Polar classification with correspondence analysis for fault isolation

Sonia Pusha, Ravindra Gudi, Santosh Noronha

Department of Chemical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai 400076, India

Keywords:
Polar angle base classification
Correspondence analysis
Fault detection and isolation

Abstract

Data collected from operating plants can be mined to extract information related to both normal and fault modes of operation. Correspondence analysis (CA), that decomposes a measure of row–column association, to generate the lower dimensional space has been recently proposed [1] for this task. CA represents the association between samples and variables in terms of angle based measures on a biplot. Thus, toward clearer resolution of the faults, polar clustering and classification procedures are necessary. In this paper, we develop a methodology to mine the operating data and build such clusters. We demonstrate the application of this methodology on data generated from simulations and experiments involving representative systems, for detecting parametric changes and resolving sensor and actuator biases.

1. Introduction

Data based approaches to fault detection and diagnosis (FDD) in the process industry typically involve either a comparison of plant operation with an a priori established template of normal operation, or involve matching of plant signatures with known fault modes of operation. In the former approach, accurate representation of normal operation is an important requirement for efficient detection. In the second approach, discriminants between the known fault modes are first identified and subsequently a data signature obtained on plant operation is classified to one of the modes.

Principal component analysis (PCA) is typically used to project multivariate operating data onto fewer dimensions for analysis [2]. Relevant process information is described using a mutually orthogonal linear combination of the original variables. From this lower dimension statistical model, faults can be easily detected using $T^2$ and $Q$ statistics and diagnosed using contribution plots. However, PCA has been shown to be predominantly representation oriented and perhaps not well suited for classification [3]. Algorithms such as Fisher discrimination analysis (FDA) have been shown to be better suited for resolution of normal and faulty data [3,4]. However, FDA is a supervised algorithm that requires labeling of the training samples and is therefore relatively difficult to use; further PCA and FDA require expansion of the column space to accommodate dynamic or serial correlation in the data.

Towards addressing the above problems, correspondence analysis (CA, [5]) was proposed for enhanced discrimination, while retaining the unsupervisory nature of the algorithm [1]. Unlike PCA, CA uses a measure of row–column association to represent the sample–variable relationship and has been shown to be effective at capturing cause and effect relationships present in the data [1]. CA also enables the joint projection of variables and samples in a reduced dimensional space called the biplot (for two retained dimensions).

It has also been shown that the biplot enables the analysis of association between the rows (samples) and columns (variables) of the data matrix $X$ in terms of the angles that these rows and column points subtend at the origin, when projected onto the biplot. As mentioned earlier, when the training data contains information for both normal and fault modes of operation, resolution and discrimination between the signatures related to these modes is an important requirement. Further more, such training data can contain information related to the same fault but with different intensities. For example, the inlet reactant concentration in a stirred tank reactor could be a faulty variable that can assume different values and could therefore result in different fault clusters. In such cases it is important to classify the fault to be originating from the same cause but as having a different intensity.

In this paper, we address the above issues and propose a new FDD methodology that uses CA and a novel polar (angle based) classification procedure, for clearer fault resolution. Analysis of similar cause–effect relationships by CA using biplot involves an angle based evaluation of similarity. The relationship between variables and samples is manifested in terms of angles subtended on the CA biplot, with similarly behaving variables, or strongly correlated samples and variables, lying along a particular direction. Conventional classification approaches involve the use of scaled distances such as Euclidean or Mahalonobis distances as proximity measures. However, these measures are not suitable when the data is transformed such that directional trends are more informative than relative distances. For example, when the earlier mentioned
fault related to inlet concentration occurs with two different intensities, the samples would project to two different radial locations on a biplot; however they would both subtend the same angle with respect to the centroid of the biplot. Traditional clustering methods would flag this scenario as resulting from two different faults while ideally this should be identified as the same fault occurring at different intensities. To overcome this drawback, a novel polar clustering algorithm proposed in this paper is shown to unify clusters containing points that exhibit similar variable–sample relationships. It is also important that these relationships are discriminated to the maximum extent to prevent overlap between clusters. Toward this end we show that the CA algorithm although being unsupervised, enables maximal discrimination between the different operating modes. The proposed CA methodology along with the polar classification procedure has been validated on data generated from simulations and experiments on a quadruple tank set up [6] and a distillation column [7].

This paper is organized as follows. In the next section, we briefly define the CA algorithm and highlight the row column association on the biplot. We then present the development of the polar clustering methodology wherein we also evaluate a validity measure of cluster integrity. Results and discussions of the systems analyzed are subsequently presented.

2. Overview of correspondence analysis (CA)

Correspondence analysis [5] is a dimensional reduction technique which is designed to analyze correspondence or associations between the rows, the columns, and the joint variation between rows and columns of a data matrix \(X \in \mathbb{R}^{n \times m}\). It detects row column associations and has also been shown to identify nonlinear dependencies present in the data matrix [1]. For each row point that spans the \(n\)-dimensional space and each column point that spans the \(m\)-dimensional space, this multivariate technique identifies a set of coordinate axes spanning a lower dimension subspace that represents the joint association between these row and column points. When two coordinate axes are chosen, the projection of the row and column points results in a biplot. This joint visualization of the row (sample) and column points on the same plot enables the analysis of their association. A brief development of the CA formulation is presented below.

In CA, the expected value \(E_{ij}\) of each element \(x_{ij}\) in \(X\) is first estimated under the assumption of statistical independence of rows and columns as

\[
E_{ij} = \frac{r_i c_j}{g}
\]

(1)

where, \(r_i\) is the sum of \(i\)th row, \(c_j\) is the sum of \(j\)th column and \(g\) is the sum of all the entries in \(X\). Next, the corresponding observed values \(x_{ij}\) are compared with these estimated values \(E_{ij}\) using the statistic \(t\) defined as

\[
T = \sum \sum \frac{(x_{ij} - E_{ij})^2}{E_{ij}}
\]

(2)

It can be shown that [5], under the hypothesis of independence, the variable \(t\) has a \(\chi^2\) distribution and, therefore, any significant departure from the assumption of independence (i.e., due to the presence of row–column associations) can be systematically analyzed. This is typically done by constructing a matrix \(C\) of the deviation values for each element in \(x_{ij}\), which is written as

\[
C_{ij} = \frac{(x_{ij} - E_{ij})}{E_{ij}}
\]

(3)

The elements \(C_{ij}\) may be viewed as measuring the weighted departure between the observed \(x_{ij}\) and the predicted values, \(E_{ij}\), under the assumption of statistical independence. The elements \(C_{ij}\) thus measure the association between the rows and columns of \(X\). They also represent a value of the inertia, which is an equivalent of variance in PCA, scaled by the grand sum \(g\). To assess the associations represented by the scaled inertia values in \(X\), the matrix is subjected to singular value decomposition to yield

\[
C = AB^T
\]

(4)

In the above decomposition \(A\) and \(B\) contain eigenvectors of \(CC^T\) and \(C^TC\), respectively. \(\lambda\) is a diagonal matrix whose entries represent the singular values of matrix \(C\). If \(k\) is the rank of \(C\), then it can be shown using matrix theory that

\[
\text{trace}(CC^T) = \sum_{i=1}^{k} \lambda_i = \sum_{j=1}^{m} C_{ij}^2 = t
\]

(5)

which justifies the choice of the statistic \(t\) and also shows that the SVD of this measure of deviation can be interpreted as decomposing the total \(\chi^2\) value, rather than the variance.

The vectors corresponding to \(A\) and \(B\) give the principal axes (loading vectors) for row points and column points respectively. The coordinates/score values of row points and column points can be computed by projecting them onto matrices \(A\) and \(B\) and are given by vectors of \(F\) and \(G\) matrices [8,1], which represent the score values for the row and column clouds respectively. These are given by

\[
F_{ij} = \frac{A_{ij} \sqrt{\lambda_i}}{\sqrt{f_j}}
\]

(6)

\[
G_{ij} = \frac{B_{ij} \sqrt{\lambda_i}}{\sqrt{g_i}}
\]

(7)

After calculation of new loading vectors and score values, the next important step is to fix the number of principal axes to be retained. This number can be calculated on the basis of the inertia explained by each axis and is denoted by

\[
\text{INexp(jth axis)} = \frac{\lambda_j^2}{\sum \lambda_i^2} \quad \forall i = 1, 2, \ldots, m
\]

(8)

The total inertia explained by the axes retained provides a measure of the accuracy of the lower dimensional space representation. In most practical cases, the first two principal axes account for major variations in the original data leading to dimension reduction [9]. A graphical representation is obtained by plotting the first two columns of \(F\) (representing row points) and \(G\) (representing column points). The resulting biplot facilitates simultaneous analysis of associations between rows, columns, and between rows and columns. A simplified representation of the biplot is shown in Fig. 1. The angle between row and column points on the biplot represents the association between them.

The angle measure for a row points can be computed as follows:

\[
\theta_i = \tan^{-1} \left( \frac{F_{i2}}{F_{i1}} \right) \quad \forall i = 1, 2, \ldots, n
\]

(9)

where \(F_{i1}\) and \(F_{i2}\) are the first and second principal vectors of matrix \(F\). Similarly, the angle measure for a column point can be computed as

\[
\psi_i = \tan^{-1} \left( \frac{G_{i2}}{G_{i1}} \right) \quad \forall i = 1, 2, \ldots, m
\]

(10)

where \(G_{i1}\) and \(G_{i2}\) are first two principal axes of matrix \(G\).

The unsupervised classification of associated points is usually not applicable to the data in \(X\). However, as mentioned earlier on a biplot, groups of associated points may be identified by their common direction. An angular measure is clearly more appropriate therefore, for measuring the association between row and column points [10]. The angular distance between two points \((a,b)\), given an origin, may be defined as [11]
where \( a \) and \( b \) can be either a row or a column point.

Smaller the angle between variables and samples, stronger is the relationship between them. Similarly behaving variables (or strongly correlated variables and samples) therefore line up along one particular angular direction on the biplot. If this angle is acute, then two data points (row or column) are positively correlated; if the angle is obtuse, the two are negatively correlated. If the row and column subtend a right angle, then there is no association between them [5]. Next, we discuss the implementation of a polar clustering technique which identifies groups of strongly correlated data on the basis of their angular distance between points.

### 2.1. Polar classification algorithm

Clustering is an unsupervised classification technique [12,13] which is used to group together similar objects based on a similarity measure. Compact clusters are usually achieved by minimizing the overlap between members of a group (the intra-cluster distance) while simultaneously maximizing the separation between the groups (inter-cluster distance). The distances normally employed in such optimization efforts include Euclidean and Mahalonobis distances. However as mentioned before, such distance based metrics are not suitable on a biplot where the similarity measure is in terms of an angle metric. Therefore, in the present work, we use angular distances in a classification frame work to identify clusters that obey similar row–column relationship and look to meet a similar objective. We begin by assuming \( K \) clusters in a biplot for the given dataset. Further, we denote \( \phi_k \) \( (k = 1, 2, \ldots, K) \) to be the cluster centroid of each of these \( K \) clusters. Each of the \( K \) clusters would be represented by a cluster prototype, which is a direction that is maximally represented of all the points in that cluster. We next assign \( \theta_i \) \( (i = 1, 2, \ldots, n) \) to be the feature points that need to be assigned to these clusters. In this analysis, we use the mode to represent a cluster prototype, so as to ensure the influence of outliers are minimized, i.e.,

\[
\phi_k = \text{mode}(S^k) \tag{12}
\]

where \( S^k \) stands for the set of data points (i.e. angles) belongs to \( k \)th cluster and are initially arbitrarily assigned such that \( \{S^1 \cup S^2 \cup S^3 \cup S^K = X \} \). The points that would eventually belong to the cluster \( K \) would be optimally determined via solution of the optimization formulation presented below. First the angular distance between any two points \((a, b)\) is defined according to Eq. (11). An angular distance matrix is constructed between all possible pairings of data points \( \theta_i \) and cluster centroid \( \phi_k \) \((k = 1, 2, \ldots, K)\). From these distances, class labels which follow the following criteria are assigned:

\[
J = \arg \min_k d(\phi_k, \theta_i) \quad \forall k = 1, 2, \ldots, K \tag{13}
\]

\[
S'_i = \theta_i \quad \forall i = 1, 2, \ldots, n \tag{14}
\]

These class labels are used to classify each data point in the data set.

Next, we seek to define an objective function that minimizes the intra-partition distance within the cluster and maximizes the inter-partition distance between the clusters. Towards this end, we characterize a suitable objective function for this angle based clustering that involves the ratio of the intra-partition distances to the inter-partition distances.

The intra-partition distance is defined as the distance between the class prototype \( \phi_k \) and data set \( \theta_i \) (shown in Fig. 2b). Therefore, the total angular intra-partition distance within one cluster can be defined as

\[
d_{\text{intra}} = \sum_{i=1}^{n_i} d(\phi_k, S'_i) \tag{15}
\]
where \( n_k = \text{length}(S_i) \). The total intra-cluster distance is defined as

\[
d_{\text{intra}} = \frac{\sum_{k=1}^{K} ((n_k - 1) \sum_{i=1}^{n_k} d(q_k, S_i))}{n - K}
\]

(16)

The inter-cluster distance is defined as the distance between two cluster prototypes (shown in Fig. 2a) and can be calculated as

\[
d_{\text{inter}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{K} d(q_i, q_j)}{n K}
\]

(17)

Therefore, based on the above a suitable objective function for clustering in polar coordinates space can be defined as

\[
\min J = \min_{\phi_k} \left[ \frac{\sum_{k=1}^{K} ((n_k - 1) \sum_{i=1}^{n_k} d(q_k, S_i)^2)}{(n - K) \sum_{j=1}^{K} \sum_{i=1}^{n_j} d(q_i, q_j)^2} \right]
\]

(18)

The optimal cluster centroids are therefore determined by minimizing the objective function \( J \) subject to the constraints (12) and (19),

\[
0 \leq \phi_k \leq 360
\]

(19)

An analytical solution to this problem is difficult to obtain since both objective function and constraints are nonlinear in nature. Further, sampling distributions for the mode are difficult to obtain. However, this algorithm can be easily implemented using numerical methods for constrained optimization. In our work, we used the constrained optimization solver ‘fmincon’ available in the optimization tool box on the Matlab platform.

2.2. Cluster validation

An important aspect of cluster identification involves validation of any structure discovered in the data. In this work, we have employed a variant of the \( K \)-means clustering algorithm; however it is not always possible to correctly choose the number of cluster \( K \) in advance. Different partitions are likely to result for different choices of \( K \) and this could affect the interpretation of the cluster; which could affect the diagnosis and resolution of the fault. Towards identification of an optimal number of clusters partitions, we therefore validate the integrity of the clusters generated using the following procedure:

1. Initialize the parameters of the clustering except for the number of clusters, \( K \).
2. Run the clustering algorithm for different values of \( K \) over the range \( K = \{2, 3, \ldots, K_{\text{max}}\} \) where \( K_{\text{max}} \) is specified by the user.
3. Compute the validity index (Silhouette index) for each set of partitions obtained from the previous step.
4. Choose an optimal value for \( K \) according to the validity index.

The Silhouette width \([14]\) is an indicator of belonging of a sample \( i \) to a cluster \( j \) and is defined as

\[
S_{ij} = \frac{(b_i - a_i)}{\max(b_i, a_i)}
\]

(20)

where \( a_i \) is the average distance between the \( i \)th sample and all the samples in cluster \( j \), and \( b_i \) is the minimum average distance between the \( i \)th sample and the samples in all clusters except cluster \( j \). \( S_{ij} \) takes on a value between \(-1\) to \(1\). Compact, well separated clusters have \( S_{ij} \) value close to \(1\). An \( S_{ij} \) value is close to \(-1\) is indicative of sample misclassification. The cluster Silhouette value is calculated as

\[
S_j = \frac{\sum_{i=1}^{m} S_{ij}}{m}
\]

(21)

where \( m \) is the number of samples in cluster \( j \). The global Silhouette value that is ultimately used to identify the optimum number of clusters is given as

\[
GS = \frac{\sum_{j=1}^{K} \sum_{i=1}^{n_j} d(q_i, q_j)^2}{m}
\]

(22)

The highest value of \( GS \) gives the optimum number of clusters. A summary of the polar classification and validation approach proposed in this paper is provided in Table 1.

**Remark 1.** It is important to note that for nearly linear case, when the same fault but with different intensities occurs, the points in the biplot lie subtend the same angle at the origin but project onto different distances from the origin. This can be explained as follows.

When faults of the same type but of different intensities occur, those measurements (variables) that reflect the onset of faulty operation will more likely remain the same; the rows (samples) of the data matrix \( X \) corresponding to these instances of differing intensities are therefore linearly dependent upon each other. As a result, the eigen vectors (row vectors of \( A \)) of the matrix are similar and dependent. From Eq. (6)

\[
F_{ij} = \frac{A_{ij}}{\sqrt{r_i}}
\]

and for row \( i \),

\[
F_{i} = \frac{A_{i2}}{(A_{i1}^2 + A_{i2}^2)^{1/2}}
\]

With the eigenvectors being unchanged, row points representative of the same fault but of different intensities subtend the same angle from the principal axis as can be seen from the definition in Eq. (9). It must be noted that the \( y^2 \) distance (Eq. (3)) between row points is obviously affected by the deviation from the normal predicted values explaining the variation along the fault axis. When the variables involved are nonlinearly correlated, the eigenvectors cannot be expected to remain invariant. Consequently, for the nonlinear case, similar faults may be expected to subtend different angles from the principal axis.

3. Case studies

The CA based statistical monitoring procedure presented in Section 2 was used along with the polar classification methodology to mine the data for detecting different fault regions. In the following case studies, we first highlight the discriminative ability of CA by considering the problem of detecting parametric shifts. For this purpose, we consider data generated from simulations involving the nonlinear distillation column of \([7]\). Next, using the experimental data generated from the quadruple tank set up, we evaluate the joint application of CA and the polar classification methodology for the task of classifying faults of differing intensities.
3.1. Detection of parametric changes

The nonlinear distillation column [7] has 40 theoretical stages and separates a binary mixture having a relative volatility of 1.5 into products having a purity of 99%. The nonlinear equations governing the behavior of the distillation column can be found in [7]. Under the assumptions of a (i) binary mixture being present, (ii) constant pressure, (iii) constant relative volatility, (iv) constant molar flows, (v) no vapor holdup, (vi) linear liquid dynamics, (vii) equilibrium on all stages, and with the use of a total condenser, the matlab code for the nonlinear simulation is provided in [7]. The input variables for the distillation column are feed flow (F), boil up (V), reflux ratio (R) and the output variables are condensate composition (X_2) and distillate composition (Y_2). For our study, data related to nominal operation was first generated by simulating the above dynamic model for nominal variations in the input and output variables by generating random binary signals of magnitude 0.0005. Next towards generating fault data, the following parametric perturbations were made in the nonlinear model:

(1) Parametric shifts to the extent of ±13% and ±26% of the nominal value of the relative volatility.

(2) Parametric changes to the valve coefficient on the reflux flow valve to the extent of ±0.37% and ±0.22%.

The data generated during the above nominal and fault modes was considered further for the analysis using the CA algorithm. The data matrix X was constructed using measurements of the input and output variables mentioned above. A total of five hundred steady state data points for each mode of operations (nominal and each fault mode) were included in the data matrix X. The application of CA algorithm resulted in the generation of a lower two dimensional representation of the matrix X. The two principal coordinates were cumulatively able to explain 99% of the total inertia in the system. This lower two dimensional representation (also known as the biplot) is shown in (Fig. 3a). The nominal region of operation is indicated by cluster 1 and accounts for the nominal random variations in the process. Clusters 2 and 3 consist of points corresponding to a parametric variation of negative shift of 13% and 26% in relative volatility whereas cluster 4 and 5 corresponds to the positive shift of 13% and 26% for the same parameter. As can be seen from the (Fig. 3a) an increase in the extent of parametric variation results in an outward migration of the cluster along the straight line that connects the cluster centroid. Thus, it can be inferred that larger distances from the centroid on the biplot imply higher intensities of the relationship (in this case the parametric uncertainty). Fig. 3a also shows the clusters corresponding to the parametric variation on the reflux flow valve. As can be seen from the figure clusters 6 and 7 corresponding to these points are clearly resolved from the earlier clusters. This clear resolution can be attributed to the inherent discriminative ability of the CA algorithm resulting from its objective function [8].

We next consider the case when the data also contains samples corresponding to a simultaneous application of the above two parametric changes. Specifically we consider the cases corresponding to a simultaneous ±26 in the relative volatility and ±0.22 in the reflux valve coefficient. The new clusters corresponding to this simultaneous parametric changes are seen in Fig. 3b. The location of these new clusters are such that the line joining the centroid of these clusters and the nominal cluster approximately bisects the angle formed between the lines relating to single faults. This behavior further reinforces the angle based similarity in the relationships.

3.2. Quadruple tank setup: simulation and experiment evaluation

The quadruple tank process is a multivariate laboratory process with an adjustable zero [6]. Here we validate the proposed methodology involving both CA and polar angle base classification algorithms for detecting faults with varying intensities. We first carry out analysis based on data generated from simulations involving several different fault scenarios. Subsequently, to evaluate the performance using real time data we corroborate our analysis using experimental data generated from the laboratory setup of the quadruple tank system at the Department of Chemical Engineering, IIT Bombay. The nonlinear model derived for the quadruple tank set up can be written as

\[
\frac{dh_i}{dt} = -\frac{A_i}{hi} \sqrt{2gh_1} + \frac{A_i}{hi} \sqrt{2gh_3} + \frac{A_i}{hi} f_1
\]

\[
\frac{dh_2}{dt} = -\frac{A_i}{h_2} \sqrt{2gh_2} + \frac{A_i}{h_2} \sqrt{2gh_4} + \frac{A_i}{h_2} f_2
\]

\[
\frac{dh_3}{dt} = -\frac{A_i}{h_3} \sqrt{2gh_3} + \frac{A_i}{h_3} (1 - \beta) f_2
\]

\[
\frac{dh_4}{dt} = -\frac{A_i}{h_4} \sqrt{2gh_4} + \frac{A_i}{h_4} (1 - \beta) f_1
\]

In the above mentioned model, for tank \(i\), \(h_i\) is the water level, \(A_i\) is the cross-section of the tank and \(a_i\) is the cross-section of the outlet.

Fig. 3. Clusters on CA biplot with different fault intensities for distillation column. (a) Clusters on CA biplot with different fault scenario (both positive negative). (b) Clusters on CA biplot for different fault scenario in relative volatility, valve coefficient and in both relative volatility and valve coefficient.
hole. $\gamma_1$ and $\gamma_2$ are the fractions of the total flow going to the bottom tanks. A constrained MPC controller [15] was used to control the level in the tank by changing the manipulated variables (flow to the control valves). There are two input variables ($U_1$ and $U_2$) representing the power supplied to the control valves, and two output variables ($Y_1$ and $Y_2$) which represent the water levels for the two lower tanks. For the simulations, measurement noise having normal distribution $N(0,0.005)$ is added to all the sensors. To generate fault data, the following biases were introduced in the nonlinear model:

(1) Sensor bias for output variable ($Y_1$ and $Y_2$) with shifts of $\pm 20\%$ and $\pm 30\%$ of the nominal value.
(2) Actuator bias for input variable ($U_1$ and $U_2$) with shifts of $\pm 40\%$ and $\pm 30\%$ of the nominal value.

Simulations were carried out for both normal and different fault scenarios using the above mentioned nonlinear model. The data matrix thus formed contained two inputs and two outputs as column profile points. Normal operating data as well as data from different fault scenarios were taken as row profile points. This data matrix was subjected to CA for identifying the lower dimensional space (on a biplot). For all the cases considered below, it was found that two principal coordinates were able to explain approximately 99% of the total inertia. Thus, it was concluded that an analysis of the relative variable sample relationships can be performed on a biplot (two principal coordinate). To identify different faults and discriminate them with faults having different intensities, the polar classification methodology was next used to perform classification of the points on the biplot. The procedure given in Table 1 was followed to find the optimum number of clusters (faults) and their corresponding angular measures. Next, we discuss the identification of the two faults (i) induced separately with different intensities and (ii) occurring in combination and with different intensities.

3.2.1. Case 1: positive bias in actuator and sensor

Fig. 4 shows the CA biplot when a positive bias is separately introduced into all the inputs and outputs to simulate the occurrence of single fault with varying intensities. The row points around the center of the biplot (Fig. 4) represents the normal operating condition. The outward migration of the data points from the normal operating conditions along one direction represent different faults, and their distances from origin is representative of their intensities. These cluster directions (arrows shown in 4) were obtained by the polar classification method. It is seen from the Fig. 4 that, instances of the same fault, occurring with different intensities, share the same direction; however different faults (represented by actuator or sensor biases) show up along distinct clusters directions. As is also seen in the Fig. 4, the 4 cluster centroid (angular measure) obtained for this case were found to occur at angles $[4^\circ, 26^\circ, 148^\circ, 170^\circ]$ and is shown by appropriate rays originating from the origin.

3.2.2. Case 2: positive and negative bias in sensor

Next, we evaluate the classification of data resulting from a positive and negative bias of magnitude 20% and 30% in each of the sensor $Y_1$ and $Y_2$-four cluster directions were identified by the polar classification method and occurred at angles of $[14^\circ, 165^\circ, 196^\circ, 340^\circ]$. Since the biases are negatively correlated, it is expected that the cluster directions would lie approximately on diagonally opposite side of the origin. This behavior is also seen in Fig. 5 for the positive and negative biases.

3.2.3. Case 3: simultaneous bias in two sensors

We next test the suitability of approach for the identification of simultaneous fault. Specifically we compare the location of the cluster for the simultaneous fault relative to the cluster location corresponding to the individual faults. Fig. 6a shows the behavior of the cluster for the case of simultaneous fault of equal magnitude of 20% and 30% in each of the sensor bias (vertical ray originating from the centroid of the biplot). Also seen in Fig. 6a is the behavior of the cluster on the introduction of single fault of magnitude 20% separately in each sensor. It can be seen that the simultaneous fault cluster approximately bisects the angle subtended by the cluster corresponding to the two biases. On the other hand, when the bias in $Y_1$ is large than bias in $Y_2$ for the simultaneous fault case, Fig. 6b shows the cluster of point migrate toward the cluster of points corresponding to the bias in $Y_1$. This behavior is interesting in that it suggests the possibility of decomposing signature of double (simultaneous) fault in terms of the individual faults. The optimum number of clusters were determined from an analysis of the Silhouette index as a validity measure. In all of the above cases results of cluster validation are presented in Table 2. In all four simulations, the correct number of clusters was identified.

Remark 2. It is important to note that although the same amount of samples were used in the above analysis, the proposed CA based analysis is general enough to accommodate different sample size in each cluster. This has been illustrated in Detroja et. al. [8.1].
3.2.4. Experimental evaluation

Real time data, from the Quadruple tank setup under close loop constrained multivariate control action was collected for all the following scenarios: (i) normal operation, (ii) sensor bias in $Y_1$ and $Y_2$ and (iii) actuator bias in valves $U_1$ and $U_2$. The total real time data having 8000 measurements were collected for all of the above scenarios and accumulated into the data matrix $X$. The data matrix was subjected to correspondence analysis which again revealed that approximate 97% inertia captured in two principal coordinates. The row and column points were further subjected to polar classification analysis, which as expected revealed the existence of four clusters, the number of cluster were validated by the Silhouette index. These four clusters corresponding to each of above faults scenarios occurred at angles $[26^\circ, 31^\circ, 130^\circ, 160^\circ]$ as seen in Fig. 6. As in the simulation case study an outward migration of the clusters for increasing intensity of faults was observed. These experimental results further verify the fault resolution and classification methodology.

It can therefore be inferred that the polar classification method proposed in the paper was capable of clearly distinguishing different faults, and also faults of different intensities.

4. Conclusions

In this paper, we have studied the utility of a polar classification procedure involving the application of correspondence analysis (CA) coupled with an angle based classification. CA is used to generate a biplot from the original data; which achieve good discriminative ability and sharp resolution between the fault clusters while permitting the dimensionality reduction. Hence this method is more suited for fault detection over traditional methods including PCA. A classification objective of minimization of the intra-cluster to inter-cluster angular distance ratio facilitates the partitioning of data into compact and well separated clusters. The proposed fault detection methodology has been validated for quadruple tank system and distillation column via simulations, and also has been validated on experimental data from a quadruple tank system.

The Polar classification method proposed is appropriate when directional trends are more informative than Euclidean-type distance measures. This is in particular true when data is transformed using correspondence analysis for the purpose of identifying correlated variables. As evident from the systems studied, this methodology has direct application in fault detection and isolation (FDI). In addition to classifying data to the appropriate fault, information about the relative intensity of the fault is retained in the form of the radial distance from the biplot origin; this information is related to fault intensity can be useful in deciding on further compensatory action.
References


