Effects of phase errors and shot noise on asynchronous coherent optical CDMA

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Abstract: Effects of phase errors and shot noise on asynchronous coherent optical code division multiple access systems are examined here. It is shown that asynchronous systems have better bit error performance than the synchronous systems.

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1. Introduction

The performance of a coherent optical code division multiple access (CDMA) system is affected by laser phase noise, phase lock errors and shot noise. A study of synchronous CDMA systems assuming perfect phase locking has been presented in the past [1]. However, the asynchronous systems with laser phase drift and phase lock errors have not been investigated so far. In this paper, we examine the bit error rate (BER) performance of an optical heterodyne binary phase shift keying (BPSK) based asynchronous CDMA system in the presence of laser phase noise, phase lock errors and the shot noise.

2. System Model

Let us consider an asynchronous CDMA system with M users. The spreading codes $c_k^{(i)}$ are modeled as independent equiprobable sequences of ± 1s of length G. For the i^{th} user, $s_i(t)$ is the signature, b_i is the data bit and w_i is the bit energy. The signal obtained at the intermediate radio frequency of the coherent optical heterodyne receiver is given by

$$r(t) = \sum_{i=1}^{M} b_i \sqrt{w_i} s_i(t-\tau_i) \cos(\omega_0 t + \theta_i + \phi_t^{(i)}) + n(t) \quad (1)$$

where for the i^{th} user, θ_i is the initial phase uniformly distributed in $[0, 2\pi)$ and $\phi_t^{(i)}$ is a Brownian motion with variance $2\pi\beta t$ which accounts for the laser phase drift. β is the sum of linewidths of transmitter and local oscillator lasers. τ_i represents the delay offset for the i^{th} user and is uniformly distributed in $[0, GT_c)$. $n(t)$ represents the shot noise.

3. Theoretical Analysis

Without loss of generality, the first user can be considered to be the desired user. Thus, τ_1 can be taken to be *zero*. The phase of the first user is tracked. The output of the filter matched to the first user's signature can be calculated to be

$$\begin{aligned} y_1 &= b_1\sqrt{w_1}\sum_{k=1}^{G}\cos(\phi_{e_k}) + \sum_{i=2}^{M} b_i\sqrt{w_i}\sum_{k=1}^{G}\int_{(k-1)T_c}^{kT_c} s_i(t-\tau_i)s_1(t)\cos(\bar{\theta}_i + \bar{\phi}_k^{(i)} - \phi_{e_k})dt + n_1 \\ &= y_d + \sum_{i=2}^{M} b_i\sqrt{w_i}\bar{\gamma}_i + n_1 \end{aligned} \quad (2)$$

where T_c is the chip width, $\bar{\theta}_i = \theta_i - \theta_1$ is uniformly distributed over $[0, 2\pi)$ and $\bar{\phi}_t^{(i)} = \phi_t^{(i)} - \phi_t^{(1)}$ is a Brownian motion with twice the variance of $\phi_t^{(i)}$. Thus the effective linewidth is $\beta' = 2\beta$. We assume that $1/\beta' > T_c$

which implies that phase drift is approximately constant over a chip duration. ϕ_e is the phase error due to imperfect phase locking (assumed constant over a chip interval) and is modeled as zero mean Gaussian with variance σ_e^2, that is, $\phi_e \sim \mathcal{N}(0, \sigma_e^2)$. $\phi_e(t)$ and $\tilde{\phi}_t^{(i)}$ are referred to as ϕ_{e_k} and $\tilde{\phi}_k^{(i)}$ respectively in the duration $(k-1)T_c \leq t < kT_c$. For a synchronous system, $\tilde{\gamma}_i$ is defined as $(1/G)\sum_{k=1}^{G} c_k^{(1)} c_k^{(i)} \cos(\tilde{\theta}_i + \tilde{\phi}_k^{(i)} - \phi_{e_k})$ and is asymptotically Gaussian [1]. The first term y_d in the summation is the desired user's signal, the second term represents the multiple access interference (MAI) due to the other $M-1$ users and the third term is the shot noise, $n_1 \sim \mathcal{N}(0, \sigma^2)$. The decision statistic for the first user is given by $\hat{b} = sgn(y_1)$.

We consider synchronous CDMA (S-CDMA) and asynchronous CDMA (CDMA) below. The details of the proofs are in [2].

3.1 Synchronous CDMA

With an analysis similar to the one used for two user synchronous system in [1], the bit error rate for M users can be derived using the fact that the interference due to different users is independent. We now consider the case of imperfect phase locking. Using the central limit theorem [3], it can be shown that $y_d \sim \mathcal{N}(b_1\sqrt{w_1}e^{-\sigma_e^2/2}, w_1.(1-e^{-\sigma_e^2})/2)$. Variance of $\tilde{\gamma}_i$ accounting for the phase lock error is given by

$$r_i^2 = E[\tilde{\gamma}_i^2] = \frac{1}{2G} + \frac{1}{G^2}\sum_{K=1}^{G}\sum_{j>k}^{G} c_k^{(1)} c_k^{(i)} c_j^{(1)} c_j^{(i)} e^{-(\sigma_e^2 + \pi\beta' T_c(j-k))} \tag{3}$$

Representing mean and variance of y_d by m_{y_d} and $\sigma_{y_d}^2$ respectively, the bit error probability is given by

$$P_e = Q\left(\sqrt{\frac{m_{y_d}^2}{(\sum_{i=2}^{M} w_i r_i^2) + \sigma^2 + \sigma_{y_d}^2}}\right) \tag{4}$$

3.2 Asynchronous CDMA

For this case, we write $\tau_i = l_i T_c + \tau_i'$ where $l_i \in \mathbb{Z}$ and $0 \leq l_i \leq G-1$; $\tau_i' \in \mathbb{R}$ and τ_i' lies in $[0, T_c)$. Now, we define $p_i = \tau_i'/T_c$. p_i is uniformly distributed in $[0, 1)$.

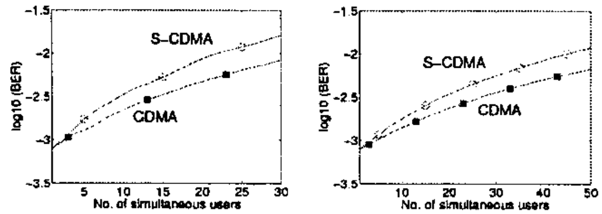
$\tilde{\gamma}_i$ is now $(1/G)\sum_{k=1}^{G} c_k^{(1)}(p_i c_{k-1}^{(i)} + (1-p_i)c_k^{(i)})\cos(\tilde{\theta}_i + \tilde{\phi}_k^{(i)} - \phi_{e_k})$ and is no longer Gaussian. It has a variance of

$$\begin{aligned} r_{ai}^2 &= \frac{1}{3G} + \frac{2}{3G^2}\sum_{k=1}^{G}\sum_{j>k}^{G} c_k^{(1)} c_k^{(i)} c_j^{(1)} c_j^{(i)} e^{-(\sigma_e^2 + \pi\beta' T_c(j-k))} \\ &\quad + \frac{1}{3G^2}\sum_{k=1}^{G}\sum_{j=1}^{G} c_k^{(1)} c_k^{(i)} c_j^{(1)} c_{j-1}^{(i)} e^{-(\sigma_e^2 + \pi\beta' T_c|j-k|)} \end{aligned} \tag{5}$$

The bit error probability for asynchronous CDMA is given by (4) with r_i^2 replaced by r_{ai}^2. The MAI from $M-1$ users is Gaussian for large number of users which justifies the use of (4). It should be noted that r_{ai}^2 has a constant $(1/3G)$ instead of $(1/2G)$ of r_i^2. The remaining terms have an even distribution and depend on the spreading codes of first and i^{th} user. Simulations show that for two spreading codes, on an average $r_{ai}^2 < r_i^2$ which leads to better performance of asynchronous CDMA over synchronous CDMA.

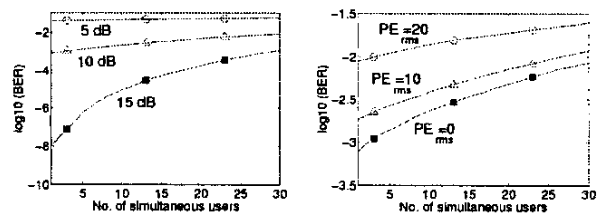
4. Simulations and Results

Monte Carlo simulations were performed with $G = 127$ and $G = 255$. The bit energy w_i of all users was kept equal to 1. It can be observed from Fig. 1 that asynchronous CDMA performs better than synchronous CDMA as the number of simultaneous users increases. Fig. 2(a) demonstrates the impact on the bit error probability as the shot noise variance σ^2 increases. Fig. 2(b) shows the effect on the system performance as rms phase lock error (PE_{rms} given in degrees) increases. It should be noted that the laser phase noise affects the bit error probability by increasing the phase lock error [4]. Keeping the phase lock error constant, the effect of increasing laser phase noise is small.



(a) $G = 127$ (b) $G = 255$

Fig. 1. Comparison of BER in synchronous and asynchronous CDMA($\beta T_c = 0.02,\ \sigma^2 = 0.1,\ \sigma_e^2 = 0$)



(a) Shot noise effects ($\beta T_c = 0.02,\ \sigma_e^2 = 0$) (b) Phase lock error effects ($\beta T_c = 0.02,\ \sigma^2 = 0.1$)

Fig. 2. Effects of shot noise and phase lock error on BER in asynchronous CDMA.

5. Conclusions

The bit error probability of synchronous and asynchronous CDMA in the presence of laser phase drift, shot noise and phase lock error has been derived. It has been shown that asynchronous CDMA performs better than synchronous CDMA. Effects of varying shot noise and phase lock error have been demonstrated.

6. References

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