Evolution of bubble size distribution in baked foods

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Abstract

A model to predict the bubble size distribution in baked foods and the oven rise during baking process is presented, and the effect of changing various model parameters on the model output is studied. The model incorporates a population balance equation for the formation and expansion of bubbles coupled with reaction kinetics and heat and mass transfer equations. Nucleation and growth of bubbles occur because of production of carbon dioxide in the dough due to decomposition of baking soda, which is assumed to follow a first-order kinetics. The results are compared to experimental bubble size distributions in bread samples, obtained using confocal laser scanning microscopy. The sensitivity of the results to different parameters has been analyzed. The model provides a framework to predict the dynamic evolution of the foam structure, which is an important characteristic in defining the quality of baked foods.

1. Introduction

Leavened baked foods like bread, cake, etc. owe their “quality” mainly to their aerated structure. Many of these food products are characterized by a distinctive aerated structure which is achieved through manipulation of bubbles in the dough during mixing and baking and their subsequent transformation into interconnected gas cells in the crumb. Thus, foam formation is one of the most important features of the baking process. Several studies have attempted to model the bubble growth inside the dough, but most of these are single bubble growth models and do not take into account the bubble size distribution inside the dough.

Shimiya and Yano (1988) analyzed the kinetics of single bubble growth and shrinkage in the dough, at a temperature below 45 °C, by considering the diffusion-induced growth of an isolated bubble resisted by surface tension forces; the viscous resistance of the fluid was neglected. Bloksma (1990) took into account the variations in the solubility of CO2 and water vapor pressure with temperature and produced expressions to estimate the separate contributions of CO2 and water vapor, to oven rise of the dough, at given temperatures. He showed the total relative volume increase in oven rise was by a factor of 3 for a temperature rise of 100 °C, with water vapor making a dominant contribution at high temperatures.

De Cindio and Correra (1995) developed a comprehensive model aiming to predict three ultimate properties: softness (specific volume), firmness (water content), and acidity (pH) in baked products like bread. For bubble growth they assumed the driving force to be diffusion of both CO2 and water vapor and the dough to be characterized by linear viscoelasticity. The model involved growth of a single bubble but the initial bubble size was determined by averaging the experimentally obtained bubble size distribution in the dough. Similar bubble growth models have been developed by Shah et al. (1998) and Fan et al. (1999). Shah et al. (1998) took into account the diffusion of only CO2 and neglected dough rheology, because according to them, the surface tension effects predominate over rheological factors. On the other hand, Fan et al. (1999) assumed a temperature dependent power law rheology, a linear variation in temperature with time and diffusion of both CO2 and water vapor.

Though, Chiotellis and Campbell (2003) extended the work of Shah et al. (1998) to model a bubble size distribution, they did not take into account the continuous nucleation of new bubbles. The objective of the present work is therefore to model the dynamic evolution of the bubble size distribution inside the dough using a general population balance approach which takes into account both nucleation and growth of bubbles. Such an approach will also allow, at some stage, the incorporation of more complex features of the baking process such as coalescence of individual cells into an interconnected network, breaking of bubbles near surfaces and so on. The phenomenon of direct bubble nucleation incorporated in the model might not be very important in case of yeast leavening (Baker and Mize, 1937) as the degree of supersaturation of CO2 is low. However, leavening by chemical leavening agents, such as baking soda, being faster produces a greater driving force and so nucleation can become important, especially, when
2. Model

In the present work we consider a situation where the ingredients of a chemically leavened baked food are mixed and placed, in a cylindrical cup, inside an oven with temperature \( T_s \). The mixture has a temperature \( T_c \) contains baking soda with initial concentration \( C_b \) and has an initial size distribution \( n(a,t=0) \), where \( n(a)da \) describes the number of entrained bubbles of diameter between \( a \) and \( a + da \) at any time \( t \). The baking mixture is subjected to external heating in the oven, due to which its temperature starts rising. Reaction of baking soda and the acids present in baking powder causes evolution of CO\(_2\). As a result, the liquid mixture gets supersaturated with CO\(_2\) which leads to nucleation and growth of bubbles. With time, the baking soda depletes, bubble nucleation and growth slow down and the dough attains its final volume. Thus, the main elements of the model are, heat transfer from surroundings, generation of CO\(_2\), nucleation of new bubbles and growth of existing bubbles. As the age and growth histories of bubbles are not identical, they will not be of equal size in the final product. A population balance equation coupled with the expressions for heat transfer, reaction kinetics, nucleation and growth is solved to obtain the resulting bubble size distribution. The theoretical treatment of the model has been simplified by making the following assumptions:

1. The temperature varies only with time and is uniform throughout the dough. There are no radial or axial variations in temperature and concentration profiles. This “uniform” temperature is obtained by averaging the mild radial temperature profile obtained from transient heat transfer equation. This assumption is based on the fact that properly baked foods are reasonably uniform in physical properties including cellular structure.
2. The growth of the bubbles is only due to the release of CO\(_2\), which is assumed to be an ideal gas, and its transfer from the liquid dough to the bubble is controlled by liquid-side resistance. This is a simplifying assumption based on the dominant role played by the blowing agent as compared to water vapor.
3. The pressure of the whole mold is constant and is equal to atmospheric pressure. The rise in pressure inside the gas bubbles because of the viscoelastic resistance to its growth is negligible. This assumption is true when the extent of dough rise is controllable by the exhaustion of blowing agent rather than hardening of baked material.
4. There is no coalescence or breakage of the bubbles/cells, though this is likely to be a good assumption only in the early part of the baking process.
5. CO\(_2\) does not escape out of the dough and all of it is used for the expansion of the foam.

2.1. Mathematical formulation

2.1.1. Heat transfer

Heat from an external source is provided to the dough in a cylindrical cup of radius \( R \), maintained at a constant surface temperature \( T_s \), giving rise to a transient temperature profile inside the dough (Fig. 1). The unsteady state heat transfer equation for radial heat flow into an infinite cylinder is:

\[
\frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \alpha \frac{\partial T}{\partial r} \right) \tag{1}
\]

The thermal diffusivity, \( \alpha \), is a function of temperature \( T \), which in turn is a function of both time \( t \) and radial position \( r \) inside the dough. Thus, \( \alpha \) is a function of both \( r \) and \( t \). But as a simplification \( \alpha \) is assumed to be dependent on time only as its variation with position is rather small. Now, \( T(r,t) \) in the above equation can be
shown to be a Bessel’s function of zero order (Kreyszig, 1999). The series solution for this zero order Bessel’s equation can be obtained as:

\[ T(r, t) = T_s - 2(T_s - T_i) \sum_{n=1}^{\infty} \frac{J_n(\xi_n)}{\xi_n^n} e^{t \frac{3}{a^2} \int_{0}^{\infty} \xi^2 d\xi} \]  

where, \( \xi = r/R \) is the dimensionless radius of the cup, \( T_i \) is the initial temperature of the mixture, and \( \xi_k \) is the kth root of \( J_0(\xi) = 0 \). The spatial average of temperature as defined by:

\[ T_{avg}(t) = \int_{0}^{R} \frac{T(r, t) r^2}{\pi R^2} dr \]  

can be shown to be:

\[ T_{avg}(t) = T_s - 4(T_s - T_i) \sum_{n=1}^{\infty} \frac{1}{\xi_n^3} e^{t \frac{3}{a^2} \int_{0}^{\infty} \xi^2 d\xi} \]  

2.1.2. Overall mass balance

Since the transport of material across the system boundary (gas escape) has been assumed to be negligible, mass of the system (dough and bubbles) remains constant and it will be equal to initial mass, i.e.,

\[ m_0 = m(t) \Rightarrow \rho(t) = \frac{\rho_0 H_0}{H(t)} \]  

2.1.3. Mass transfer coefficient

For a spherical bubble of diameter \( a \), the mass transfer coefficient, \( k_m \), for transfer of CO2 from liquid to the bubble phase is given by:

\[ Sh = \frac{k_m a}{D} \]  

where, \( Sh \) is the Sherwood number and \( D \) is the diffusion coefficient of CO2 in the liquid phase.

2.1.4. Bubble growth

Let \( Q(a) \) be the overall rate of transport of CO2 into a bubble of diameter \( a \). If the number of bubbles between \( a \) and \( a + da \) is \( n(a) da \), the rate of change of mass of blowing agent in the dispersed phase (bubbles) is given by:

\[ \frac{dm}{dt} = \frac{d}{dt}(V_B M CO_2 C_B) = \int_{a_0}^{a} M CO_2 Q(a) n(a) da \]  

where, \( M CO_2 \) is the molecular weight of CO2, \( C_B \) is kmol of CO2 in dispersed phase per m³ of liquid phase having volume \( V_B \) and \( da \) is the critical bubble nucleation size. The rate of mass transfer of CO2 into a bubble of diameter \( a \) can be expressed as:

\[ Q(a) = k_m \pi a^2 (C_B - C_g) = k_m \pi a^2 \Delta C_B \]  

where, \( C_B \) and \( C_g \) are the concentrations of CO2 in the bulk liquid phase and at the bubble–liquid interface, respectively. The growth rate of a bubble can be obtained from the mass balance over a single bubble. Assuming ideal behavior of the vapor phase:

\[ \frac{d}{dt} \left( \frac{\pi a^4 P}{RT} \right) = Q(a) = k_m \pi a^2 \Delta C_B \]  

\[ \frac{da}{dt} = G = 2 \frac{ShD RT}{a} \Delta C_B + \frac{a}{3T} \frac{dT}{dt} - G_c + \frac{a}{3T} \frac{dT}{dt} \]  

The first term in the above expression corresponds to bubble growth due to supersaturation and the second term corresponds to growth due to temperature rise. \( G_c \) is a fitted parameter called the growth coefficient.

The gas bubble contains CO2 as the blowing agent and the pressure \( P \) inside it is assumed to be constant. The amount of CO2 at the interface is given by a modified Henry’s Law (Fan et al., 1999). The value of the interfacial concentration, \( C_g \), is given by the following equations:

\[ P = k_s X_B \]  

\[ k_n = \frac{\Delta H_c}{R} \left( \frac{1}{T} - \frac{1}{T_0} \right) \]  

where, \( X_B \) is \( C_B \) expressed as a mole fraction and \( k_s \) and \( k_n \) are the Henry’s constants at temperatures \( T \) and \( T_0 \), respectively, \( \Delta H_c \) being the heat of solution of CO2 in dough.

2.1.5. Reaction kinetics

The generation of CO2 from baking soda is modeled as a first-order reaction:

\[ \frac{dC_{NaHCO3}}{dt} = -k_C n_{NaHCO3} \]  

where, \( C_{NaHCO3} \) is the kmol of CO2 produced per m³ volume of liquid phase. The reaction constant \( k \) varies with temperature, following the Arrhenius equation, having an activation energy \( E_a \) (Laidler, 1997). The overall mass balance for the liquid phase is:

\[ \frac{d(V_B C_B)}{dt} = kV_B C_{NaHCO3} - \int_{a_0}^{a} Q(a) n(a) da \]  

where, \( V_B \) is the volume of the bulk liquid containing the dissolved soda and having CO2 concentration \( C_B \).

2.1.6. Nucleation rate

The classical theory of nucleation suggests that the rate of nucleation depends on the free energy change due to formation of a nucleus of critical size. Han and Han (1990a,b) have provided an expression for nucleation rate incorporating the extent of supersaturation, into the classical nucleation theory, for systems like polymer foam. A similar situation applies to baking mixture supersaturated with CO2. The rate of nucleation \( J \) is given by (Han and Han, 1990a,b):

\[ J = MB_{exp} \left( \frac{\Delta F'}{nkT} \right) \]  

where, the net free energy required for the formation of a critical nucleus is:

\[ \Delta F' = \frac{16 \pi \sigma^3}{3(P_c - P)^2} - nkT \ln \left( \frac{C_B}{C_g} \right) \]
The average temperature profile is obtained from Eq. (4). First 95 terms of the series in Eq. (4) were included in the computation which led the temperature profile to converge within less than 1 × 10^{-3} K. The value of thermal diffusivity \( \alpha \) varies very slowly with time relative to the change in temperature itself. Thus \( \alpha \) can be assumed to be constant at any instant. To relate the heat transfer with the properties of the dough at any given time, a functional dependence of thermal conductivity \( k \) on density, temperature and moisture content has been taken. For this, a neural network model developed by Sablani et al. (2002) is used. The thermal diffusivity can be found from the following definition:

\[
\alpha = \frac{k}{\rho C_f} \tag{21}
\]

The specific heat \( C_f \) (in J kg^{-1} K^{-1}) was calculated using the following model developed for specific heats of different bread samples by Baik et al. (1999):

\[
C_f = 7107w + 18.7T - 45.3wT \tag{22}
\]

where, \( T \) is the product temperature in °C and \( w \) is the wet basis moisture content in mass fraction.

The growth rate of a bubble of a given volume at a given time is obtained by solving Eq. (10). The analytical expressions for \( dT/dt \) in this equation can be obtained by differentiating Eq. (4) as:

\[
\frac{dT}{dt} = \alpha(t)\frac{4(T_s - T_i)}{R^2} \sum_{i=1}^{\infty} \frac{1}{\tau_i} \int_{0}^{t} g_i(y)dy \tag{23}
\]

The nucleation rate is obtained from Eq. (15). For both, the nucleation rate and the bubble growth rate equations, we need \( \Delta C_m \). The rate of generation of CO\(_2\) is given by Eq. (13) while Eq. (11) gives the interfacial concentration of CO\(_2\). The amount of CO\(_2\) transferred into the bubbles, calculated from the present volume, is subtracted from the total CO\(_2\) generated to obtain CO\(_2\) concentration in the dough phase.

The next step is to simplify the problem of evaluating the total volume. For this a discretization method, which considers particles of different sizes to exist in groups and interact collectively with particles in other groups (Kostoglou and Karabelas, 1994), has emerged as accurate and computationally efficient. The continuous bubble size distribution has been discretized into bins. The \( i \)th bin containing all the bubbles between diameters \( a_i \) and \( a_{i+1} \) has an average diameter \( x_i \) which is the representative volume of that bin. The population balance equation at time instant \( t \) now reduces to:

\[
\frac{dp_i}{dt} = \int_{a_{i+1}}^{a_i} J(a, t)V_s da \tag{24}
\]

with

\[
\frac{dp_i}{dt} = G(a_i, t) \tag{25}
\]

\[
\frac{dx_i}{dt} = G(x_i, t) \tag{26}
\]

\[
\frac{da_{i+1}}{dt} = G(a_{i+1}, t) \tag{27}
\]

\( p_i \), the number of bubbles in the \( i \)th bin, is the discretized form of \( p(a) = n(a) da \). The expression for \( G \), the bubble growth, is given by Eq. (10).

In each time iteration, the size of the mean diameter and of the limits defining the bin is varied according to the growth function. Also at each iteration the amount of bubbles nucleated in each bin is evaluated. Thus, the problem of simultaneous nucleation and growth is solved. Size distribution of bubbles of different ages is found by numerical integration of Eq. (24). The normalized bubble size distribution is defined as:

\[
N_i = \frac{n_i}{p_{a_{i+1} - a_i}} \sum_{i=1}^{\infty} p_{a_{i+1} - a_i} \tag{28}
\]

The average bubble size \( \langle a \rangle \) is defined as:

\[
\langle a \rangle = \frac{\sum_{i=1}^{\infty} p_{a_{i+1} - a_i} x_i}{\sum_{i=1}^{\infty} p_{a_{i+1} - a_i}} \tag{29}
\]
Image-Pro Plus 6.0 image analysis software. Fig. 2 shows the CLSM micrograph of the dough and the baked sample.

4. Results and discussion

Initial dough temperature is taken to be 27 °C and the cylindrical cup surface is assumed to be at a constant temperature of 200 °C. Table 1 shows the physical properties of the system and values of model parameters. The moisture content inside the dough, needed as an input for thermal conductivity computation, was taken to be 38.5%. This represented an average value of the moisture content of the sample during the baking process with the wet dough (moisture content, 43.5–46.1%) gradually transforming to a baked bread (moisture content, 28–36%) and is within the range reported by Sablani et al. (2002). The thermal conductivity calculated from this moisture content was used as an average k value of the sample during the baking process. The critical nucleation diameter is taken to be equal to the smallest bubble size in the dough samples, estimated roughly from the CLSM micrographs. According to McGee (1984), when 5 g of baking soda is added to 240 g of flour, 2.5 g of CO₂ is produced in about 6 min. The pre-exponential factor and the activation energy in the Arrhenius expression for the first-order rate constant for generation of CO₂ have been chosen such that the rate of CO₂ generation matches this experimental observation. The frequency factor, B in nucleation rate expression Eq. (15) and the growth coefficient, G in Eq. (10) were fitted to match the rate of oven rise from literature (Junge and Hoseney, 1981) and the average bubble size obtained from CLSM imaging. The soda concentration was chosen such that final volume matches with that reported by Junge and Hoseney (1981).

The computed variation of temperature with time is shown in Fig. 3. The temperature of the walls of the cylindrical cup is constant and thus the driving force for heat transfer decreases as the system temperature increases. Also, the thermal conductivity varies with density and temperature. Both these changes give rise to the observed variation in temperature with time. It is important

![Fig. 2. CLSM micrograph of (a) fermented dough (b) baked product. Scale bar = 50 μm.](image)

![Fig. 3. Variation of average temperature inside the dough with time during the baking process.](image)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface tension</td>
<td>α</td>
<td>0.04</td>
<td>N m⁻¹</td>
</tr>
<tr>
<td>Initial dough density</td>
<td>ρ</td>
<td>1100</td>
<td>kg m⁻³</td>
</tr>
<tr>
<td>Moisture content</td>
<td>w</td>
<td>38.5</td>
<td>%</td>
</tr>
<tr>
<td>Henry's constant at 300 K</td>
<td>Kₐ₀</td>
<td>7.03 x 10⁹</td>
<td>Pa</td>
</tr>
<tr>
<td>Heat of solution of CO₂ in dough</td>
<td>ΔHₕ</td>
<td>1.94 x 10⁴</td>
<td>kJ mol⁻¹</td>
</tr>
<tr>
<td>Critical nucleate diameter</td>
<td>aₙ</td>
<td>5 x 10⁻⁶</td>
<td>m</td>
</tr>
<tr>
<td>Frequency factor in nucleation rate expression</td>
<td>B</td>
<td>1.0 x 10⁻¹²</td>
<td>m⁻¹ s⁻¹</td>
</tr>
<tr>
<td>Bubble growth coefficient</td>
<td>Gᵥ</td>
<td>2.46 x 10⁻¹¹</td>
<td>m³ mol⁻¹ K⁻¹ s⁻¹</td>
</tr>
<tr>
<td>Initial concentration of baking soda</td>
<td>C₀</td>
<td>0.14</td>
<td>kmol m⁻³</td>
</tr>
<tr>
<td>Leavening reaction rate constant at 300 K</td>
<td>k₀</td>
<td>3.0 x 10⁻⁶</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>Activation energy of leavening reaction</td>
<td>E₀</td>
<td>20</td>
<td>kJ mol⁻¹</td>
</tr>
</tbody>
</table>

a From Vliet et al. (1992).
b From Sablani et al. (2002).
c From Hibberd and Parker (1976).
d Obtained from data on solubility of CO₂ in water (Loomis, 1928).
to mention here that the dough temperature rise, as determined by the model, is higher than the expected maximum temperature in the dough during baking on account of neglecting heat consumption by water evaporation.

Figs. 4–6 show rise in relative dough size with time, normalized bubble size distribution at different times and change in average bubble size with time, respectively. Initially, as there are no bubbles in the system and since the concentration of CO₂ is also very low, both nucleation and growth are very slow and the volume increases slowly as seen in Fig. 4. As soon the as the CO₂ concentration in the bulk liquid phase increases beyond the CO₂ concentration at bubble–liquid interface, mass transfer and nucleation increase leading to a faster rate of increase in the volume. At large times the rate decreases again due to exhaustion of the blowing agent that generates CO₂. Total increase in relative volume was approximately 5.3 and so the final volume attained is about six times the initial volume of the dough (Fig. 4). This is typically in agreement with the experimental results obtained by Junge and Hoseney (1981). The initial relative volume was taken to be 1.0 in this study, and not 3.6 (Moore and Hoseney, 1985) as the proposed model assumes no air bubbles to be present in the system initially (but were nucleated during ovens rise itself). Rapid increase in both nucleation and growth due to production of CO₂ and their gradual decrease due to exhaustion of the blowing agent gives rise to the observed bubble size distribution. The mean bubble size and also the spread around the mean increases with time (Figs. 5 and 6). Fig. 5 also compares the final bubble size distribution function to the experimentally observed distribution in the baked product. It can be seen that although the two distribution are generally in agreement with each other, the model results do not account for relatively large bubbles (greater than 40 µm) seen in the experimental distribution. This clearly shows the importance of bubble coalescence, especially during the higher baking time, which is not included in the population balance equation for the present model.

Fig. 7(a) shows the variation of the nucleation rate with time, which is initially low, rises with an increase in the concentration of CO₂ and falls again because of CO₂ depletion. It was observed that if the bubbles present initially (incorporated during mixing) are accounted for in the model, the nucleation rate decreases to a

![Fig. 4. Comparison of volume rise predicted by the model with the published experimental data (Junge and Hoseney, 1981). Best fit values: $G = 2.46 \times 10^{-11} \text{m}^2\text{kmol}^{-1}\text{K}^{-1}\text{s}^{-1}$ and $B = 1.0 \times 10^{-12} \text{s}^{-1}$. For experimental data, the origin of time axis has been shifted to the point where the model volume becomes equal to the initial experimental volume (3.6).](image)

![Fig. 5. The normalized bubble size distribution function at different times during baking. The curve for $t = 2000$ s represents the distribution at the end of baking and is compared with experiments. The experimental data shown is the typical bubble size distribution obtained from CLSM images of bread samples prepared using straight dough process and baked at 200 °C.](image)

![Fig. 6. Variation of average bubble size in the dough with time during the baking process.](image)

![Fig. 7. (a) Variation of bubble nucleation rate with time and the effect of bubbles present in the system at $t = 0$ on the nucleation rate. (b) Effect of bubbles present in the system at $t = 0$ on volume rise during baking.](image)
great extent, as expected. This happens because the supersaturation of \( \text{CO}_2 \) is used up in the growth of the bubbles already present. Fig. 7(a) and (b) shows the effect of bubbles that are already present in the system (at \( t = 0 \)) on nucleation rate and volume rise, respectively. Four different cases are shown. Case 4 corresponds to the situation where, at \( t = 0 \), the relative volume of dough is 3.6 (same as that in the experimental data by Junge and Hoseney, 1981) For this case total number of bubbles in the system, at \( t = 0 \), is calculated to be \( 3.97 \times 10^{16} \) bubbles/m\(^3\) of liquid dough.

The variation of \( \text{CO}_2 \) concentration at bubble interface and in the liquid phase, with time, is shown in Fig. 8. Initially, the rate of generation of \( \text{CO}_2 \) is very high because of high soda concentration, but, because the number of bubbles in the system is very small, the rate of its transfer to the gas phase is rather low. Thus, \( \text{CO}_2 \) concentration in the liquid phase starts rising rapidly. Later, depletion of baking soda and increase in number of bubbles lead to a lower generation rate and a higher consumption rate of \( \text{CO}_2 \), respectively, which leads to a fall in its concentration. Further, the concentration at the bubble interface decreases with time due to the increase in temperature leading to reduced solubility.

4.1. Effect of different parameters

The effect of change in various model parameters on the model output has been studied. The parameters are changed one at a time while keeping the others constant and the consequent effect on the rise in relative volume and the average bubble size is analyzed. Different parameters that have been studied are: Rate constant at \( 300 \text{ K} \) \( (k_0) \) for first-order rate of generation of \( \text{CO}_2 \), activation energy of this reaction \( (E_a) \), initial concentration of baking soda \( (C_0) \), frequency factor in the nucleation rate expression \( (B) \), bubble growth coefficient \( (G_c) \) governing the mass transfer from the liquid phase to the bubble, and, critical nucleation diameter \( (a_\text{c}) \). Except \( E_a \) and \( C_0 \), all other parameters have been varied over a span of one order of magnitude.

A decrease in the values of \( k_0 \) and \( E_a \) effectively slows down the rate at which \( \text{CO}_2 \) is generated in the system. A slower rate of \( \text{CO}_2 \) generation slows down the initial nucleation and growth and shifts the volume rise versus time curve towards right on the time axis. It was found that a decrease in activation energy by 20 \( \text{kJ/mol} \) or a decrease in \( k_0 \) by one order of magnitude delayed the volume rise by about two minutes. However, neither the final average bubble size nor the rate of volume rise (slope of the linear part of volume rise versus time curve) are affected much by these parameters. A change in \( C_0 \) leads to an almost proportional change in volume rise but only a small change in average bubble size. \( G_c \) and \( B \) are important parameters which control the rates of growth and nucleation of bubbles, respectively. The effect of these parameters on volume rise and average bubble size in the dough is shown in Fig. 9. Higher values of these parameters lead to higher rates of volume rise, but the final volume achieved is the same (soda concentration being the same). An increase in \( G_c \) means higher bubble growth rate leading to higher average bubble size. However, a higher value of \( B \) means more nucleation, which decreases the growth rate and thus leads to bubbles of smaller size. Also, an increase in \( G_c \) will make the bubble size distribution broader whereas an increase in \( B \) will make it narrower. Changing critical nucleate size from 10 \( \mu \text{m} \) to 1 \( \mu \text{m} \) causes almost no change in volume rise and only about a 5% decrease in final average bubble size.

This study of the effect of model parameters shows that, while final volume depends only on blowing agent concentration, parameters that have the most direct effect on rate of volume rise and average bubble size (i.e., also the actual distribution) are the nucleation frequency factor \( B \) and the bubble growth coefficient \( G_c \). Other parameters have a relatively small effect on the output. This means that the rate of \( \text{CO}_2 \) generation is relatively fast and the rate limiting factors in volume rise are diffusion in the dough and activation energy for nucleate formation.

4.2. Discussion on model assumptions

Although the model developed in this paper accounts for some of the important features of dough expansion and provides the basic framework to predict the bubble size distribution, it is necessary to examine some of the major assumptions made in model development. Firstly, it was assumed that the growth of bubbles is only due to \( \text{CO}_2 \) and the effect of evaporation of water vapor was neglected. This caused the temperature rise to be over-predicted. Experimental works (Hastani et al., 1992; Zanoni and Peri, 1993; Lostie et al., 2002) show that the crumb temperatures show a plateau around 100 °C after initial heating up. If evaporation of
water is taken into account, the predicted dough temperature will be lower because of the heat absorbed by the evaporating water. This will also reduce the contribution of CO₂ to bubble growth because, at a lower temperature, a given amount of CO₂ inside the bubble will occupy lesser volume and its solubility in liquid phase will also increase. However, this is compensated by water vapor also playing a role in bubble growth and thus the overall rate of oven rise and bubble size distribution may not change much. The above mentioned experimental studies also suggest liquid phase diffusion of water within the dough and formation of an evaporation front close to the outer surface of the bread which acts as a boundary between the dry crust and the crumb. These phenomena cause significant temperature drop at the crust leaving the temperature within the crumb to be nearly uniform. Thus, assuming uniform temperature within the dough (crumb) is a reasonable assumption at this stage.

Secondly, the viscoelastic resistances offered by the liquid dough to the bubble growth, effects of surface tension and bubble coalescence have not been taken into account. While, viscoelastic effects may lower the bubble growth, coalescence may alter the bubble size distribution by formation of bigger cells making the bubble size distribution broader. Towards the end of the baking process, the dough begins to set and the bubble boundaries break up to form an open network of pores, releasing the trapped gas (Autio and Laurikainen, 1997). This effect is not incorporated into the present model. Our model describes the situation where there is no gas loss from the dough (just enough blowing agent). Additives such as high melting point fats affect dough rheology and improve bubble stability and gas retention. A mechanism for this has been described by Brooker (1996). Such systems are likely to be closer in behavior to the physics described by the proposed model.

5. Conclusion

A population balance based mathematical model of bubble nucleation and growth during baking driven by chemical leavening agents, such as baking soda, has been proposed to overcome shortcomings of the previous models which are based mostly on single bubble growth and do not use a bubble size distribution.

In absence of any structural or rheological constraints, which is a simplification, the oven rise mainly depends on the amount of blowing agent present and temperature of the dough. Its rate of rise and the actual bubble size distribution depend upon the nucleation and growth rate of the bubbles while baking. Other parameters such as rate of CO₂ generation, nucleation frequency, mass transfer coefficient, etc. also affect the bubble size distribution and time in which final volume and bubble size distribution are reached.

References