Variable property effects in single-phase incompressible flows through microchannels ✩

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Abstract

In a systematic approach we address the question of how important variable property effects are for flows in micro-sized channels. Due to heat transfer, the temperature dependence of fluid properties like viscosity and thermal conductivity results in deviations in a solution which accounts for that dependence compared to a solution only considering constant fluid properties. Compared to flows through macro-sized geometries, it turns out that two distinct scaling effects lead to a strong influence of variable fluid properties in micro-sized channels; Reynolds numbers are \( \text{Re} = \mathcal{O}(1) \) and not large, and axial temperature gradients are no longer small. These scaling effects can be identified after the basic equations are nondimensionalized properly. Examples are given in which Nusselt numbers differ by up to 30% depending on how the property behaviour is accounted for.

Keywords: Heat transfer; Microchannels; Variable property effects; Dimensional analysis; Scaling effects

1. Introduction

The influence of variable properties on momentum and heat transfer has been extensively studied for various kinds of flows in the past and correction formulae have frequently been derived that can correct results gained under the assumption of constant properties. Methods like the property ratio method or the reference temperature method are two examples in which a constant property result in terms of a friction coefficient \( f_{cp} \) or a Nusselt number \( \text{Nu}_{cp} \) (\( cp \): constant properties) is a posteriori corrected to account for the influence of temperature- or pressure-dependent physical properties. For an overview with respect to this kind of approach, see Kays and Crawford [1].

All these methods assume that the influence of variable properties is small in some sense. Under this assumption it can be accounted for by corrections that are linear in nature. For example, the property ratio method in which the constant property result is multiplied by a ratio of some property at different temperatures taken to some power. For for example, \( (\mu_{wall}/\mu_{mean})^n \) is linear in nature as long as the exponent is a constant (for details Ref. e.g. Schlichting and Gersten [2]). A method that explicitly exploits this linearity of the problem with respect to the influence of variable properties is the asymptotic method that treats the influence of variable properties as a regular perturbation problem (Ref. Herwig [3] and Herwig [4] for details). Since for laminar flows this method is completely analytical and the results are exact in the framework of the underlying assumptions, it is convenient to expect them as “direct candidates” for application to micro flows (which in most cases are laminar).

However, it turns out that the situation in micro flow systems is special and different from what is usually found in macro flows. Therefore, a special treatment of the problem of variable properties in micro devices is mandatory. What is so special will be outlined in the following chapter. There is a dearth of investigations that report conceptual understanding of micro
flow convection, considering additional identified mechanisms, which increasingly surface towards the microscale. It is only recently that Mahulikar et al. [5] numerically demonstrated the significance of induced radial flow and radial convection due to steep density gradients in continuum-based laminar gas micro flow convection. In another recent investigation Mahulikar and Herwig [6] theoretically demonstrated the increasing significance of induced radial flow and radial convection due to fluid thermal conductivity variations along the flow. In the following sections, the answer/s to the question, “What is so special about fluid property variations in laminar micro flows?” will be outlined.

2. Heat transfer with micro tubes

The special properties associated with heat transfer in micro tubes can best be seen when typical heat transfer situations are compared for micro and macro dimensions.

Hence a comparison is made on how a certain laminar flow with a unique mass flux per cross sectional area, \( \dot{m}^*/A^* \), in a tube or channel that is heated from \( T^* \) to \( T^* + \Delta T^* \) by a constant and unique wall heat flux density \( \dot{q}_w^* \). The pumping power required is \( P^* = \dot{m}^* \rho^* u^* L^* \) so that \( P^*/\dot{m}^* = L^* \rho^* / \dot{q}_w^* \) holds when \( L^* \) is the length of the tube.

It then would just take a large number of micro tubes to have the same mass flux \( \dot{m}^* \) as in one macro tube. The crucial difference between both cases (micro and macro) is the much higher surface area per length of the tube(s), \( S^*/L^* \), of the bundle of micro tubes compared to the one macro tube.

Since order of magnitude estimations are of interest, we assume fully developed flow and temperature profiles and minor differences that probably will exist can be neglected. Assuming the same friction and heat transfer laws for both cases the following relations are used:

\[
\begin{align*}
\lambda_R &= \frac{(-d\rho^*/dx^*)D^*}{(\rho^*/2)u^2_m} = \frac{64}{Re} \quad (1) \\
Nu &= \frac{(-\dot{q}_w^*)D^*}{k^*(T_w^* - T_m^*)} = 4.36 \\
\end{align*}
\]

The mean velocity \( u^*_m \) is the same for both cases due to the assumption \( \dot{m}^*/A^* = \) const. Therefore

\[
Re \equiv \frac{\rho^* u^*_m D^*}{\mu^*} \sim D^*; \quad \frac{dp^*}{dx^*} \sim 1/D^2; \quad \frac{dT^*}{dx^*} \sim 1/D^*; \quad (T_w^* - T_m^*) \sim D^*
\]

Here \( \sim \) means “of the order of”. Since \( \dot{q}_w^* = \dot{Q}_w^*/S^* = \) const and the surface area is \( S^* = \pi D^*L^* \), we have \( \dot{Q}_w^* \sim D^*L^* \). With \( \dot{Q}_w^* = \dot{m}^* c_p^* \Delta T^* \) (energy balance), \( \Delta T^* = \) const, \( \dot{m}^*/A^* = \) const and \( A^* \sim D^* \), finally \( L^* \sim D^* \).

Since \( \dot{q}_w^* = \) const (with the same constant for the micro and macro cases) there is homogeneous heating downstream of the inlet, i.e. \( dT^*/dx^* = \) const, i.e. \( dT^*/dx^* = \Delta T^*/L^* \sim 1/D^* \), since \( \Delta T^* = \) const and \( L^* \sim D^* \).

In Table 1 the order of magnitude estimations in the limit \( D^* \to 0 \) are summarized. For the analysis of the influence of variable properties in micro flow situations in contrast to those for macro flows, i.e. when going from macro to micro scales, three aspects are important and are marked by a grey shading in Table 1.

<table>
<thead>
<tr>
<th>Reynolds number</th>
<th>Pressure gradient</th>
<th>Temperature gradient</th>
<th>Necessary tube length</th>
<th>Cross section temp. diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Re \sim D^* )</td>
<td>( dp^<em>/dx^</em> \sim 1/D^2 )</td>
<td>( dT^<em>/dx^</em> \sim 1/D^* )</td>
<td>( L^* \sim D^* )</td>
<td>( (T_w^* - T_m^<em>) \sim D^</em> )</td>
</tr>
</tbody>
</table>
(1) Reynolds numbers are small \((Re \sim D^*)\). That is why micro flows almost always are laminar. Conventional methods to account for variable property effects assume large Reynolds numbers, however.

(2) Axial temperature gradients are large \((dT^*/dx^* \sim 1/D^*)\). Consequently, a constant property solution cannot be a good approximation over an appreciable downstream length \(\Delta L^*\).

(3) Cross sectional temperature differences are small \((T_w^* - T_o^* \sim D^*)\). In macro flow situations, there is an appreciable temperature difference over the cross section and a very small one over finite axial distances \(\Delta L^*\). In micro flow situations this is vice versa, or at least both are of equal importance. Since for \(\tilde{q}_r^* = k^*(\partial T^*/\partial r^*)_w = \text{const the radial temperature gradient is independent of } D^*\), we find \((\partial T^*/\partial x^*)(\partial T^*/\partial r^*) \sim 1/D^*\), i.e. the axial temperature gradient becomes more and more important when \(D^*\) gets smaller \((D^* \to 0)\).

These special features in micro flow devices give rise to the so-called scaling effects which becomes obvious when the problem is treated systematically by a dimensional analysis approach as shown next.

3. Nondimensional basic equations

Since this study is restricted to incompressible flow, property variations may occur due to the temperature and pressure dependence of \(\mu^*\) (viscosity), \(k^*\) (thermal conductivity) and \(c_p^*\) (specific heat capacity). Taking water as a typical fluid, it turns out that the pressure dependence for all three properties is very small and that the temperature dependence of \(c_p^*\) is almost negligible. This follows from the sensitivity coefficients

\[
K_{\alpha T} = \left( \frac{T^* \partial \alpha^*}{\alpha^* \partial T^*} \right)_x, \quad K_{\mu p} = \left( \frac{p^* \partial \alpha^*}{\alpha^* \partial p^*} \right)_p
\]

For \(\alpha^* = \mu^*, k^*, c_p^*\), their numerical values at the reference temperature \(T_r^* = 293 \text{ K}\) and the reference pressure \(p_r^* = 1 \text{ bar}\) are shown in Table 2.

With \(\mu^*(T^*)\) and \(k^*(T^*)\) as the most prominent and important property variations, the basic equations for momentum and heat transfer in a tube are provided in a nondimensional form (according to Table 3). These equations are for those flows that are not a priori assumed to be fully developed but undergo certain changes due to a changing geometry or due to heat transfer, for example.

\[
\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial (r v)}{\partial r} = 0
\]

Table 3

<table>
<thead>
<tr>
<th>(\alpha^*)</th>
<th>(r)</th>
<th>(\mu)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\partial \alpha^<em>}{\partial x^</em>})</td>
<td>(\frac{\partial \alpha^<em>}{\partial x^</em>})</td>
<td>(\frac{\partial \alpha^<em>}{\partial \mu^</em>})</td>
<td>(\frac{\partial \alpha^<em>}{\partial v^</em>})</td>
</tr>
<tr>
<td>(\rho^*)</td>
<td>(\mu^*)</td>
<td>(k^*)</td>
<td>(T^*)</td>
</tr>
<tr>
<td>(\rho^<em>\beta^</em>_r)</td>
<td>(\mu^<em>\beta^</em>_r)</td>
<td>(k^<em>\beta^</em>_r)</td>
<td>(T^<em>\beta^</em>_r)</td>
</tr>
</tbody>
</table>

*: dimensional quantity

o: reference state

Table 4

<table>
<thead>
<tr>
<th>(\mu)-variations accounted for</th>
<th>Model to account for (\mu(T))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\partial \mu}{\partial x})</td>
<td>(I) constant properties</td>
</tr>
<tr>
<td>(\frac{\partial \mu}{\partial \mu^*})</td>
<td>(II) quasi-constant properties</td>
</tr>
<tr>
<td>(\frac{\partial \mu}{\partial T})</td>
<td>(III) weakly variable properties, perturbation approach (linear)</td>
</tr>
<tr>
<td>(\frac{\partial \mu}{\partial T})</td>
<td>(IV) strongly variable properties, fully coupled problem (non-linear)</td>
</tr>
</tbody>
</table>
4. Heat transfer with variable $\mu$ and $k$

Fig. 1 shows the heat transfer situation that was selected to demonstrate the influence of variable properties on the heat transfer performance of a micro pipe. The fluid is water with the viscosity and thermal conductivity relations

$$\mu^*(T^*) = \mu_r^* \left(\frac{T^*}{T_r^*}\right)^n \exp\left[\frac{B^*}{T^* - T_r^*}\right]$$ \hspace{1cm} (8)

with: $\mu_r^* = 1.005 \times 10^{-3}$ kg m$^{-1}$ s$^{-1}$ and the constants $n = 8.9$, $B^* = 4700$ K

$$k^*(T^*) = \left(-1.51721 + 0.0151476|T^*| - 3.5035 \times 10^{-5}|T^*|^2 + 2.74269 \times 10^{-8}|T^*|^3\right) W m^{-1} K^{-1}$$ \hspace{1cm} (9)

Fig. 1. Changing the theoretical model from "constant properties" (model I) to "strongly variable properties" (model IV) at $\tilde{x} = x^*/D^* = 0$; $D^* = 100 \mu m$. taken from Sherman [8] and Holman [9], respectively. The model situation chosen shows the influence of variable properties on the solution of the basic equations by "switching on" variable properties at $x^* = 0$. This situation, though somewhat unrealistic, can clearly reveal the role played by variable fluid properties. Specific results of that kind, therefore, cannot be adopted immediately for real flow situations. They can, however, be used to get a clear picture of what has to be expected when variable property effects are accounted for in a heat transfer analysis of micro flow problems. At the end of this chapter, an example for a real flow situation is given.

Due to the constant property behaviour for $\tilde{x} < 0$ the initial conditions at $\tilde{x} = 0$ are

$$u = 2(1 - r^2)$$ \hspace{1cm} (10)

for the flow, and

$$T = 1 + \dot{q}_w \left(\frac{r^2 - r_d^4}{4} - \frac{7}{24}\right); \hspace{0.5cm} Nu = \frac{48}{11} = 4.36$$ \hspace{1cm} (11)

for the temperature profile. The Nusselt number at $\tilde{x} = 0$ in this situation is the well known value of $Nu = 48/11$.

For $\tilde{x} > 0$ variable property effects "set in" and the Nusselt number changes with $\tilde{x}$ as shown in Fig. 2. Numerical calculations have been performed with the CFD code CFX-4.4 which for this investigation solves Eqs. (4)–(7) subject to the inlet boundary conditions (10) and (11). For details of the numerical solution see Mahulikar et al. [10].

For $\dot{q}_w^* > 0$ the flow is heated, and for $\dot{q}_w^* < 0$ it is cooled. With $|\dot{q}_w^*| < 30 W cm^{-2}$ the water neither boils nor freezes at the pipe exit ($\tilde{x} = 50$). For small values of $|\dot{q}_w^*|$ the heating and cooling cases are almost symmetrical around $\dot{q}_w^* = 0$. For larger values of $|\dot{q}_w^*|$ this symmetry ceases to exist due to the highly non-linear character of this heat transfer situation. Nusselt number deviations ($\Delta Nucp$) $= (Nu - Nucp) / Nucp$ with respect to the constant property case are up to 10%. With higher heat transfer rates, covering the whole temperature range between freezing and boiling of water, $\Delta Nucp$ values as high as 28% may occur (Ref. Mahulikar et al. [10]).

Regarding the flow field, it turns out that the radial velocity $v$, though small compared to the axial velocity (and completely neglected for $Re \to \infty$), plays an important role. This is due to the fact that $v$ is multiplied by $\partial T/\partial r$ (which may
be large in a micro pipe flow) in the thermal energy equation. Hence the ratio
\[ \frac{\int_0^1 v \frac{\partial T}{\partial r} r \, dr}{\int_0^1 u \frac{\partial T}{\partial \tilde{x}} r \, dr} \]
which is the ratio of radial to axial heat advection, is of order \( O(1) \), i.e. not asymptotically small as it would be for \( Re \to \infty \).
As a consequence, there exists an appreciable length over which the flow develops under the influence of an imposed heat transfer [10].

To show how strong these effects are in real flow situations, Fig. 3 shows the entrance region of a heated pipe. Velocity and temperature at \( \tilde{x} = 0 \) now are constant across the entrance cross section, which leads to high values of the Nusselt number for small values of \( \tilde{x} \) with \( Nu \to \infty \) for \( \tilde{x} \to 0 \). Fig. 3 shows that there is a strong influence of variable properties “activated” by different heating rates \( \dot{q}_w^* \). For example, the Nusselt number at the pipe exit (\( \tilde{x} = 50 \)) for \( \dot{q}_w^* = 200 \text{ W cm}^{-2} \) is almost 30% higher than the value which a constant property solution (which is independent of \( \dot{q}_w^* \)) would predict.

5. Conclusions

While variable property effects in pipe and channel flows through macro-sized conduits are often of minor importance, they have a strong influence in micro-sized pipe and channel geometries. This increased importance of considering the temperature dependence of physical properties when heat transfer occurs is due to scaling effects with respect to different orders of magnitude of the Reynolds number and the axial temperature gradient. When characteristic lengths are changed from macro to micro size constant property results are always only approximations. For micro flows with high heat transfer rates these approximations are obviously very crude.

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References