Optimal Objective Functional Selection for Image Reconstruction in Diffuse Optical Tomography

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Abstract

Diffused Optical Tomography (DOT) is a powerful noninvasive functional imaging technique. Interparameter crosstalk and near source detector artifacts are major source of inaccuracies in DOT images. In this work we investigate the effect of various objective functional definitions and measurement types on performance of image reconstruction algorithm. Special attention is paid to measurement data types appropriate for handling experimental limitations and inaccuracies. We propose a method of selecting optimal objective functional by visualizing objective functionals in two parameter space using single inclusion DOT problem. Using our method we synthesize a new objective functional for our sample DOT problem. The proposed objective functionals provide minimum inter-parameter crosstalk with negligible near source detector artifacts. Limited memory quasi-Newtonian algorithm is used for image reconstruction. Synthetic data is used to demonstrate effect of various objective functionals on image reconstruction and the superiority of the proposed objective functional.

1. Introduction

Diffused Optical Tomography (DOT) is emerging as a low cost noninvasive functional imaging technique. Objective of DOT image reconstruction is to reconstruct spatio-temporal map (image) of optical properties in the region of interest. Various medical applications of the DOT are proposed in literature like, brain functional studies [1], breast cancer [2], study of bones and joints [3] and, muscle metabolism [4]. The DOT Images are reconstructed from a set of optical measurements called measurement set. The measurements are taken on region boundary with a source of NIR (Near-InfraRed) light at known position on the boundary. Multi source, multi detector configuration is used to increase information content of the measurement set. Limited measurement data, inhomogeneous nature of tissue, and nonlinear absorptive diffused light transport process make DOT image reconstruction a nonlinear ill posed inverse problem.

The ill-posedness of DOT gives rise to image reconstruction artifacts which can create ambiguity in image interpretation. It is essential to develop DOT image reconstruction techniques which minimize the artifacts to such an extent that it can be used as a routine clinical diagnostic technique. Two commonly seen artifacts in DOT are 1) absorption-scattering crosstalk and, 2) Near source detector artifacts. Both of these artifacts create ambiguity in DOT images. We address the issue of removing both of these artifacts using appropriate objective functional and measurement type. Our work is mainly on frequency domain DOT where the source intensity is sinusoidaly modulated (at 100-200MHz) and, phase and amplitude of the optical signal exiting from boundary are measured by the detection system. Various groups have reported details of the frequency domain DOT instrumentation [5]. In this paper we deal only with image reconstruction algorithm. The key contribution of the paper is proposing a new method for selecting optimal objective functional that provides minimum inter-parameter crosstalk with negligible source detector artifacts. Moreover, the objective functional selected by proposed method entails superior reconstructed images as compared to images reconstructed using the existing objective functionals.

2. Image Reconstruction algorithm

Iterative algorithms are most widely used for image reconstruction in DOT [6, 7, 8]. These algorithms iteratively use a model of light transport called the forward model. The forward model is used to compute measurement set for an arbitrary image. Closeness of arbitrary image from the solution image is quantified using an objective functional which is a metric on computed and experimentally observed measurement sets. An iterative scheme is set up to generate ‘guesses’ of images, if the proper convergence is provided by the
algorithm, closeness of guess images with actual image increases as number of iterations increase.

2.1 Forward model

Most widely used forward model for the frequency domain optical tomography is the diffusion equation in frequency domain [9].

\[-\nabla \cdot D \nabla \Phi(r,w) + \left( \frac{\mu_a + \frac{i\omega \mu_s}{c}}{c} \right) \Phi(r,w) = q_0(r,w) \quad ... (1)\]

Where, \( \Phi \) is the complex optical fluence at the position \( r \) and the modulation frequency \( \omega \); the diffusion coefficient \( D = \frac{3}{2} \left( \mu_a(r) + \mu_s'(r) \right) \); \( \mu_a(r) \) is the absorption coefficient; \( \mu_s'(r) \) is the reduced scattering coefficient; \( c \) is the velocity of light in the medium. \( q_0 \) is the source function. We use the finite element method (FEM) to solve eq. 1 in two dimensional region with Robin boundary condition. Optical properties are approximated using piecewise linear basis functions \( u_i \) as,

\[ \mu_a(r) = \sum_{i=1}^{N} \mu_{a,i} u_i(r) \quad \text{and} \quad \mu_s'(r) = \sum_{i=1}^{N} \mu_{s,i}' u_i(r) \]

The absorption and reduced scattering coefficient image vectors for FEM mesh with \( N \) nodes are \( \mu_a = [\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,N}] \) and \( \mu_s' = [\mu_{s,1}', \mu_{s,2}', ..., \mu_{s,N}'] \). Let \( m^s = [m_{s,1}, m_{s,2}, ..., m_{s,N}] \) be vector of the computed measurement set for \( d \) detectors and \( s \) sources. A measurement is outward normal component of the complex valued optical flux at detector position on the boundary. This forms the forward model \( F: [\mu_a, \mu_s'] \rightarrow m^s \) for our reconstruction algorithm.

2.2 Measurement data types

Main measurement data types in the frequency domain DOT are AC amplitude, DC amplitude and, phase. Measurement of the absolute values of these measurement types is not always feasible due to experimental limitations. Normalized measurements are proposed to avoid this problem [10]. We propose two types of normalized measurement sets as, 1) measurement set normalized by one of the measurements and, 2) measurement set normalization by the average of the measurement set. The normalization scheme also eliminates requirement of finding source strength which further requires knowledge of parameters like refractive index mismatch at the boundary and scattering coefficient [11]. Both of these parameters are unknown and are not feasible to measure. Measurement types of interest in our analysis are 1) the complex flux \( m \), 2) the AC amplitude \( Mod(m) \) and, 3) the phase \( Arg(m) \). DC amplitude is neglected in our analysis.

2.3 Objective Functional

The DOT image reconstruction problem is formulated as an optimization problem where, an objective functional is minimized for image parameters. As given by eq. 1 in case of frequency domain DOT the optical fluence is a complex valued function. Other measurement types are derived form the complex fluence. We define the objective functional \( f \) as squared norm based metric for complex or real valued measurement sets. Definitions of various objective functionals are.

\[ f^s = \left\| \text{Mod}(m^s) - \text{Mod}(m^*) \right\| \]
\[ f^s = \left\| \ln \left( \text{Mod}(m^*) \right) \right\| \]
\[ f^s = \left\| \text{Arg}(m^s) - \text{Arg}(m^*) \right\| \]
\[ f^s = \left\| (m^s - m^*) \right\| \]
\[ f^s = \left\| \ln (m^s) - \ln (m^*) \right\| \]

where, \( m^s = [m_{s,1}, m_{s,2}, ..., m_{s,N}] \) is the observed (experimental) measurement set with \( s \) sources and \( d \) detectors. We refer above objective functionals as the basic objective functional types.

2.4 Limited memory quasi-Newtonian (lm-QN) method

Lm-QN algorithm is reported as a feasible quasi-Newtonian technique for DOT [12]. We use similar notations as used in [12]. Update formula for the lm-QN technique is

\[ \mu^{k+1} = \mu^k + \Delta \mu^k \quad ...(3) \]
\[ \Delta \mu^k = \alpha u^k \quad ...(4) \]

where, \( \mu^k = [\mu_{a,k}, \mu_{s,k}'] \) is the \( k \)th parameter vector; \( \Delta \mu^k \) is the \( k \)th image update vector; \( \alpha \) is the search length
obtained by line search; \( u^k \) is the \( k^{th} \) search direction obtained as,

\[
u^k = -\nabla_{\mu} f(\mu^k) + \gamma s^k + \lambda y^k
\]

where, \( y^k = \nabla_{\mu} f(\mu^{k+1}) - \nabla_{\mu} f(\mu^k), s^k = \mu^{k+1} - \mu^k \), \( \lambda = \frac{(s^k)^T \nabla_{\mu} f(\mu^{k+1})}{(s^k)^T y^k} \) and,

\[
\gamma = \left(1 + \frac{(y^k)^T y^k}{(s^k)^T y^k}\right) \frac{(s^k)^T \nabla_{\mu} f(\mu^{k+1})}{(s^k)^T y^k} + \frac{(y^k)^T \nabla_{\mu} f(\mu^{k+1})}{(s^k)^T y^k}
\]

\( \nabla_{\mu} f(\mu^k) \) is gradient of the objective functional \( f \) with respective to parameter vector \( \mu^k \). The initial guess (at \( k=0 \)) is homogeneous image with optical properties equal to background optical properties of the solution image for \( k=0, \gamma=0 \) and, \( \lambda=0 \). Computational and mathematical details for DOT are reviewed in [13]

3. Image Artifacts in DOT

In the image reconstruction process it is observed that the solution absorption image can cause artifacts in the reconstructed scattering image; similarly, the solution scattering image can cause artifacts in the reconstructed absorption image. Phenomenon of generation of these types of artifacts is called inter-parameter crosstalk. The inter-parameter crosstalk is undesired because it leads to ambiguous image reconstruction. The last row in fig.2 shows typical inter-parameter crosstalk effect. The inter-parameter crosstalk is basically due to coupling of absorption and scattering parameter information in the experimentally observed data \( m^x \) and hence, in objective functional \( f \).

Other type of artifacts appearing in DOT images are unwanted sharp artifacts near source and detector locations. These artifacts appear mainly due to higher sensitivity of the measurements to the change in optical property near source detector locations. Selection of appropriate objective functional leads to almost negligible near source detector artifacts. Typical near source detector artifacts are shown in images in the last row in fig. 2.

4. Objective Functional Selection

As mentioned earlier, the image artifact problem is directly related to appropriate objective functional synthesis or selection. Direct visualization of objective functional can provide details about uniqueness of minima and hence ability of the objective functional to separate absorption and scattering images. However, in the multidimensional parameter space, visualization of the objective functional is not feasible. We choose to examine the objective functional in two parameters space. Coordinates of the two parameter space are a scattering parameter and, an absorption parameter. The objective functionals are evaluated in the two parameter space by setting up a single inclusion problem. Guess images are generated by varying absorption and reduced scattering coefficient values of the inclusion in the range of interest. The solution image is set-up with absorption and scattering values of the inclusion equal to the central value of the range of interest. This selection of solution image ensures the minima of objective functional at the center of the two parameter space. A computed measurement set is generated for each guess image using the forward model. Synthetic experimental measurement set is generated for the solution image. Using these computed and experimental (synthetic) measurement sets objective functional values are computed for all guess images. During the whole procedure background optical properties are kept invariant. Objective functional values are then plotted as a surface in the two dimensional parameter space. For the case of time resolved DOT similar study of objective functional behavior has been reported [14], with less emphasis on objective functional for frequency domain DOT.

We hypothesize that the objective functional plot in the two parameter space gives idea about eccentricity of the objective functional in the general multidimensional parameter space of inhomogeneous DOT problem. By visual observations about eccentricity of objective functional the objective functional with minimum eccentricity is selected and examined for its ability to separate absorption and scattering images by minimizing inter-parameter crosstalk. It is well established in the convergence theory of optimization that less eccentricity of objective functional ensures good convergence [15]. Since we select the objective functional with single minima, uniqueness is also ensured and hence inter-parameter crosstalk is expected to reduce.

For the case of our example problem a circular domain of radius 3.8cm is used. Solution image inclusion values are taken as \( \mu_a = 0.5 cm^{-1}, \mu_s = 40 cm^{-1} \). Fixed background optical properties are \( \mu_a = 0.25 cm^{-1}, \mu_s = 20 cm^{-1} \). Objective functionals are evaluated for, \( \mu_a, \mu_s \) values of perturbation, in the range 0.25 - 0.75 cm$^{-1}$ and, 20 - 60 cm$^{-1}$ respectively. Various objective functional definitions in eq. set 2 and their linear combinations are evaluated for analysis.
4.1 Optimal Objective Functionals

For the absolute measurement data set, synthesized objective functional \( (f^2 + f^4) \) has less eccentricity as compared to other synthesized objective functionals (shown in fig. 1). Using similar analysis for the normalized measurement sets it is found that \( (f^3 + 500f^4) \) is optimal objective functional since it has minimum eccentricity compared to other objective functionals. The objective functional surface plots for normalized measurement sets are not shown here due to space constrain. The objective functionals stated here are not necessarily optimal for all ranges and geometries since the reconstruction problem is a nonlinear problem. However the methodology proposed is useful to identify or synthesize the optimal objective functional.

5. Results

We present results for following three types of measurement data sets 1) absolute measurements, 2) measurements normalized using one of the measurement and, 3) measurements normalized by average value of measurement set. Images reconstructed with proposed optimal objective functionals i.e. \( (f^2 + f^4) \) for absolute measurements and, \( (f^3 + 500f^4) \) for both types of normalized measurements are presented. For comparison we also include images reconstructed with objective functional \( f^7 \) which is a commonly used type of objective functional in DOT image reconstruction. Due to space constraint it is not feasible to show results for all objective functionals. All results are for a circular geometry (7.6 cm dia.). Synthetic measurement data is generated using forward model on the sample phantom images shown in first row of fig. 2.

The Image reconstruction results shown are not quantitatively comparable to the phantom image because they are shown at 33rd iteration. It is observed that measurement data normalized by average value of the measurement set show minimum inter-parameter crosstalk and negligible near-source detector artifacts whereas, normalization of the measured data by one of the measurements leads to severe near-source detector artifacts. Use of the basic type of objective functional leads to reconstruction with artifacts as can be seen from reconstruction results in last row of fig. 2. By visual comparison of results it is verified that the proposed objective functionals indeed generate superior images as compared to the basic objective functionals.
6. Conclusion

Form the above results we conclude that appropriate linear combination of the amplitude and phase data based objective functional leads to good quality DOT image reconstruction. Coefficients for the linear combination can be found from our proposed method that is, observations on eccentricity of the objective functional in two parameter space. From the results our hypothesis that the objective functional synthesized using proposed method provides image reconstruction with minimum inter-parameter crosstalk and negligible near source detector artifacts is verified.

7. References


