Steady State Performance of Polyphase Permanent Magnet Synchronous Motor Fed From Single Phase Supply System

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Abstract: The operation of a three phase induction motor from a single phase supply with a capacitor in circuit is well known. While, detailed investigations on induction motor performance under such operating condition are reported in the literature, no such work is reported for synchronous motor. This paper presents analytical and experimental evaluation of the performance of three phase permanent magnet synchronous motor (PMSM) fed from a single phase supply. A capacitor is connected across two of stator phases during starting and running. Symmetrical component concept is used for the analysis. Estimation of capacitor value to meet the desired objective such as minimum unbalance or maximum efficiency is carried out. Experimental load tests are conducted on the machine, which validate the simulation results.

Keywords: Permanent Magnet, Synchronous Motor.

I. INTRODUCTION

The polyphase line start permanent magnet synchronous motor (LSPMSM) has been successfully demonstrated in various sizes as a high efficiency alternative to the induction motor. Its technical and economical feasibility is well tested in literature. Different issues such as its design, performance evaluation, parameter estimation, FEM Modeling, starting performance, transient and stability analysis are widely reported in the literature. These motors may become an attractive choice in India where cost of electrical energy is sharply increasing and efficient utilization of electrical power is needed due to acute shortage of power in some areas.

In some practical situations it becomes necessary to use three-phase motors on a single-phase supply system. Rural electrification in remote and hilly regions, where, single wire earth return systems are installed is one of the examples. The load in such cases is mainly irrigation pump sets, which may use single phase or three-phase motors with phase converters. The later has the advantage that it can be directly used when the three-phase supply is installed in future. Similarly, single phase synchronous motors, whatever be the type, are almost sub-fractional or fractional horsepower motors. Three-phase PMSM run on single phase supply can be the solution where both synchronous speed and high ratings are essential. In view of these, it is necessary to test the performance of three-phase PMSM when operated from single-phase supply.

This paper deals with performance evaluation of three-phase PMSM run on single phase supply. The paper presents the analytical and experimental evaluation of performance characteristics for a three-phase PMSM fed from a single phase supply. Two value capacitor is used, one for starting and one for running. Capacitor value is estimated to obtain desired performance characteristics. The performance of the machine is compared with fixed and variable capacitor. Load tests are carried out with fixed value of capacitor and the results are analyzed.

II. ANALYSIS

A. Condition For Balanced Operation

It is possible to obtain a three phase balanced voltages provided certain conditions are met. Let all the phases, in Fig.1a, have total impedance $Z_1$ per phase and impedance of phase balancer be $Z_c$. Assuming loss-less phase balancer,

$$Z_{BN} = \frac{Z_1(Z_1 + Z_c)}{2Z_1 + Z_c}$$

$$Z_{RB} = \frac{Z_1(Z_1 + Z_c)}{2Z_1 + Z_c}$$

If $V \sin \omega t$ is the supply voltage, then the current $I$ is given by

$$I = \frac{V(2Z_1 + Z_c)}{3Z_1^2 + 2Z_1Z_c}$$

Various phase currents are:

$$I_R = I, \quad I_B = I \frac{Z_1 + Z_c}{2Z_1 + Z_c} \quad \text{and} \quad I_Y = I \frac{Z_1}{2Z_1 + Z_c}$$

For balanced condition, phase currents should be equal. Hence,

$$I = \left| I \frac{Z_1 + Z_c}{2Z_1 + Z_c} \right| = \left| I \frac{Z_1}{2Z_1 + Z_c} \right|$$

$$\therefore \quad Z_1 + Z_c = 2Z_1 + Z_c$$

i.e. $R + jX = jX_c = \sqrt{\frac{2R + 2jX - jX_c}{2jX}}$

$$\therefore X_c = \frac{3(R^2 + X^2)}{2X}$$
Also, the phase difference between the current phasors should be 120°.

For branch NYB, \( \tan \phi_1 = \frac{X - X_C}{R} \)

For branch NB, \( \tan \phi_2 = \frac{X}{R} \)

\[
\tan(\phi_1 - \phi_2) = \frac{X_C/X}{1 + X^2/R^2 - XX_c/R^2}
\]

\( X_C/R = -\sqrt{3} (1 + X^2/R^2 - XX_c/R^2) \) \( (7) \)

From (6) & (7) conditions obtained for balanced operation are,

\[
X/R = \sqrt{3} \quad \text{and} \quad X_c = 2X
\]

However, these conditions are rarely met in practice. The values of X and R are load dependent. Thus, balance operation for entire load range may not be possible even if the capacitor value is varied continuously. Hence, stator is subjected to unbalanced excitation and symmetrical component theory can be used to study the performance of machine.

### B. Steady State Analysis

![Connection Diagram](image)

For the connections shown in Fig.1b, the zero, positive and negative sequence components of capacitive reactance will be \(-jX_c/3\). Since, the applied voltages are \( V_{ab} = 0, V_{bc} = V \) & \( V_{ca} = -V \), sequence voltages become,

\[
V_{a1} = \frac{1}{3}[0 + aV - a^2V] = \frac{jV}{\sqrt{3}}
\]

\[
V_{a2} = \frac{1}{3}[0 + a^2V - aV] = -\frac{jV}{\sqrt{3}}
\]

The sequence voltages for phase R are,

\[
V_a = V_{a1} = \frac{V}{\sqrt{3}} \quad \text{and} \quad V_a = V_{a2} = -\frac{V}{\sqrt{3}}
\]

The voltage equations for phase ‘R’ can be written as

\[
V_a = I_p(Z_p - jX_c/3) - jX_c/3 I_n
\]

\[
V_a = I_n(Z_n - jX_c/3) - jX_c/3 I_p
\]

where, \( Z_p \) and \( Z_n \) are positive and negative sequence impedances of phase R, respectively. Sequence currents can be obtained by solving (11) and (12) as,

\[
I_p = \frac{\frac{V}{3}(Z_{n} \leq 60 - \frac{X_c}{\sqrt{3}})}{Z_p Z_n - j \frac{X_c}{3} (Z_p + Z_n)}
\]

\[
I_n = \frac{\frac{V}{3}(Z_{p} \leq 60 - \frac{X_c}{\sqrt{3}})}{Z_p Z_n - j \frac{X_c}{3} (Z_p + Z_n)}
\]

The current in phase ‘Y’ is the line current. It is given by,

\[
I_c = I_{e1} + I_{e2} = a I_p + a^2 I_n
\]

\[
I_c = \frac{3}{3} Z_p Z_n - j X_c (Z_p + Z_n)
\]

The current unbalance factor can be calculated using (13) and (14). The sequence voltages are given by,

\[
V_p \equiv I_p Z_p = \frac{\frac{V}{3}(Z_{n} \leq 60 - \frac{X_c}{\sqrt{3}})}{Z_p Z_n - j \frac{X_c}{3} (Z_p + Z_n)}
\]

\[
V_n \equiv I_n Z_n = \frac{\frac{V}{3}(Z_{p} \leq 60 - \frac{X_c}{\sqrt{3}})}{Z_p Z_n - j \frac{X_c}{3} (Z_p + Z_n)}
\]

Using (17) & (18), voltage unbalance factor can be determined. After some algebraic manipulations, the expressions for sequence voltages can be written as,

\[
v_p = \frac{\sqrt{3}}{\sqrt{3}} Y_{30} + 30 + Y_{30} + Y_{30} = \frac{\sqrt{3}}{3} Y_{30} + 30 + Y_{30} + Y_{30}
\]

\[
v_n = \frac{\sqrt{3}}{\sqrt{3}} Y_{30} - 30 + Y_{30} - 30 + Y_{30} - 30 = \frac{\sqrt{3}}{3} Y_{30} - 30 + Y_{30} - 30 + Y_{30} - 30
\]

where \( Y_1 \) and \( Y_2 \) are admittances of phase balancer, positive sequence and negative sequence respectively. These are similar to those given in [3-5]. Once the sequence voltages and impedances are known, synchronous performance can be easily evaluated. The sequence impedances are calculated in the following manner.

The positive sequence impedance is calculated using the positive sequence phasor diagram of the PMSM, which is shown in Fig.2. It consists of a torque and excitation dependent resistance and reactance. The negative sequence impedance is calculated in a manner similar to that of induction motor. However, the only difference is that for induction motor, the ‘d’ and ‘q’ axis magnetic circuits are similar. In PMSM the presence of magnets inside the rotor makes these circuits magnetically different. There are various ways to represent negative seq. impedance of such machine[5]. In this paper the averaging of ‘d’ and ‘q’ axis
circuits is done to obtain negative sequence impedance as (22)
\[
Z_n = R_x + jX_x = \frac{X_{mn} \cot \delta - \frac{X_{m}^2}{X_{mn}} \cot \theta}{1 + \frac{X_{m}^2}{X_{mn}} \cot^2 \theta} + j \frac{X_{m}^2 \cot \delta + \frac{X_{m}^2}{X_{mn}} \cot \theta}{1 + \frac{X_{m}^2}{X_{mn}} \cot^2 \theta}
\]
where \( \theta = \tan^{-1} \left[ \frac{E_n \sin \delta}{E_n \cos \delta - E_0} \right] \) (21)

Thus, knowing the sequence voltages and impedances, sequence currents can be calculated.

### C. Choice of Phase Balancer

The capacitor required for both starting and running conditions can be obtained in number of ways. The capacitor during starting can be chosen for getting maximum starting torque or maximum starting torque/amp ratio, where as, for steady state operation, its selection can be based on minimum or zero unbalance, maximum efficiency, maximum power factor, maximum torque. It can also be optimized for best overall performance. The analytical expressions to find out capacitor value to meet different objectives are derived. The analysis is approximate as the parameters used are assumed to be constant.

**For perfect balanced condition:**

In order to get perfect balanced operation, \( I_n \) must be zero. Thus, \( X_c / \sqrt{3} = Z_p \leq 60 \)

This implies that, \( X_p / R_p = \sqrt{3} \) and \( X_c = 2X_p \).

**For minimum negative sequence current:**

\[
I_n = \left| 1 \right| \left[ \begin{array}{c}
\frac{Z_p^2 + X_c^2 - 2X_c Z_p \sin(\phi_n - 60)}{\sqrt{3} Z_p^2 + X_c^2 + 2X_c Z_p \cos \phi_n - 2X_c Z_p \cos(\phi_n - 60)}
\end{array} \right]^{1/2}
\]

Differentiating \( I_n \) with respect to \( X_c \) and equating to zero yields a second order equation of the form

\[
AX_c^2 + BX_c + C = 0
\]

where,
\[
A = \frac{2}{9} Z_p^2 Z_n \cos \phi_n - \frac{2}{9} Z_p^2 \cos \phi_p - \frac{2}{3} Z^n \sin(\phi_p - 60)
\]
\[
B = \frac{2}{9} Z_p^2 Z_n \sin(\phi_p - 60) - \frac{4}{9 \sqrt{3}} Z^n Z_p \cos \phi_p \sin(\phi_p - 60)
\]
\[
C = \frac{2}{3} Z_p Z_n \sin(\phi_p - 60) + \frac{2}{5} Z^n Z_p \cos \phi_n + \frac{2}{5} Z^n Z_p \cos \phi_p
\]

Since all parameters are known, value of \( X_c \) for minimum negative sequence current can be calculated.

**For maximum \((I_p/I_n)\) condition:**

The ratio between positive and negative sequence currents is given by,
\[
\frac{I_p}{I_n} = \frac{Z_n \leq 60 + X_c}{\sqrt{3}} \frac{1}{Z_p \leq 60 - X_c}
\]

(24)

This yields,
\[
U = \left[ \frac{I_p}{I_n} \right] = \left[ \begin{array}{c}
\frac{Z_p^2 + X_c^2 - 2X_c Z_p \sin(\phi_n - 60)}{\sqrt{3} Z_p^2 + X_c^2 + 2X_p Z_p \cos \phi_n - 2X_c Z_p \cos(\phi_n - 60)}
\end{array} \right]^{1/2}
\]

Differentiating \( U \) with respect to \( X_c \) and equating to zero gives second order equation of the form

\[
AX_c^2 + BX_c + C = 0
\]

where,
\[
A = \frac{2}{3} Z_p \sin(\phi_p - 60) + \frac{2}{3} Z_p \sin(\phi_p + 60)
\]
\[
B = \frac{2}{3} Z_p - \frac{2}{3} Z_p \cos \phi_p
\]
\[
C = \frac{Z_p^2 Z_n \sin(\phi_p - 60) - Z_p^2 Z_n \sin(\phi_p - 60)}{\sqrt{3}}
\]

The roots of the equation give the value of \( X_c \) for maximum \( U \). For maximum efficiency:
\[
\eta = \frac{P_{\text{output}}}{P_{\text{output}}} = \frac{I_p R_p - I_n^2 R_n}{U^2 R_p - R_n}
\]

(26)

Then,
\[
\frac{d\eta}{dX_c} = \frac{dU}{dX_c} \times \frac{dU}{dX_c}
\]
Thus, maximum efficiency occurs at a specific value of 'Xc', resulting, maximum ratio of positive sequence to negative sequence current. Though, the constant losses (mechanical and iron) are not included in the output expression, it is found that the deviation between Xc value corresponding to maximum efficiency and maximum U is negligible.

III. SIMULATION RESULTS

In view of complexity involved, in predicting the value of phase balancing capacitor and finding out the steady state performance of the machine with the selected capacitor, simulation studies are carried out using MATLAB. The rating of the machine whose parameters are used for simulation are 1.67 hp, 4 Pole. 192 V, 5.3 A, 1500 rpm. 3 phase PMSM. The machine parameters are R1=1.17 Ω, Xc=1.74 Ω, Xr=25.8 Ω, Xe=9 Ω, Xe2=1.75 Ω and Excitation Voltage (Phase) =62 V. The values of capacitor are found out for different load angles to satisfy various criteria. The variation of capacitance for different load angles for minimum CUF (i.e. maximum U), maximum efficiency, maximum power factor, minimum negative sequence current, and maximum torque are given in Fig.3. It is observed that the value of capacitance required to obtain minimum CUF and maximum efficiency is almost the same. Similarly, the size of capacitor required, for minimum negative sequence current, is closer to that required for obtaining minimum CUF. The variations in capacitance needed for different load angles in normal working range, is not much except for max. torque criteria. The capacitor values required to obtain maximum torque are significantly larger compared to those obtained for other criteria. Also, a wide variation in its value is needed, to obtain maximum torque at different load angles. This rules out the selection of capacitor for maximum torque condition.

In order to check whether continuous variation of the capacitor with load angle is necessary or not, the performance of the machine is evaluated using fixed capacitor. It is compared with the performance obtained when capacitor is continuously varied. The capacitor value is chosen such that the phase current does not exceed 110% of rated current. The value of capacitor required is 62 μF and 75 μF for maximum efficiency and maximum power factor conditions respectively. The variation in efficiency with load angle are shown in Fig.4, when capacitor is 62μF. The results show that the variation of capacitor is not required for the machine if any one of the three conditions namely minimum unbalance, maximum efficiency or maximum power factor is chosen. Machine performance is obtained for different values of fixed capacitor and compared with the performance of a balanced 3 phase machine. The variations of few performance indices are shown in Fig.5-6. Efficiency at any load angle for any value of capacitor is less than that at corresponding load angle in case of balanced three phase operation. The negative seq. current at a particular load angle minimum for 62μF capacitor. From the above simulation studies, maximum permissible load (For which the current does not exceed 110% of rated current) is given in Table.1.

Table 1: Simulation Results For Fixed Capacitors

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Case</th>
<th>Rating</th>
<th>Efficiency</th>
<th>Tmax (Max.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Balanced</td>
<td>100</td>
<td>86.32</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>with 40μF</td>
<td>52.14</td>
<td>82.32</td>
<td>56.41</td>
</tr>
<tr>
<td>3</td>
<td>with 50μF</td>
<td>58.08</td>
<td>83.19</td>
<td>58.33</td>
</tr>
<tr>
<td>4</td>
<td>with 62μF</td>
<td>64.89</td>
<td>84.37</td>
<td>60.76</td>
</tr>
<tr>
<td>5</td>
<td>with 75μF</td>
<td>68.56</td>
<td>83.95</td>
<td>63.52</td>
</tr>
</tbody>
</table>

IV. EXPERIMENTAL RESULTS

Load tests are carried out for six different values of capacitors. The experimental results are given in Fig.7-9. As seen from the Fig.7, the efficiency obtained is higher with the capacitor of 62μF. The efficiency for the load angle range of 60-90° is very closer to the efficiency of a balanced three phase machine. For lower load angles, however, there is significant drop in efficiency when operated from single phase supply. The power factor of the machine when operated from single phase supply is higher compared to that in the case of balanced three phase operation. This power factor increases with the increase in capacitor. This feature is observed in Fig.8. For all the cases, the variation in developed torque is shown in Fig.9. With 40μF capacitor machine develops almost 63 % of the rated torque while with 100μF it develops 76%. Table [2] shows the maximum torque developed by the machine required derating and corresponding efficiency for above cases.
Table 2: Experimental Results For Fixed Capacitors

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Case</th>
<th>Rating %</th>
<th>Efficiency %</th>
<th>$\frac{T_{\text{max}}}{T_{\text{max}[\text{Bal.}]}} \times 100%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Balanced Operation</td>
<td>100</td>
<td>85.3</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>40uF</td>
<td>63.88</td>
<td>82.32</td>
<td>63.74</td>
</tr>
<tr>
<td>3</td>
<td>50uF</td>
<td>65.55</td>
<td>83.19</td>
<td>68.70</td>
</tr>
<tr>
<td>4</td>
<td>62uF</td>
<td>66.67</td>
<td>84.37</td>
<td>69.46</td>
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<tr>
<td>5</td>
<td>75uF</td>
<td>67.77</td>
<td>83.95</td>
<td>72.51</td>
</tr>
</tbody>
</table>

V. CONCLUSION

Simulation and experimental studies are carried out on the operation of three phase machine on single phase supply. Expressions are derived for determining optimum size of the capacitor to obtain minimum current unbalance factor, minimum negative sequence current, maximum efficiency, maximum power factor and maximum torque conditions. It is observed that, capacitor should be selected either for maximum efficiency or for maximum power factor. In these cases, variation of capacitor with load is not required and a fixed value of capacitor can be employed. Simulation and experimental results confirm that, on single phase supply, machine rating can be as high as 70% of the balanced rating.

V. REFERENCES

VI BIOGRAPHIES

B.N. Chauhadi was born in Jalgaon, India on 13 Sept. 1967. He received B.E. Electrical and M.E. Electrical degree from Dr. B.A. Marathwada University at Aurangabad, in 1987 and 1989 respectively. He has joined Government College of Engineering Aurangabad in March 1990 as faculty in Electrical Engineering. Presently he is pursuing Ph.D. program at Indian institute of Technology, Mumbai. At presents he holds the post of Assistant Professor at Government College of Engineering Aurangabad.