Hybrid Control of a Tri-state Boost Converter

Sreekumar C. 1, Vivek Agarwal 2
Applied Power Electronics Lab
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Powai, Mumbai-400076, INDIA
Email: 1 sreeku@sc.iitb.ac.in, 2 agarwal@ee.iitb.ac.in

Abstract- A tri-state converter has three controllable switches and hence has more operating flexibility. This also brings additional control design flexibility when viewing the converter as a hybrid dynamical system. The operational and control design flexibility of a tri-state boost converter is explored in this paper to control its output voltage. The analysis and control design for the converter is based on a hybrid automaton model. The hybrid control design is carried out to ensure the required voltage regulation and switching stability. Using a system theoretical approach, a geometrical region in the state plane around the set point is determined as a stable ‘safe-set’ by ensuring that a vector field of the system pointing towards the set point always exists to ensure the stability of switching in the converter. Within this stable region, the guards for the hybrid automaton model are determined as the switching control law to satisfy the regulation requirements. The control algorithm is tested using a model of the tri-state boost converter constructed in MATLAB/SIMULINK environment. Modeling, analysis, control design and simulation results are presented. Using the proposed control scheme, it is found that the tri-state converter can work with a wide range of loads and input voltages without compromising on regulation requirements.

I. INTRODUCTION

A tri-state converter has a constant inductor current mode, where the inductor current freewheels through a short circuiting switch connected across the inductor [1-3]. It has been proved that a wide range of operation is possible with such a converter, because of the elimination of the right half plane zero in a state averaged tri state converter [1]. The cost for this is an additional controlled switch connected across the inductor to enable a constant current to freewheel through the inductor. This paper explores the control design for such a converter using an exact hybrid automaton model which exactly portrays the circuit behavior without making any approximations, compared to the conventional state averaged model. Using this model, the large signal analysis and design can be carried out accurately. The exact model is considered as a continuously interacting parallel combination of continuous system and discrete system [4-7]. The hybrid automaton model of the pseudo operated converter is quite similar to its DCM operated counterpart except for the control algorithm. The details of modeling, analysis and control design are presented in the following sections. The system is simulated in MATLAB/SIMULINK and the results are discussed. The merits and demerits of the proposed scheme are investigated.

II. HYBRID MODELING OF A TRI-STATE CONVERTER

A tri-state converter has three controlled switches as shown in Fig. 1. This control flexibility helps to shape the current and voltage waveforms easily as per the user requirement. During its operation, a tri-state converter assumes three different configurations or structures. It is by switching between these structures, that the desired control objective is achieved.

During the first sub-interval of a period, switch S1 is closed and all other switches remain open. The inductor current ramps up during this interval. During the second interval, S2 is closed while other two switches are open. As a result, the inductor transfers its energy to the load and its current ramps down. In the third interval, switch S3 is closed and S1 and S2 are open. The inductor current freewheels through the switch. S3 and remains constant assuming the parasitic resistance of the inductor to be negligible. A typical inductor current waveform for the operation described above is shown in Fig. 2.
In hybrid modeling and analysis, the dynamics of the individual configurations are used rather than sticking on to an approximated single state averaged model. Thus, the identity of the individual configuration is maintained. Referring to Fig. 1, the three switching elements may lead to eight different configurations. Some of these configurations are not feasible or not used in the normal operation of a tri-state converter. As mentioned above, only one switch is closed at a time interval, while the other two switches are open. So, the set of possible discrete states is, \( Q = (q_1, q_2, q_3) \) where \( q_1 = \) (S1 on, S2 off, S3 off), \( q_2 = \) (S1 off, S2 on, S3 off) and \( q_3 = \) (S1 off, S2 off, S3 on). The set of feasible events is \( E = \{ (q_1, q_2), (q_2, q_3), (q_3, q_1) \} \).

The continuous dynamics corresponding to each discrete state can be defined using the state equations for the individual configuration. For the control design, let the states of the system be defined as \( x = [i_L, v_o] \), where \( i_L \) is the instantaneous inductor current and \( v_o \) is the instantaneous output voltage. The configurations corresponding to \( q_i \) (\( i = 1, 2, 3 \)) and their dynamics represented by three state equations are described below.

For \( q_1 \), the circuit configuration is as shown in Fig. 3.

The state equation in mode-1 is:
\[
\dot{x}(t) = A_1 x(t) + B_1 = f_1(x(t))
\]  

where, \( A_1 = \begin{pmatrix} 0 & 0 \\ 0 & -1/RC \end{pmatrix} \) and \( B_1 = \left( \frac{V_m}{L} \ 0 \right)^T \)

For \( q_2 \), the circuit configuration is as shown in Fig. 4.

The state equation in mode-2 is:
\[
\dot{x}(t) = A_2 x(t) + B_2 = f_2(x(t))
\]  

where \( A_2 = \begin{pmatrix} 0 & -1/L \\ 1/C & -1/RC \end{pmatrix} \) and \( B_2 = \left( \frac{V_m}{L} \ 0 \right)^T \)

For \( q_3 \), the circuit configuration is as shown in Fig. 5.

The state equation in mode-3 is:
\[
\dot{x}(t) = A_3 x(t) + B_3 = f_3(x(t))
\]  

where, \( A_3 = \begin{pmatrix} 0 & 0 \\ 0 & -1/RC \end{pmatrix} \) and \( B_3 = \left( 0 \ 0 \ 0 \right)^T \)

The linear state models (1), (2) and (3) portray all possible modes of operation of a tri state boost converter. In the present work, a hybrid automaton model is framed and used for analysis and design [4-6]. The hybrid automaton can be represented as a 6-tuple collection \( H = (Q, X, E, I, G) \) where \( Q = q_{1}...q_{N} \) is a set of discrete states; \( X \subseteq \mathbb{R}^n \) is the continuous state space; \( f : Q \times X \rightarrow R^0 \) assigns to every discrete state a Lipschitz continuous vector field on \( X \); \( I : Q \rightarrow 2^X \) assigns each \( q \in Q \) an invariant set; \( E \subseteq Q \times Q \) is a collection of discrete transitions; \( G : E \rightarrow 2^X \), \( e = (q, q') \in E \), is a guard.

For a tri-state boost converter hybrid automaton, \( Q = \{q_1, q_2, q_3\}, x \in X \) where \( x = [i_L, v_o] \), \( f = (f_1, f_2, f_3) \) as defined by (1), (2) and (3). Each discrete state is defined to be characterized by a discrete symbol, \( \sigma = (\sigma_1, \sigma_2, \sigma_3) \) which corresponds to the three different modes of operation. The guards, \( G_{12}, G_{23} \) and \( G_{31} \) defined in terms of the continuous states, governs the evolution from model1 to mode2 (event 1), mode2 to mode3 (event 2) and mode3 to mode1 (event 3) respectively. Thus the three individual modes, three events, and three guard conditions constitutes the hybrid automaton of the evolutions in a tri-state boost converter. Using the above definitions, a generalized hybrid model [4-6, 8] represented as
a parallel combination of interacting discrete and continuous automaton is shown in Fig.6.

![Diagram](image)

Fig. 6. Hybrid automaton representation of a tri-state boost converter.

III. CONTROLLER DESIGN

The controller design is done in two steps. First a geometric area in the state space is determined as the safe-set for the pseudo mode operation. Then a controller is designed inside the safe-set as described in the following sections.

A. Safe-set synthesis

To mathematically track the problem, a suitable shape for the safe-set in the phase plane which justifies the various modes of operation of the converter, is to be selected. Fig 7 shows the assumed safe-set for a given operating condition.

![Diagram](image)

Fig. 7 Assumed shape of safe set for tri-state boost converter operation.

From Fig 7, the safe-set can be viewed as a chopped circle. So, to fully define the safe-set, a circle along with a sectioning surface has to be identified. In this work, the circle is defined around the set point and the sectioning surface is treated as the line defined by the average load current of the converter for a given operating condition. The safe-set around the set point is determined as described below [4-6].

At first, the part of the safe-set is viewed as a circular region around the set point, co-ordinate transformed to origin. Then a small increment is made in the radius vector and it is moved from zero angle position towards 360 degree in steps to cover the whole state space in the admissible region. For each point, x on the tip of the radius vector, the possibility of the vector field corresponding to any mode pointing towards the set-point, $x_i$ is checked by computing the inner product,

$$I_{P_i} = <x - x_d, f_i(x)>$$

(4)

For a vector field pointing towards the steady state set point, the inner product given by (4) is negative. So, the existence of a negative inner product guarantees a stable switching towards the set point. The radius of the circle is increased in small steps till the condition in (4) is violated or the boundary of the admissible set is reached. This procedure is repeated for various input and loading conditions. The largest radius thus obtained gives the boundary of the safe-set. If such a boundary can not be determined, this method fails.

B. Control synthesis

As the stability of the system is guaranteed on the safe boundary, controllers for the interior of the safe set can be designed based on certain performance requirement. In the current problem, the output voltage regulation and the switching frequency are the system properties related to control design. The voltage regulation is already taken care off by defining the safe-set inside the admissible set. In order to have minimum switching, the guard conditions are checked at the boundary of the safe-set corresponding to a given operating condition. In the present example, the system states are allowed to move inside the safe set, if the guard conditions are true. When the guard condition is false on the circular region defined by the boundary, control is selected to minimize the cosine of the angle between $x - x_d$ and $f_i(x)$ as:

$$\sigma_i = \arg \left( \min_i \frac{<x - x_d, f_i(x)>}{\|f_i(x)\|} \right)$$

(5)

Similarly, on reaching the sectioned line boundary, the system is allowed to move through the constant current mode. The selection of the guard, $G_{33}$ is done by fixing a current level to freewheel through S3. The mathematical analysis on the selection of this current level is already discussed in literature [1]. In the proposed control scheme, it is done by maintaining the inductor current at the average load current value. The guard causing transition from mode 3 to mode 1, $G_{31}$ is chosen by checking for the vector field corresponding to mode-1 to be negative for an output voltage less than the set voltage. Hence, all guard conditions are defined. Using the ideas presented in this section, a switching controller based on regulation requirement is defined and simulated in MATLAB/SIMULINK to test the suitability of the proposed
algorithm. Details of the computer simulations and results are included in the next section.

IV. MATLAB SIMULATIONS AND DISCUSSIONS.

The safe-set representing all possible operating conditions is determined using a MATLAB program as per the algorithm described in Section III. The parameter values and rated load condition assumed to test the algorithm are: \( V_m = 15V \), \( L = 80\mu F \), \( C = 100\mu F \) and \( R = 80\Omega \). It is desired to design the control law to regulate the output voltage at \( V_o = 30V \) with a tolerance of \( \pm 0.5V \). Assuming that the load resistance can vary from \( 80\Omega \) to \( 300\Omega \), the inductor current can be in the range \([0-1.25A]\) approximately. Also an input voltage variation in the range \(15\pm 5V\) is considered for simulation. This implies an admissible set \( F = \{ x \in R^2: 0 \leq x_1 \leq 1.25; 29.5 \leq x_2 \leq 30.5 \} \). Steady state operation requires that \( I_L = \frac{V_o}{V_m} (V_m R) \), where \( I_L \) and \( V_o \) are the steady state inductor current and output voltage respectively. With the help of a MATLAB program it is found that the radius of the safe set is 0.5 for the assumed load and input voltage variation.

To test the control algorithm described in the previous section, a tri-state converter model and controller is constructed in MATLAB/SIMULINK and simulations are carried out using the variable step ode45 algorithm with a time step of 0.01 \( \mu s \) and a tolerance of 1e-4. A load variation from \( 80\Omega \) to \( 300\Omega \) and input voltage disturbance from \( 15V \) to \( 10V \) are considered for simulation. The simulation results are plotted in Fig. 6. The waveforms under various operating conditions are discussed below:

The circuit is switched on with the rated load of \( 80\Omega \) to an input source of \( 15V \). Though an inrush of current is seen, the system soon settles in the steady state operation. A further load change of \( 60\Omega \) each is applied at \( t=3ms \) and \( t=6ms \). In both the cases, the controller is able to keep the output voltage constant without proceeding through complex behavior [9-10]. The transients at these disturbances are shown in Fig 7. To check the suitability of the proposed hybrid control scheme over a wide range, an input voltage variation of \( -5V \) is considered at \( t=9ms \). Due to the controller action, the variation in output voltage, owing to the variation in input voltage and load resistance, is restricted to a narrow band and settles quickly to the set value of 30V. On each disturbance, the safe-set gets modified to suit the new operating condition. Typical safe-set for two different operating conditions are shown in Fig 8.

Fig. 7 Enlarged view of waveforms under disturbances: (a) Variation in output voltage due to load disturbance of \( 60\Omega \) at \( t=3ms \); (b) Corresponding inductor current; (c) Variation in output voltage variation due to a further load change of \( 60\Omega \) at \( t=6ms \); (d) Corresponding inductor current.

Fig. 8 Safe set corresponding to (a) \( R=80\Omega \); (b) \( R=300\Omega \).

The phase plane diagram representing the various operating conditions corresponding to different input voltage and load resistance conditions are shown in Fig 9. It may be observed
that the trajectory for a particular operating condition is moving through the same oscillations in a cyclic manner.

![Fig.9 Phase plane trajectories representing various operating conditions of the tri-state boost converter.](image)

**CONCLUSIONS**

A hybrid control scheme for the tri state boost converter has been discussed in this paper. The regulation problem is formulated as a hybrid control problem and the control design is done in two steps. A safe set, contained in the admissible region, is first determined and the controller is then designed to satisfy the regulation criteria. As the switching between various modes of operation is done inside the safe set, the system is always stable. The designed controller is able to maintain constant output voltage under various input voltage and load resistance conditions. The controller action is quick enough to settle the system in a stable condition following a disturbance.

**REFERENCES**


