RESTORATION OF COLOR IMAGES SUBJECTED TO INTERCHANNEL BLURRING

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ABSTRACT

This paper deals with the restoration of colored images, distorted by both intra- and inter-channel blur, and corrupted by additive white Gaussian noise. The image is modeled as a Markov Random Field (MRF), and color image restoration is cast as a maximum a posteriori (MAP) estimation problem. We propose a First Order Interchannel Interaction (FOII) model for image restoration. Simulated annealing algorithm is then used to minimize the posterior energy function. We compare the simulation results of conventional non interaction (NI) approach and the proposed FOII approach. Proposed model is fairly general, and the results are satisfactory even when interchannel degradation parameter is unknown.

1. INTRODUCTION

Restoration of monochrome images, when the blurring function is known, is well addressed in literature ([1], [2]). Some of the techniques are listed in ([3], [4] and [5]). Results for color image restoration are not all that satisfactory mainly because, psychology of color image perception in human beings is not fully understood.

In this paper, we assume color planes interact with each other. To account for the interchannel blurring, we propose a new model for interchannel interaction, namely, the first order interchannel interaction (FOII) model. Then, a probabilistic approach is used by modeling the color image as a Markov Random Field (MRF). Restoration problem is then cast as a maximum a posteriori (MAP) estimation problem. In general, the energy function will be non-convex with multiple local minima and non-unique global minima. We use simulated annealing (SA) algorithm with inverse log cooling schedule for energy minimization.

2. IMAGE MODEL

Let X be the lexicographically ordered (row transposed stacking) vector for an $M 	imes M$ image ([1]).

Definition 1 $X$ is a Markov random field if and only if

$$
P[X_{i,j} = x_{i,j} | X_{k,l} = x_{k,l}, \forall (k,l) \neq (i,j)] = P[X_{i,j} = x_{i,j} | X_{k,l} = x_{k,l}, (k,l) \in \eta_{i,j}]$$

(1)

Where $P[ ]$ is the conditional probability and $\eta_{i,j}$ is the neighborhood of $(i,j)$. The neighborhood condition is translation independent except at boundaries, where a free boundary assumption is made.

Now, according to the Hammersly and Clifford theorem ([6], [7]), $P[X = x]$ can be written as:

$$P[X = x] = \frac{1}{Z} \exp(-U(x))$$

(2)

The normalizing constant $Z$ (the partition function) is given by

$$Z = \sum_{\text{all config.} x} \exp(-U(x))$$

(3)

and $U(x)$ is the (Gibbs) energy function, given by

$$U(x) = \sum_{c \in \mathcal{C}} V_c(x)$$

(4)

with $\mathcal{C}$ being the set of all cliques ([7]). A typical unconditional problem would be to estimate a configuration $x$, such that $P[X = x]$ is maximized, or equivalently, $U(x)$ is minimized ([8]).

We extend the monochrome image observation model as given in ([1]) for the color image as:

$$Y = HX + N$$

(5)

where, $\mathbf{Y}$ is the observed image, $\mathbf{X}$ is the original image, and $\mathbf{N}$ is the corrupting Gaussian noise vector which is assumed to be independent of $\mathbf{X}$, $H$ is the blurring matrix. Note that For an image of size $M \times M$, $\mathbf{X}, \mathbf{Y}, N$, all are lexicographical ordered column vectors of size $3M^2 \times 1$.

Structure of $\mathbf{X}$ will be

$$\mathbf{X} = [X_{0,0}, X_{0,1} \ldots X_{M-1,M-1}]^T$$

(6)

where,

$$X_{i,j} = [x^r(i,j), x^g(i,j), x^b(i,j)]^T, \quad 0 \leq i, j \leq M - 1$$

(7)
Structure of $Y$ and $N$ will be similar to that of $X$. $H$ will be $3M^2 \times 3M^2$ matrix, whose structure is somewhat similar to the one given in (9)) and (10)). Precise structure of $H$ will be

$$H = \begin{pmatrix} H_1 & H_2 & H_3 \\ H_4 & H_5 & H_6 \\ \vdots & \vdots & \vdots \\ H_{3M^2} & H_{3M^2+1} & H_{3M^2+2} \end{pmatrix}$$

and $H_1$ is $3 \times 3$ identity matrix.

Structure of $H$ will be same as that of $H_1$ with $H_1$ replaced by $H_1$. The $H$ matrix is appropriately normalized. The term $\xi$ decides the amount of interchannel blurring operation.

With $X$ and $Y$ as defined in (5), we assume that $X$ is a MRF; thus $X$ has the probability distribution given in (2). With the color image observation model as described above, the color image restoration problem can be cast as: Estimate $X$ such that $P[X = x|Y = y]$ is maximized with respect to $x$.

We further make the assumptions: $N$ is normally distributed with zero mean and covariance matrix $\sigma^2 I$ ($I$ being a $3M^2 \times 3M^2$ identity matrix). Moreover, $N$ is statistically independent of $X$.

In general, the a priori energy function $U(x)$ will have three color planes, as given in (7), non-linearly interacting with each other. Moreover, to take care of discontinuities in the color planes, one could also incorporate horizontal and vertical line fields like corresponding to each color plane. Thus, the most general form for a posteriori energy function could be expressed as

$$U(x, l, v) = f(x^c, x^h, x^v, l^c, l^h, l^v, v^c, v^h)$$

where $l^c$ and $v^c$ represent horizontal and vertical line fields corresponding to the color plane $c$.

We propose two different forms for $U(x, l, v)$:

In the first form we assume that the colors planes are uncorrelated. That is,

$$U_1(x, l, v) = \sum_{c=r,g,b} U_1(x^c, l^c, v^c)$$

and,

$$U_1(x^c, l^c, v^c) = \sum_{i,j} \mu ((x^c_{i,j} - x^c_{i,j-1})^2 (1 - v^c_{i,j}) + (x^c_{i,j} - x^c_{i-1,j})^2 (1 - v^c_{i,j}))$$

where

$$H_\xi = \begin{pmatrix} H_1 & H_1 & H_1 \\ H_1 & H_1 & H_1 \\ \vdots & \vdots & \vdots \\ H_1 & H_1 & H_1 \end{pmatrix}$$

where

In the second form (FOII) we take into account first order interchannel interaction between color planes.

$$U_2(x, l, v) = \sum_{c,r,g,b} \mu ((1 - l_{i,j})(1 - l_{i,j})(x^c_{i,j} - x^{c-1}_{i,j}))((x^c_{i,j} - x^{c-1}_{i,j}))$$

$$+ \gamma [l_{i,j} + v^c_{i,j} + 1/l_{i,j} + l_{i,j} + v^c_{i,j}]$$

for $c, d = r, g, b$

Minimization of the a posteriori cost function can be done using a variety of minimization algorithms. We opted for the simulated annealing algorithm because it guarantees convergence in probability.

3. SIMULATION

Here, we report simulation results for the FOII energy functions given by (13) and compare its performance with the usual NI energy function given by (12). Original image was degraded with different values of $\xi$, but, during restoration $\xi$ was assumed to be 0, that is, we do not have any knowledge of $\xi$. For simulation purpose, we used a uniform, space invariant blur of size $5 \times 5$. White Gaussian noise was added (with $\sigma = 10$) separately on R, G, and B. Reported here are the results for $64 \times 64$ checkerboard type synthetic and $128 \times 128$ Lisa image. Values of $\mu$, $\gamma$ and $\theta^c = \theta^h = \theta^v = \theta$ respectively were 0.025, 200.5 and 0.75, 150 and 20 respectively for Lisa image. Stopping criterion for SA was 1250 for both images. Initial temperature was 5.5 and inverse log cooling schedule was used. The value $\xi$ selected was 0. All these parameters were selected such that we get acceptable results with both the methods.

To validate the performance, signal to noise ratio (SNR) of degraded and estimated images are computed as:

$$SNR = 10 \log \frac{||x||^2}{||x - \hat{x}||^2}$$

with $\hat{x}$ being the estimated image. Results of our simulation is tabulated. Few of the sample images are given at the end.

$$^1 \alpha(z) = 1 \text{ if } z > 0 \text{ and } 0 \text{ otherwise}$$
Table 1: SNR Values for Lisa image

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<th>ξ</th>
<th>R</th>
<th>G</th>
<th>B</th>
<th>R</th>
<th>G</th>
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Table 2: SNR Values for Synthetic image

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4. DISCUSSIONS AND CONCLUSIONS

As can be seen from the table, the proposed FOII performs better than the NI model, for different values of ξ. Restored images seem to be sharp and look better. Also, the model is robust to some extent, since we are able to obtain these results without any knowledge of ξ. Performance of NI and FOII were found similar for ξ = 0, which is expected.

Proposed model also worked satisfactorily when degradation was done in other color coordinates. For example, we linearly degraded in the YIQ and Ohta's ([11]) \( I_1, I_2, I_3 \) coordinates, and then transform the resulting image to RGB domain, then, this will effectively do the interchannel blurring with unknown ξ. Encouraging results were obtained with FOII model. These results are reported in [12].

However, proposed model is not an optimal model. SNR improvements are not all that satisfactory for highly textured images (like wings of parrot). This may be due to the fact that some of the informations may be irrecoverably lost because of the 5 \( \times \) 5 blur. However, for comparatively smooth image (faces, for example) the proposed algorithm does satisfactorily.

5. REFERENCES


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Results for $\xi = 0.8$

Original Degraded

Original Degraded

Restored Images

Restored Images

Results for $\xi = 0.6$

Original Degraded

Original Degraded

NI FOII

NI FOII

Figure 1: Simulation Results Lisa image

Figure 2: Simulation Results synthetic image

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