figs. 4 and 5 show the real and imaginary parts of $Y_{\text{NQS}}^{\text{QP}}$ and $Y_{\text{NQS}}^{\text{QP}}$, respectively, when the device is kept in linear region ($V_S = 0 \text{ V}, V_D = 0 \text{ V}, V_G = 0.05 \text{ V}, V_D = 1 \text{ V}$). Here, we observe similar trends for the source and drain. The real and imaginary parts of both increase with frequency. In the deep triode region, there is hardly any difference between source and drain. The real and imaginary parts of $Y_{\text{NQS}}^{\text{QP}}$, $Y_{\text{NQS}}^{\text{QP}}$, and $Y_{\text{NQS}}^{\text{QP}}$, respectively.

### IV. Conclusion

A completely new approach to model small-signal NQS effects in the MOS transistor has been presented. The new model shows an excellent match with both exact analytic solution and
device simulation. The NQS effect has been modeled by $Y_{THS}$ parameters which have a simple description. Each pole in $Y_{THS}$ parameters corresponds to a harmonic which is used to model the NQS charge profile. We have found that, by using only 6 harmonic terms, it is possible to model accurately the $Y$ parameters up to 100 times the cut-off frequency. We also observed that, in linear region, it is possible to go up to 100 times the cutoff frequency using only 3–4 harmonic terms. This is because of the fact that, in linear region, the small-signal NQS charge profile is simple compared to that in saturation; and fewer number of
The solution of (22) can be given in terms of modified Bessel function of the first kind.

\[ Q_{\text{small}} = C_1 I_{2/3} \left( \frac{4}{3} \sqrt{\frac{s}{\omega_0}} (1 - \frac{x}{L})^{3/4} \right) + C_2 I_{-2/3} \left( \frac{4}{3} \sqrt{\frac{s}{\omega_0}} (1 - \frac{x}{L})^{3/4} \right). \] (23)

\[ C_1 \text{ and } C_2 \text{ can be determined from boundary condition [see (24), (25), shown at the bottom of the page] } Q_{\text{small}} \text{ consists of both QS and NQS parts} \]

\[ Q_{\text{small}} = Q_{\text{small,qs}} + Q_{\text{small,ns}}. \]

Using (21), it can be shown that

\[ Q_{\text{small,qs}} = \delta Q(0) \sqrt{\frac{1 - \frac{x}{L}}{L}} + \frac{1}{L} Q(L) \delta Q(L) - Q(L) \delta Q(0) \frac{Q(0)}{Q(0)^2} \] (26)

After finding \( Q_{\text{small,ns}} \), we can easily obtain the small-signal NQS current. Using the relations given in Section III of [1], in s domain, we have

\[ i_{\text{NPS,qs}} = sW \int_0^L \frac{x}{L} Q_{\text{small,qs}} dx \] (27)

\[ i_{\text{NPS,ns}} = sW \int_0^L \frac{1 - \frac{x}{L}}{L} Q_{\text{small,ns}} dx \] (28)

\[ i_{\text{NPS,g}} = sW \frac{1}{\eta} \int_0^L Q_{\text{small,qs}} dx \] (29)

\[ i_{\text{NPS,b}} = -(i_{\text{NPS,qs}} + i_{\text{NPS,ns}} + i_{\text{NPS,g}}). \] (30)

We notice that \( Q_{\text{small,ns}} \) is a function of \((x/L)\) and \((s/\omega_0)\). We now make two changes of variables, \( \xi = (x/L) \) and \( s' = (s/\omega_0) \). If we express \( Q_{\text{small,ns}} \) in terms of \( \xi \) and \( s' \) and \( i_{\text{NPS}} \) in terms of \( s' \), then every \( Y \) parameter term will have a common factor

\[ Y_0 = \frac{\omega_0^2 \mu Q_{\text{bias}}(0) W}{\eta L} \] which is used to normalize the various \( Y_{\text{NPS}} \) parameters.

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