Impact of Amplitude Phase Correlation on Oscillator Phase Noise Model

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Abstract—In this paper we study how the phase noise model of an LC oscillator changes by taking into consideration the impact of amplitude phase correlations for both white noise and flicker noise. A new circuit based analytical model of phase noise is used for this purpose and the oscillator topology chosen for study is a Differential Oscillator. We compare our results with simulation results. It is found that by taking amplitude correlation into consideration there is a correction in the phase noise model for white noise, which matches simulation results better. For flicker noise the uncorrelated results are closer to the simulation results. The noise results obtained from the correlated model are lower than that obtained from the uncorrelated model. We propose that this occurs due to mutual cancellation between the impacts of amplitude deviation and phase deviation on the overall phase noise.

I. INTRODUCTION

Accurate modeling of phase noise in circuit simulators is very important in order for the designer to have a good idea of the circuit’s performance. Phase noise output of an oscillator is a measure of the total deviation from ideal oscillation. In other words it is a measure of the total power dissipation over a range of frequencies around the frequency of oscillation. To simplify the simulation of phase noise, often only the phase jitter or phase deviation is considered. Commercial simulators like SpectreRF in Cadence follow this approach. However the amplitude deviation also causes dissipation of power. Simulators like HP EESofF (ADS) provide two separate measures of Phase Noise, one due to amplitude deviation and the other due to phase deviation.

In this paper we intend to study the impact of amplitude and phase correlation on the total phase noise for both white and flicker noise i.e. how the amplitude deviation and phase deviation combined impact the Phase Noise. We describe a general circuit based Phase Noise theory in section 3. Further the circuit topology we follow is a differential topology as shown in Fig 2.

II. RELATED WORK

One of the earliest practical Linear Time Invariant (LTI) models of Phase Noise was put forward by Leeson, [3] where he used a combination of circuit based parameters and fitting parameters for the phase noise model. No specific separation between the impact of amplitude deviation and phase deviation on the total phase noise is made in this model due to its empirical nature. Further improvements were made by Edson [4], where he obtains expressions for phase noise due to amplitude deviation and phase deviation separately. Kurokawa [5] obtained equations relating the amplitude and phase deviations though a correlation factor but did not give a complete description of phase noise in terms amplitude deviation component and phase deviation component. Kaertner [6] gives a rigorous expression for phase noise by splitting the oscillator response into phase and magnitude components. Hajimiri [7] developed a Linear Time Variant (LTV) model of phase noise wherein a new function called Impulse sensitivity function (ISF) is introduced. Demir [8] developed a phase noise model wherein he characterizes the oscillator output in terms of orbital deviation (similar to amplitude deviation) and phase deviation. In [1] a circuit-based model of phase noise was developed where in the contribution of amplitude and phase deviations to the total phase noise was rigorously treated.

For flicker noise there is very minimal work done regarding the impact of amplitude-phase correlation, partly due to the mathematical limitations of dealing with flicker noise. For example it is difficult to obtain an inverse Fourier Transform of flicker noise. Demir [9] describes a phase noise model for flicker noise but it does not consider the impact of amplitude deviations on phase noise. In [2] a new comprehensive model of phase noise due to flicker noise was put forward which takes into account the correlation between amplitude and phase deviations. The model assumes that the oscillator deviation from ideal behavior due to flicker noise is equivalent to that due to dc bias perturbation. Further it uses
a trap level model of flicker noise as described in [10] to obtain the phase noise expression due to a single trap, which then is integrated for all traps.

III. SHORT OVERVIEW OF THE CIRCUIT BASED PHASE NOISE MODEL

Figure 1: An Admittance model of an Oscillator

A. White Noise

Fig 1 shows an oscillator model. The basic oscillator has been divided into a linear (frequency sensitive) and a nonlinear or device part, which is both frequency and amplitude sensitive, and has impedance given by \( Y_{2N}(A, \omega) \). In a conventional LC oscillator the linear part usually represents the tank. For all derivations henceforth we shall consider the tank to be a parallel RLC circuit. At equilibrium, in the absence of noise and other perturbations, \( Y_{2N}(\omega_0) = Y_{1}(\omega_0) \) and \( Y_{2N}(A, \omega) = Y_{2N}(A_0, \omega_0) \). \( V(t) \) represents the output voltage of the oscillator. \( i_N(t) \) represents the equivalent noise current flowing across the tank (linear part) and the active part. It has to be noted that \( i_N(t) \) and \( i_{\omega}(t) \) are equivalent autocorrelation function of \( i_N(t) \) and its spectral density is given by \( |i_N(f)|^2 = \delta(f) \). It can be shown that the phase and amplitude deviations noise source, which arises due to noise sources in both the linear and the nonlinear parts of the oscillator circuit. The (\( \varphi \) and \( A \)) obey the Langevin equations (1) & (2), when the oscillator is linearized about its operating point voltage amplitude (\( A_0 \)) and frequency (\( \omega_0 \))[11].

\[
A_0 \beta \frac{dA}{dt} + \frac{d}{dt} \left[ Y_T(\omega_0) i_N(\omega_0) \right]^2 = i_{\omega_2}B_2(\omega_0) + i_{\omega_1}G_2(\omega_0) + i_{\omega_3}G_3(\omega_0)
\]

(1)

\[
A_0 \left[ \frac{dA}{dt} + \frac{dp}{dt} \right] i_N(\omega_0) = i_{\omega_2}B_2(\omega_0) + i_{\omega_1}G_2(\omega_0) + i_{\omega_3}G_3(\omega_0)
\]

(2)

Where

\[
\alpha = G_2(\omega_0)G_{12}(\omega_0, \omega_0) + B_2(\omega_0)B_{22}(\omega_0, \omega_0), \quad \beta = B_1(\omega_0)G_{12}(\omega_0, \omega_0) + G_1(\omega_0)B_{22}(\omega_0, \omega_0), \quad i_{\omega_2} \text{ and } i_{\omega_3} \text{ are two orthogonal components of } i_N(t) \text{ obtained by multiplying } i_N(t) \text{ by } \cos(\omega_0 t) \text{ and } \sin(\omega_0 t) \text{ and then integrating wrt time over a single period of oscillation. From } |i_N(f)|^2 = \frac{1}{2} |\hat{i}_N(f)|^2, \text{ the spectral density of } i_{\omega_2}(t) \text{ and } i_{\omega_3}(t) \text{ can be shown to be given by,}
\]

\[
|\hat{i}_{\omega_2}(f)|^2 = |\hat{i}_{\omega_3}(f)|^2 = \frac{1}{2} |\hat{i}_N(f)|^2
\]

Processes \( \varphi \) and \( A \) are similar to Ornstein-Uhlenbeck processes except that they are correlated. The term accounts for the correlation between \( \varphi \) and \( A \) (i.e. if \( \omega = 0 \) there is no correlation). \( Y_\varphi \) and \( Y_A \) are defined as follows,

\[
Y_\varphi(\omega_0) = \frac{\partial Y_{\varphi}}{\partial \alpha} + \frac{\partial Y_{\varphi}}{\partial \omega} = G_\varphi(\omega_0) + jB_\varphi(\omega_0)
\]

\[\begin{align*}
Y_A(\omega_0, \omega_0) &= \frac{\partial Y_{A}}{\partial \alpha} = G_A(\omega_0, \omega_0) + jB_A(\omega_0, \omega_0) \\
Y_{\varphi} \text{ and } Y_A \text{ represent the variation of the respective impedance with perturbation (noise). If the voltage across the tank is given by } A(t) = A_0 \cos(t + \varphi). \text{ Then the autocorrelation function of the output voltage is given by,}
\end{align*}\]

\[
R_\varphi(r) = \frac{1}{2} \left[ A_0^2 + R_{\omega_0} \right] \cos(\omega_0 r) \exp \left[ - \sigma^2 + R_\varphi(r) \right] \]

(3)

\( \sigma \) represents root square mean of the phase deviation \( \varphi \). \( R_{\omega_0}(r) \) & \( R_\varphi(r) \) can be obtained from the Langevin equations 1 & 2 as follows,

\[
R_{\omega_0}(r) = \frac{|\hat{i}_N|^2}{A_0} \exp \left( - \frac{\beta A_0}{|Y_T(\omega_0)|^2} |r| \right) \]

(4)

\[
R_\varphi(r) = -\frac{|\hat{i}_N|^2}{A_0} \left[ 1 + \frac{\alpha^2}{\beta^2} \right] \exp \left( - \frac{\beta A_0}{|Y_T(\omega_0)|^2} |r| \right)
\]

(5)

Hence,

\[
R_r(r) = \frac{1}{2} \left[ A_0^2 + \frac{|\hat{i}_N|^2}{A_0} \exp \left( - \frac{\beta A_0}{|Y_T(\omega_0)|^2} |r| \right) \right] \cos(\omega_0 r) \]

\[
\exp \left[ - \frac{|\hat{i}_N|^2}{A_0^2 |Y_T(\omega_0)|^2} \left( 1 + \frac{\alpha^2}{\beta^2} \right) |r| - \frac{\beta A_0}{|Y_T(\omega_0)|^2} |r| - 1 \right]
\]

(6)

From which (after taking the Fourier Transform according to Wiener Khintchin theorem) the power spectral density of output voltage can be approximated for large frequency offsets as [10],

\[
S_r(\omega) = \frac{A_0^2}{2 \left[ m_1^2 + (\omega - \omega_0)^2 \right] + \frac{c}{2} \left[ m_1 + m_2 \right]} \left[ m_1 + m_2 \right] \left[ (\omega + \omega_0)^2 \right]
\]

(7.1)

Where,

\[
m_1 = \frac{|\hat{i}_N|^2}{A_0^2 |Y_T(\omega_0)|^2} \quad \text{and} \quad m_2 = \frac{A_0^2 |\hat{i}_N|^2}{Y_T(\omega_0)^2} \quad \text{and} \quad c = \frac{|\hat{i}_N|^2}{A_0^2}
\]

Whereas for small frequency offsets the value of \( m_1 \) changes to,
\[ m_1 = \frac{|\alpha|^2}{A_2^2 |Y'_{\omega_0}|^2} \left( 1 + \alpha^2 \right) \]

A true value of \( S_r \) can only be obtained by a numerical evaluation of the Fourier Transform of \( R_n \). If the impact of amplitude phase correlation is not considered (i.e. if \( \alpha = 0 \)) then the expression for \( S_r \) will reduce to,

\[ S_r(\omega) = \frac{A_2^2}{2m_1^2 + (\omega - \omega_0)^2} \] (7.2)

Where, \( m_1 = \frac{|\alpha|^2}{A_2^2 |Y'_{\omega_0}|^2} \)

\[ B. \text{ Flicker Noise} \]

We proceed from Equations (1) and (2) above. For flicker noise \( i_{m1}(t) \) and \( i_{m2}(t) \) are correlated [5]. Since it is difficult to obtain the correlation coefficient between them, an alternate way of analysis is to assume that the perturbation of oscillation due to flicker noise is equivalent to that produced by a perturbation in the dc bias of the device part of the circuit (i.e. a perturbation in \( Y_{\omega_0} \)). The change in \( G_{\omega_0} \) and \( B_{\omega_0} \) due to \( I/f \) noise can be obtained by a Taylor series expansion given by,

\[ \Delta G_{\omega_0} = \frac{\partial G_{\omega_0}}{\partial i_N} \Delta i_N = G_{m1}^{i_N} i_N \]

\[ \Delta B_{\omega_0} = \frac{\partial B_{\omega_0}}{\partial i_N} \Delta i_N = B_{m1}^{i_N} i_N \]

Here, \( \Delta i_N \) has been substituted by \( i_N \) because \( i_N = 0 \) at \( t = 0 \). The orthogonal components \( i_{m1}(t) \) and \( i_{m2}(t) \) can hence be represented by,

\[ i_{m1} = \frac{\Delta G_{\omega_0}}{A_0} \]

\[ i_{m2} = \frac{\Delta B_{\omega_0}}{A_0} \]

\( G_{m1}^{i_N} \) & \( B_{m1}^{i_N} \) represent the variation in \( G_{\omega_0} \) & \( B_{\omega_0} \) respectively due to dc bias. Thus effectively we have expressed the noise components \( i_{m1} \) and \( i_{m2} \) by the variation of conductance and susceptance due to change in dc bias \( (G_{m1}^{i_N} \text{ & } B_{m1}^{i_N}) \). Substituting these values in Eqs 1 and 2 gives,

\[ \frac{Y'_{\omega}(\omega_0)}{A_0} \frac{d\delta i_N}{dt} + \beta \delta i_N = Bi_N \] (8)

\[ \frac{Y'_{\omega}(\omega_0)}{A_0} \frac{d\alpha \delta A}{dt} - \alpha A = -Ai_N \] (9)

Where,

\[ B = G'_{\omega}(\omega_0)B_{m1}^{i_N} (A_0, \omega_0) - B'_{\omega}(\omega_0)G_{m1}^{i_N} (A_0, \omega_0) \]

\[ A = G'_{\omega}(\omega_0)G_{m1}^{i_N} (A_0, \omega_0) - B'_{\omega}(\omega_0)B_{m1}^{i_N} (A_0, \omega_0) \]

To obtain \( R_{\omega} \) & \( R_{p} \), a charge trapping model of Flicker Noise is used [10], wherein the autocorrelation function of the noise produced by a single trap, assumed to be Wide State Stationary (WSS), is given by,

\[ R_{\omega}(\tau) = k e^{-\lambda \tau} \]

\( \lambda \) and \( k \) are constants depending on the physical structure of the trap. It can be shown that the spectral density of such a noise when averaged over all traps (i.e. for all \( \lambda \)) gives us a \( 1/f \) distribution, thereby justifying the assumption. The autocorrelation function of the phase and amplitude deviations can then be obtained from Eqn 8 and 9 as,

\[ R_{\omega}(\tau) = K_2 \frac{e^{-\sqrt{k_4}} - e^{-\sqrt{k_5}}}{2(k_5 - k_4)} \] (10)

\[ R_p(\tau) = \frac{K_2}{2(k_5 - k_4)} \left[ \frac{e^{-\sqrt{k_4}}}{\sqrt{k_4}} - \frac{e^{-\sqrt{k_5}}}{\sqrt{k_5}} \right] \] (11)

Where,

\[ K_1 = 2\lambda k x^2, K_2 = \frac{A}{|Y'_{\omega}(\omega_0)|^2}, K_3 = 2B^2 \lambda k A^2 \]

\[ k_1 = k_2, k_3 = k_4, k_5 = k_6, k_7 = k_8, k_9 = k_10 \]

\[ n_1 = (k_1 - k_2), n_2 = (k_2 - k_3), n_3 = (k_3 - k_5), n_4 = (k_4 - k_6) \]

\[ n_1 = k_4 - k_5, n_2 = k_6 - k_7, n_3 = k_8 - k_9, n_4 = k_10 - k_11 \]

Substituting these values in Eqn (3) we get the following very long expression for the autocorrelation function of noise voltage due to a single trap,

\[ \frac{A_2^2}{2} \cos(\omega_0 \tau) \exp \left[ -\frac{K_2}{2}(n_1 |\tau|) \right] + \frac{n_2}{\sqrt{k_2}} \left( e^{-\sqrt{k_5}} - 1 \right) + \frac{n_3}{\sqrt{k_3}} \left( e^{-\sqrt{k_5}} - 1 \right) + \frac{n_4}{\sqrt{k_4}} \left( e^{-\sqrt{k_5}} - 1 \right) - \sqrt{k_4} \left( e^{-\sqrt{k_5}} - 1 \right) - \sqrt{k_5} \left( e^{-\sqrt{k_5}} - 1 \right) \]

The noise density of the output voltage can be obtained by taking the Fourier Transform of the above expression and then integrating the expression so obtained for all traps (i.e. for all \( \lambda \)),

\[ S_v(\omega) = \frac{A_2^2 \kappa^2 S(1)}{4} \left[ \Delta \omega \left( k_5 + \Delta \omega \right) \right] + \frac{B_2 A_2^2}{4} \left[ \delta \omega \left( k_4 + \Delta \omega \right) \right] \]
where, $S(1)$ is the noise density of the noise current $i_N(t)$ at unit frequency. If the impact of amplitude phase correlation is not considered (i.e. if $i^2 = 0$), the expression for $S_Y$ reduces to,

$$S_Y(\omega) = \frac{A_2^0 A_2^2 S(1)}{4\Delta \omega^2} + \frac{B^2 A_2^2}{4Y_i(\omega_0)} \left( k_i + \Delta \omega^2 \right) \Delta \omega$$

(14)

IV. APPLYING THE MODEL TO A DIFFERENTIAL OSCILLATOR

A. White Noise

A Differential Oscillator as shown in Fig 2, lets us easily apply this theory since there is a clear division of the circuit into a linear part (tank representing $Y_L$) and the device part (the rest of the circuit containing the transistors representing $Y_{IN}$). The location of $i_N$ is shown in Fig (2) along with the various transistor noise sources. It can be shown [13], that the total equivalent white noise appearing across the tank $i_N(t)$ is given by, $|e|^2 = \bar{i}_n^2 = \frac{1}{2}(\bar{i}_N^2 + \bar{i}_N^2)$.

Here, $\bar{i}_n = \bar{i}_n^1 = \bar{i}_n^2 \text{ and } \bar{i}_N = \bar{i}_N^1 = \bar{i}_N^2$.

Substituting these expressions in Eqns (7.1) and (7.2) will give us the necessary values for comparison.

![Figure 2: A Differential Oscillator showing the various Transistor noise sources](image)

B. Flicker Noise

It is difficult to obtain an exact expression of $i_N(t)$ for flicker noise. However as shown in [12], the instantaneous noise produced by the core transistors (the transistors excluding the tail transistor) can be split into uncorrelated modes as shown in Fig 3. Of these the Modes A and C are most effective in upconverting noise produced by the transistors, due to the mixer like action taking place when the noise enters the oscillator core as shown in Fig 3. Tail noise also is translated to across the tank mostly by upconversion. So the overall flicker noise $i_N$ will be the sum of that produced across the tank by each of modes A and C and the tail noise source alone (which we refer to as mode E). Hence from Eqns (13), for the correlated case,

$$S_Y(\omega) = \frac{A_2^0 A_2^2 S(1)}{4\Delta \omega^2} + \frac{k_i + \Delta \omega^2}{4Y_i(\omega_0)} \left( k_i + \Delta \omega^2 \right) \Delta \omega$$

(14)

Where, $k_i$ represents the values of $k_i$ computed with the values of $V_{IN}$ for the modes A, C and E. While computing the values of $V_{IN}$ for modes A and C, the direction and positions of the dc perturbations used to calculate the deviation in $Y_{IN}$ will be the same as that shown in Fig 3 for modes A and C. Computation of $V_{IN}$ for mode E (tail noise) is done by switching all other noise sources off and the seeing the change in $V_{IN}$ due to a dc perturbation across the tail transistor. The results will also change accordingly for the uncorrelated case (Eqn 14).

![Figure 3: Oscillator Modes](image)

V. SIMULATION RESULTS AND CONCLUSIONS

The Phase Noise model developed for white and flicker noise sources and applied on a Differential Oscillator has been plotted in Fig 4 and compared with simulation results obtained using HP ADS. A BSIM3 model of transistors was used for the NMOS and PMOS transistors. As we can see from Fig 4 the Lorentzian nature of the curve has a corner at $\Delta f = \Delta \omega / 2\pi = f_m / 2\pi$. The correlated results deviate from the uncorrelated results by about 4 dB. In addition the correlated results show a better match with simulation results obtained from HPADS. The simulation results show good...
match with the phase noise model when the impact of $\Delta A$ is included. On the other hand as seen from Figure (5), the simulation results deviate from the correlated model by about 8 dB, and are closer to the uncorrelated results.

Hence in conclusion, we see that the impact of amplitude phase correlation is more prominent for the flicker noise case than for the white noise case. By taking into account this correlation there is actually a decrease in the noise. This results from mutual cancellation of noise between that due to amplitude deviation and that due to phase deviation.

VI. ACKNOWLEDGMENTS

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