Allocation of Fixed Transmission Costs by Tracing Compliant Postage Stamp Method

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Abstract—Postage stamp allocation is unfair to those users who make limited usage of the network. MW power flow tracing was invented to assess the extent of network usage. MW power tracing, a post-facto analysis of power flow solution, is amenable to multiple solutions. This implies multiplicity of solution space of cost allocation problem. Existing tracing methods enforce a ‘proportionate sharing rule’ to solve the dilemma. Case studies indicate that with such a sharing rule, extent of network usage predicted for large and distant loads can be much higher than the postage stamp allocation. To alleviate this problem, we propose a tracing compliant modified postage stamp allocation method which computes a traceable solution that minimizes deviation from the postage stamp allocation. We show that tracing problem can be formulated as a linear constrained optimization problem. Thus, fairness among all network users is ascertained. Results on actual data of central transmission utility of Western Regional Grid of India demonstrate the claims.

Index Terms—Power Flow Tracing, Transmission cost allocation, Network Optimization

NOMENCLATURE

\( \bar{c}_{lm} \) Cost of line \( lm \) per MW
\( n_b \) Total number of branches
\( n_G \) Total number of generators
\( n_L \) Total number of loads
\( P_{Ci} \) Real power injection by generator \( k \)
\( P_{Li} \) Real power load at bus \( i \)
\( P_{lm} \) Real power flow over a line \( lm \)
\( x_{ik} \) Real power fraction of generator \( k \) contributing toward load \( i \)
\( x_{km} \) Sending end real power fraction of generator \( k \) on line \( lm \)
\( y_{ik} \) Real power fraction of load \( i \) contributed by generator \( k \)
\( y_{im} \) Receiving end real power fraction of load \( i \) on line \( lm \)

I. INTRODUCTION

Traditionally electric utilities allocate the fixed transmission cost among its users having firm contracts, based on postage stamp rate [1]. Application of this principle usually leads to simplicity. However, in the postage stamp method, transmission users are not differentiated by the extent of use of transmission facilities. Review of usage based transmission cost allocation methods under open access is dealt in [1]. The paper overviews various usage-based methods including tracing based methods [2], [3] for transmission system costs under open access.

The advent of tracing principle [2], [3], [4] solves the problem of finding extent of use of a network. While the power flow solution obtained from Kirchhoff Current Law (KCL) and Kirchhoff Voltage Law (KVL) is unique, the tracing problem is amenable to multiple solutions. To solve the resulting dilemma, tracing methods invoke a proportionate sharing rule. The specific interpretation given to the proportionate sharing rule is that net incoming resource (e.g., MWs from a generator in the generation tracing problem) is shared among outflows in a proportionate manner. Application of this principle leads to simplicity. However, a major drawback associated with this approach is that geographically distant loads are overburdened with large usage costs when compared with postage stamp allocations [5].

To overcome the drawback, it is necessary that multiplicity of the solution space in tracing be exploited. Therefore, in this paper, we frame tracing problem as an optimization problem.

We develop a multi-commodity network flow model [6] which can model the complete space of tracing solutions. This leads us to introduce a class of linear constrained optimization problem known as optimal tracing problem. While network usage has multiplicity in solution space of traceable solutions, our objective is to choose the one which is nearest to the postage stamp allocation. Thus, advantages of usage based and equitable distribution cost allocation are achieved simultaneously. This leads to development of modified postage stamp method, that works under the tracing framework. The nearness of the solution to that of conventional postage stamp allocation can be measured through various vector p-norms [7]. We prefer \( l_1 \) norm (\( ||.||_1 \)) because it leads to a Least Absolute Value (LAV) optimization problem which can be easily translated into a linear programming (LP) problem.

Remark 1: Tracing compliant usage cost allocation method does not throw away the wisdom associated with postage stamp allocation. Rather, it overcomes the drawback of neglecting network usage by embedding it in a tracing framework.

The paper is organized as follows. A generic class of optimal tracing problem is introduced in section II. The space of traceable solutions for loss-less generation and load tracing problems is characterized in section II-A. It uses multi-commodity network flow with real power as the flow variable. Since, in practice, real power flow is lossy, modeling of lossy networks is considered in section III. To achieve consistency in generation and load tracing problems in an optimization
framework, a unified formulation is proposed in section IV. Explicit problem formulation is presented in section V. Results on actual data of Western Regional Grid of India are presented in section VI. Section VII concludes the paper.

II. DEFINING AN OPTIMAL TRACING PROBLEM

We now introduce a class of optimization problems, henceforth referred to as optimal tracing problem, which can be compactly defined as follows:

Problem OPT(x, y): \[ \min_{(x,y) \in S} f(x, y) \] (1)

The set \( S \) represents the set of all possible tracing solutions and a specific set of \( x \) and \( y \) vectors represents a solution to generation and load tracing problem. In later sections, we show that set \( S \) can be characterized by a set of linear equality and inequality constraints. In fact, set \( S \) is both compact and convex. This leads to a linear constrained optimization problem. It models the relationship between the flow entities and associated network usage costs.

A. Characterization of Solution Space: Lossless Network

In this section, the set \( S \) is characterized by linear equality and inequality constraints. We use ideas from multi-commodity flow decomposition. Constraint modeling is initially introduced for a lossless flow network. In the next section, modelling of lossy flow networks is discussed. For the power flow tracing problem, equality constraints are grouped into following categories:
- Flow Specification constraints for series branches i.e., transmission lines and transformers;
- Source and Sink specification constraints pertaining to shunts e.g., generators and loads;
- Inequality constraints associated with flow bounds.

B. Flow Specification Constraints

Traditionally, two types of tracing problems, viz., generation tracing and load tracing are discussed in literature. Generation tracing traces generator flows to loads, while load tracing traces load flows to generators. We first discuss modeling of the flow specification constraints for generation tracing problem.

1) Generation Tracing: Let \( P_{lm} (\text{MW}) \) be the flow on a line \( lm \). Flow \( P_{lm} \) is supplied from generators \( G_1, G_2, \ldots, G_n \), with components \( P_{G1lm}, P_{G2lm}, \ldots, P_{GnGlm} (\text{MW}) \). Therefore,

\[ P_{lm} = P_{G1lm} + P_{G2lm} + \ldots + P_{GnGlm} \] (2)

The component of generator \( G_k \) on line \( lm \) can be expressed as fraction \( x_{lm}^k \) of the total injection by generator \( G_k \) i.e., \( P_{Gk} \).

\[ P_{Gk}^m = x_{lm}^k P_{Gk} \] Thus, (3)

\[ P_{lm} = \sum_{k=1}^{ng} x_{lm}^k P_{Gk} \quad \forall \text{ set of lines} \] (4)

Remark 2: In traditional tracing [2], [3], [4], the fractions \( x_{lm}^k \) are frozen by application of proportional sharing principle. In the proposed approach, fractions \( x_{lm}^k \) are decision variables and are set as a result of optimization problem.

Since, the branch flows are known and \( x \) are unknown, flow equations for generation allocation can be written as follows:

\[ [A_{flow}] x_{flow} = [b_{flow}] \] (5)

Matrix \( A_{flow} \) has \( n_b \) rows and \( n_b \times n_G \) columns where \( n_b \) is the number of branches and \( n_G \) the number of generators.

Remark 3: All \( x \)-fractions are restricted from 0 to 1. These limits correspond to flow bound constraints. The lower limit ensures that the flow component should have same direction as the arc flow while the upper limit ensures that no flow component exceeds the corresponding generation.

Remark 4: As every equation introduces set of new variables, all the equations are linearly independent and decoupled in nature. Therefore, flow matrix has full row rank. Further, every column has a single entry and hence the resulting matrix is extremely sparse.

2) Load Tracing: The power flow \( P_{lm} \) on line \( lm \) can also be expressed as summation of load components, i.e.:

\[ P_{lm} = P_{lm}^1 + P_{lm}^2 + \ldots + P_{lm}^{nL} \] (6)

The component of load \( (P_{li}) \) on line \( lm \) is expressed as a fraction \( y_{lm}^i \) of load \( P_{li} \) as follows:

\[ P_{lm}^i = y_{lm}^i P_{li} \] (7)

Thus, \( P_{lm} = \sum_{i=1}^{nL} y_{lm}^i P_{li} \) (8)

In matrix form, the flow equations for load allocation can be written as follows:

\[ [A_{flow}] [y_{flow}] = [b_{flow}] \] (9)

Matrix \( A_{flow} \) has \( n_b \) rows and \( n_b \times n_L \) columns where, \( n_L \) is the number of loads. Remarks 3 and 4 applies to (9) also.

Remark 5: In traditional tracing, the fractions \( y_{lm}^i \) are frozen by application of proportional sharing principle. In the proposed approach, fractions \( y_{lm}^i \) are decision variables and are set as a result of optimization problem.

C. Source and Sink Specification Constraints

1) Generation Tracing: In a generation tracing problem, it is necessary to write sink (load) constraints. They express contribution of generators in loads. For a load \( P_{Li} (\text{MW}) \), the contribution of various generators is governed by the following constraint:

\[ P_{Li} = P_{G1i} + P_{G2i} + \ldots + P_{Gni} \] (10)

i.e., \( P_{Li} = \sum_{k=1}^{ng} x_{lm}^k P_{Gk} \forall j = 1, \ldots, n_L \) (11)

Where, \( P_{Gki} \) is the component of load \( P_{Li} \) met by generator \( G_k \). The \( x \)-variables as always is restricted between 0 and 1. In the matrix form, the load equations for generation allocation can be written as follows:

\[ [A_{inj}] x_{inj} = [b_{inj}] \] (12)
Matrix $A_{inj}$ has $n_L$ rows and $n_L \times n_G$ columns. Remark 4 on sparsity applies to equation (12) also.

Remark 6: In traditional tracing, the fractions $x^k_i$ are frozen by application of proportional sharing principle. In the proposed approach, fractions $x^k_i$ are decision variables and are set as a result of optimization problem.

2) Load Tracing: In the load tracing problem, it is necessary to model the share of loads in a generator. Let,

$$P_{G_k} = P_{G_{L_1}} + P_{G_{L_2}} + \ldots + P_{G_{L_{n_L}}}, \text{ i.e., } P_{G_k} = \sum_{i=1}^{n_L} y^i_k P_{L_i} \quad (13)$$

In the matrix form, generator equations for generation allocation can be written as follows:

$$[A_{inj}] [y_{inj}] = [b_{inj}] \quad (14)$$

Matrix $A_{inj}$ has $n_G$ rows and $n_G \times n_L$ columns. Again, remark 4 applies to the above equation.

Remark 7: In traditional tracing, the fractions $y^i_k$ are frozen by application of proportional sharing principle. In the proposed approach, fractions $y^i_k$ are decision variables and are set as a result of optimization problem.

D. Conservation of Commodity Flow Constraints

The conservation of flow constraints can be neatly expressed by using arc or bus incidence matrix $M$ of the underlying graph. In the matrix $M$, rows correspond to nodes and columns to arcs. The entry $M(i,j)$ is set to 1 if arc $j$ is outgoing at node $i$; it is -1 if the arc is incoming at node $i$; else it is set to zero. The shunt arcs have one node as ground which is not modeled in $M$. Corresponding entry in $M$ is either 1 or -1 depending upon whether the arc represents load or generation.

1) Generation Tracing: Let $G(V, E)$ represent the graph of network, where, $V$ represents set of all nodes and $E$ the set of arcs. Let $E$ be partitioned as:

$$E = \{e_{na}\} \cup \{e_L\} \cup \{e_G\}$$

where, subset $\{e_{na}\}$ represents the set of series branches, subset $\{e_L\}$ indicates the set of shunt branches due to generators and subset $\{e_G\}$ represents set of shunt branches due to loads.

Then, partitioning of $E$ induces following column partitions on $M$.

$$M = [M_{na}, M_L, M_G]$$

Further, let $M_d$ represent submatrix of $M$ formed by considering series branches and shunt loads.

$$M_d = [M_{na}, M_L]$$

and $f_{ld} = \left[ P_{L_{m_1}} \ldots P_{L_{m_{n_L}}} L_1 \ldots L_{n_{L}} \right]^T$. The $f_{ld}$ vector consists of flows for arcs modeled in $M_d$. Then, the conservation of flow constraint for generation tracing problem can be expressed in compact notation as:

$$[M_d][f_{ld}] = [s] \text{ where } s = [P_G(1), \ldots P_G(n)]^T \quad (15)$$

$P_G(j) = 0$ if no generator is present at node $j$ and $P_G(j) = P_{G_k}$ if $k^{th}$ generator is present at node $j$. Matrix $M_{d}$ is of dimension $n \times (n_b + n_L)$, where, $n$ represents number of nodes.

Let, the corresponding flow component vector for generator $G_k$ be, $f_{ld}^k = \left[ P_{G_{L_1}} \ldots P_{G_{L_{n_L}}} P_{L_1} \ldots P_{L_{n_L}} \right]^T$.

From decomposition modeled in (2) and (10), flow vector $f_{ld}$ can be represented as summation of generation commodity flows. Then,

$$f_{ld} = \sum_{k=1}^{n_G} f_{ld}^k$$

Let, $s^k$ represent MW injection vector for $k^{th}$ generator.

$$s^k = [0, 0, \ldots, P_{G_k}, 0, \ldots, 0]^T = P_{G_k} e_k$$

Where, $f_{ld}$ is the node at which $k^{th}$ generator is connected and $e_k$ is the $k^{th}$ column of identity matrix. Then in (15),

$$s = \sum_{k=1}^{n_G} s^k$$

In terms of flow components, equation (15) can be expressed as:

$$\sum_{k=1}^{n_G} \{[M_d][f_{ld}^k] - s^k\} = [0] \quad (16)$$

In addition to conservation of flow at a node, in tracing, each commodity flow at a node also has to be conserved. Therefore, in equation (16), each individual group should be identically zero, i.e.,

$$[M_d][f_{ld}^k] = [s^k] \quad k = 1 \ldots n_G \quad (17)$$

Dividing (17) by $P_{G_k}^k$ leads to following equation:

$$[M_d][x^k] = [e_k] \quad k = 1 \ldots n_G \quad (18)$$

where $x^k$ represents the set of $x$-variables for lines and loads associated with the $k^{th}$ generator.

Instead of partitioning $x$ variables by generator numbers, they can be partitioned by series branch flow ($x_{flow}$) and shunt flow ($x_{inj}$) variables. The set of the continuity equations (18) can be rearranged and written in block matrix notations with $x_{flow}$ and $x_{inj}$ variable partition as follows:

$$\begin{bmatrix} A_{cont\_flow} & A_{cont\_inj} \end{bmatrix} \begin{bmatrix} x_{flow} \\ x_{inj} \end{bmatrix} = \begin{bmatrix} b_{cont} \end{bmatrix} \quad (19)$$

2) Load Tracing: For the load tracing problem, the corresponding conservation of the flow equation is given by

$$[M_u][f_{lu}] = [l] \text{ where } l = [P_{L(1)}, P_{L(2)}, \ldots P_{L(n)}]^T \quad (20)$$

where,

$$M_u = [M_{na}, M_G]$$

$$f_{lu} = \left[ P_{L_{m_1}} \ldots P_{L_{m_{n_L}}} P_{G1} \ldots P_{G_{n_G}} \right]^T$$

Matrix $M_u$ is of dimension $n \times (n_b + n_G)$. Loads are modeled by $l$-vector specification on the right hand side.

Following similar arguments as in generation tracing, the continuity equations for load tracing are given as follows:

$$[M_u][y] = [e_l] \quad i = 1 \ldots n_L \quad (21)$$
Where, \( i \) is the node at which \( i^{th} \) load is connected. Set of equations given by (21) can be rearranged into the following form:

\[
\begin{bmatrix}
A_{cont,flow_{u}} & A_{cont,flow_{w}}
\end{bmatrix}
\begin{bmatrix}
y_{flow_{u}} \\
y_{flow_{w}}
\end{bmatrix} = [b_{cont}]
\]

(22)

Remark 8: Since tracing is a post-facto analysis of power flow solution, it does not violate the KCL and KVL.

III. MODELING OF LOSSY FLOW NETWORK

A. Exact Approach

1) For generation tracing: For a lossy flow network, flow reduces along the arc from sending end to the receiving end. Under such situations, it is necessary to model two flow equations per line, one for the sending end and one for the receiving end. Also, two sets of variables are defined for sending end and the receiving end. Hence, sending end and receiving end variables are distinguished by superscripts \( s \) and \( r \) respectively. The flow equations for sending end and receiving end for a line \( lm \) are as follows:

\[
P_{lm}^s = \sum_{k=1}^{n_G} x_{lm}^{k,s} P_{G_k} \quad \text{at node } l
\]

(23)

\[
P_{lm}^r = \sum_{k=1}^{n_G} x_{lm}^{k,r} P_{G_k} \quad \text{at node } m
\]

(24)

To write down the continuity equations for the lossy flow networks, the bus incidence matrix has to be modified to discriminate between the sending end and the receiving end. This doubles the number of columns. The continuity equations have to be written using the modified matrices \( M_d \) and \( M_u \). In the modified matrix, for a lossy branch, there are two columns, one to model flow at the sending end and other to model flow at the receiving end. Hence, there is only one entry per column. In addition, the constraint that ‘sending end fraction \( x_{lm}^s \) cannot be smaller than the receiving end fraction \( x_{lm}^r \)’ has to be modeled. The number of equations in the lossy formulation increases by \( n_b \times n_G \), and the number of variables by \( n_b \times n_G \).

B. Simplified Approach

1) For generation tracing: An alternative approach has been developed to restrict the number of variables and equations in the optimization problem to the corresponding lossless formulation. For this purpose, receiving end flow fractions are fixed as follows:

\[
x_{lm}^{k,r} = \frac{P_{lm}^r}{P_{lm}^s} x_{lm}^{k,s} \quad \forall \, lm \in \text{set of lines}
\]

(25)

Consequently, modeling of (24) becomes redundant. Thus, for the generation tracing problem, series branch flow at sending end only is specified. Further, constraint \( x_{lm}^{k,s} \geq x_{lm}^{k,r} \) is also satisfied by (25). The substitutions given by (25) can be used to reduce the \( M_d \) matrix to the reduced matrix \( M_{d}^{loss} \) which has identical dimension and structure as \( M_d \). First, the matrix \( M_{d}^{loss} \) is initialized to matrix \( M_d \). Then, the nonzero entries of matrix \( M_{d}^{loss} \) are modified as follows. Let \( o_i \) be the origin and \( d_i \) the destination of an arc \( i \). Then for all arcs representing series branches, set:

\[
M_{d}^{loss}(o_i,i) = 1
\]

(26)

\[
M_{d}^{loss}(d_i,i) = -\frac{P_{lm}^r}{P_{lm}^s} \forall \, lm \in \text{set of lines}
\]

(27)

2) For load tracing: For the load tracing problem, transmission line flows at the receiving end are specified. On the similar lines to generation tracing formulation, it can be shown that for the load tracing, the nonzero entries of the modified bus incidence matrix \( M_u^{loss} \) has structure identical to \( M_u \). Further, modifications in the non zero entries for series arcs are as follows:

\[
M_u^{loss}(o_i,i) = +\frac{P_{lm}^r}{P_{lm}^s} \forall \, lm \in \text{set of lines}
\]

(28)

\[
M_u^{loss}(d_i,i) = -1
\]

(29)

This approach has been formally programmed in this work.

IV. UNIFIED FORMULATION

Even in a lossless system, “generator-\( k \) contribution in load-\( i \)” obtained from the generation tracing problem may not match with “load \( i \) share in generator \( k \)” obtained from the load tracing problem. This implies that for consistent generation and load allocation, coupling or boundary constraints should be modeled. This leads to the unified generation-load tracing problem formulation where in, arc flows are simultaneously decomposed into generation and load commodity flows.

The integrated approach involves following three steps:

1) Formulate the generation tracing problem.
2) Formulate the load tracing problem.
3) Formulate the coupling or boundary constraints to achieve consistency between generation and load tracing results.

A. Modeling of Boundary Conditions

In a MW flow network, if the results are consistent then, this difference i.e. power dispatched by a generator \( k \) to load \( i \) minus power received from generator \( k \) by load \( i \) should correspond to the loss incurred in the generator-\( k \)-load-\( i \) interaction. Let us define this loss as \( \text{loss}_i^k \) where:

\[
\text{loss}_i^k = y^i_k P_{L_i} - x^k_i P_{G_k}
\]

(30)

For a lossless system, \( \text{loss}_i^k = 0 \) i.e.,

\[
y^i_k P_{L_i} - x^k_i P_{G_k} = 0
\]

(31)

Equation (30) will model loss in the generator-\( k \)-load-\( i \) interaction if it also satisfies the following loss properties of a network:

\[
y^i_k P_{L_i} - x^k_i P_{G_k} \geq 0 \quad \forall \, k \in \{1,2,\ldots,n_G\}, \forall \, i \in \{1,2,\ldots,n_L\}
\]

(32)

\[
P_{loss} = \sum_{k=1}^{n_G} \sum_{i=1}^{n_L} (y^i_k P_{L_i} - x^k_i P_{G_k})
\]

(33)

where \( P_{loss} \) is the total loss in the system. Inequality (32) models the constraint that loss is a non-negative number while...
equation (33) models the requirement that total generator-load interaction losses should be equal to the power loss in the system.

**Proposition 1:** Equation (33) is redundant.

For proof, see appendix I. To conclude, the boundary conditions for unified formulation are given by

\[ \text{loss}_i^k = y_i^k P_{Li} - x_i^k P_{Gi} \] (34)

\[ \text{loss}_i^k \geq 0 \quad \forall i = 1, \ldots n_L \quad k = 1, \ldots n_G \] (35)

**V. OPT: EXPLICIT FORMULATION**

A. Modified postage stamp formulation

The transmission system usage cost per MW paid by a load $i$ can be worked out as follows:

\[ tr\text{price}^i_{pu} = \sum_{lm} y_{lm}^i P_{Li} c_{lm} = \sum_{lm} y_{lm}^i c_{lm} \]

where, $c_{lm}$ is the cost of line $lm$ per MW. The product $y_{lm}^i \times P_{Li}$ (discussed in section II-B) represents the MW power of load $i$ flowing on line $lm$. Let, average transmission cost per MW of the system, $k_T^*$, be given by

\[ k_T^* = \frac{\sum_{lm} y_{lm}^i c_{lm}}{\sum_{i=1}^{n_T} P_{Li}} \] (36)

Aim of tracing compliant postage stamp method is to compute the closest traceable solution to the proportionate distribution of transmission system usage costs. Therefore, the objective function $f\{x, y\}$ in equation (1) is written as:

\[ f\{x, y\} = \sum_{i=1}^{n_T} |tr\text{price}_{pu}^i - k_T^*| \] (37)

Now, the OPT problem which was defined in section II can be explicitly formulated as follows:

\[ \min f(x, y) \quad \text{(38)} \]

\[ \begin{bmatrix} A_d & 0 \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_d \\ b_u \end{bmatrix} \quad \text{(39)} \]

\[ y_i^k P_{Li} - x_i^k P_{Gi} \geq 0 \quad \forall k \in \{1, 2, \ldots, n_G\} \]

\[ y_i^k P_{Li} - x_i^k P_{Gi} \leq 0 \quad \forall i \in \{1, 2, \ldots, n_L\} \quad \text{(40)} \]

\[ 0, 0, \ldots, 0]^T \leq [x] \leq [1, 1, \ldots, 1]^T \quad \text{(41)} \]

\[ [y] \geq [0, 0, \ldots, 0]^T \quad \text{(42)} \]

The constraints $A_d x = b_d$ are the equality constraints for generation tracing formulation. The constraints $A_u y = b_u$ represent corresponding constraints for load tracing formulation. Constraints (40) model non-negativity of loss characteristics. Inequality constraints (41, 42) model the limits on $x$ and $y$ variables.

**Remark 9:** Equations (39-42) model the set $S$ of feasible solutions. Every feasible solution of multi-commodity optimization problem satisfies all requirements of tracing. Proportionality based tracing models a specific instance in this feasible space.

**Remark 10:** This problem is LAV minimization problem that can be converted to LP problem as explained in [8].

**VI. RESULTS**

We now present results obtained on the Central Transmission Utility (CTU) network of Western Regional Grid of India. Optimal tracing is carried out on actual data of one day. The system is 33 bus, 12 generator EHV network. Special energy meter (SEM) data, that gives power flows at both ends of a line is used to obtain line flows. SCADA gives generation of each generating unit for every minute. The whole exercise is carried out on averaged out data of 28th July 2004. More details of the system can be obtained from [5]. The details about the geographical location of various constituent loads can be obtained from [9].

Transmission Service Charge (TSC) is the fixed transmission cost to be recovered from the constituent states. Table I compares the TSC allocation to various states obtained by following three methodologies:

1. Prevailing practice (Postage stamp allocation)
2. Tracing compliant postage stamp method (Optimal tracing)
3. Network usage by proportional tracing

**TABLE I**

<table>
<thead>
<tr>
<th>State</th>
<th>TSC Allocation (Million Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Postage Stamp Allocation</td>
</tr>
<tr>
<td>Gujarat (GU)</td>
<td>67.68</td>
</tr>
<tr>
<td>Madhya Pradesh (MP)</td>
<td>59.12</td>
</tr>
<tr>
<td>Chhattisgarh (CS)</td>
<td>17.91</td>
</tr>
<tr>
<td>Maharashtra (MH)</td>
<td>96.28</td>
</tr>
<tr>
<td>Goa (GO)</td>
<td>11.28</td>
</tr>
<tr>
<td>Daman &amp; Diu (DD)</td>
<td>6.15</td>
</tr>
<tr>
<td>Dadra-Nagar-Haveli (DN)</td>
<td>9.9</td>
</tr>
</tbody>
</table>

* Tracing compliant postage stamp allocation

It can be seen that for states of Gujarat, Madhya Pradesh, Chhattisgarh, Maharashtra and Goa, optimal tracing results are in between the traditional postage stamp allocation and proportional tracing allocation. For example, under prevailing postage stamp allocation, Gujarat, which wheels power over large distance pays comparatively small amount (Rs. 67.68 Million), as the method does not recognize the extent of network usage. In contrast, the proportional tracing based allocation overburdens it with network usage cost (Rs. 148.6 Million). However, with tracing compliant postage stamp allocation proposed here, the cost allocation to Gujarat is in between the two extremes.

For the states of Madhya Pradesh, Chhattisgarh, Maharashtra and Goa, the situation is a bit different. Under prevailing practice, it appears that these states pay more than the extent of usage of the network. Consequently, in proportionate tracing allocation, the payment reduces significantly. The tracing compliant postage stamp allocation for these states is again in between the two extremes. The last two states, DD and DN are minor users of the network. It is seen that their cost allocations with the proposed approach are below both, prevailing method and proportionility based method. The results indicate the fairness of the proposed scheme.

It can be concluded that tracing compliant postage stamp allocation scheme reduces the positional handicap of far away
loads to the possible extent (and vice versa). It tries to bring the per unit transmission usage costs of all loads in a narrow band, as compared with the proportional tracing allocation.

VII. CONCLUSIONS

In this paper, power flow tracing problem was formulated as a linear constrained optimization problem. It was shown that power tracing problem belongs to the generic class of multi-commodity network flow optimization problem. Two formulations, one each for generation tracing and load tracing were developed. A methodology for modeling lossy flow networks was proposed. Also, a unified generation-load tracing formulation was proposed to guarantee mutually consistent generation and load tracing results. The proposed optimal tracing formulation can be applied to transmission system usage cost allocation problem.

It can be argued that postage-stamp allocation of transmission system usage cost is unfair to those who make limited usage of the network. On the other hand, case studies have shown that conventional tracing based solution can be unfair to the geographically distant loads which have a high chance of getting overburdened with usage costs. In other words, the extent of network usage predicted for such loads can be on a higher side. To overcome above problem, a tracing compliant approach for modified postage stamp allocation was proposed in this paper.

Tracing compliant postage stamp method computes the closest traceable solution to the proportionate distribution of transmission system usage costs. It is observed that the tracing compliant postage stamp method achieves fairness to both small and large users of the network. Results on CTU network of Western Regional Grid of India, presented in this paper justify the claims made.

REFERENCES


APPENDIX I

PROOF OF PROPOSITION 1

Proof: It follows equation (13) that
\[ \sum_{k=1}^{n_G} \sum_{i=1}^{n_L} y_{ik} P_{L_i} = \sum_{k=1}^{n_G} P_{G_k} \quad (43) \]

Similarly, it follows from (11) that
\[ \sum_{k=1}^{n_L} x_i^k P_{G_k} = \sum_{k=1}^{n_G} \sum_{i=1}^{n_L} x_i^k P_{G_k} = \sum_{i=1}^{n_L} P_{L_i} \quad (44) \]

It follows from identities (43) and (44), that constraint (33) is redundant and hence is not modeled in unified formulation. ■

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