Optimization of solar water heating systems through water replenishment

Govind N. Kulkarni, Shireesh B. Kedare, Santanu Bandopadhyay *

Energy Science and Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai 400 076, India

Keywords:
Water replenishment profile
Design space
Solar water heating
System optimization

ABSTRACT

In a typical solar water heating system, cold water is replenished into the storage tank as soon as the load is served. However, it is possible to determine the water replenishment profile (i.e., the quantity of the cold makeup water to be supplied to the storage tank over a day) that optimizes the overall system. In this paper, the effect of water replenishment on the system sizing is studied and a novel strategy for water replenishment is proposed to improve the design and performance of solar water heating systems.

Based on an analytical technique, an approximate water replenishment profile is proposed to size a solar water heating system near-optimally. The problem is analyzed using a methodology called design space approach. Design space of a solar water heating system identifies all possible and feasible designs on a collector area vs. storage volume diagram. For illustration of the proposed methodology, an example problem is solved. It is observed that the annualized system cost can be reduced by 13.7%. For the cost-optimal system configuration, a reduction of 12.7% in the collector area and 10.2% reduction in the storage volume are observed. The proposed methodology is particularly important and advantageous for large commercial and industrial solar water heating systems.

1. Introduction

In a typical solar water heating system, cold makeup water is added into the storage tank as soon as the hot water is drawn to serve the load. Mixing of cold makeup water with the hot water of the storage tank generate significant amount of entropy. It affects the thermodynamic quality of hot water supplied to the load. The mixing process reduces the storage temperature. The storage temperature continues to drop during successive draw offs over the day, till enough sunshine is available. Storage tank cooling due to makeup water supply is rapid when draw offs commence in the morning and evening hours. This is particularly important for large commercial and industrial systems. Solar system designers may account for this loss either by providing a larger system or by reducing the solar fraction (i.e., larger auxiliary loads). These options result either in increased capital investment or in increased operating cost. The overall system configuration may be optimized by determining the appropriate profile of water replenishment (i.e., the quantity of the cold makeup water to be supplied to the storage tank over a day) into the storage tank. In this paper, the effect of water replenishment on the system sizing is studied and a novel strategy for water replenishment is proposed to improve the design and performance of solar water heating systems. It is observed that the annualized system cost can be reduced by 13.7%. For the cost-optimal system configuration, a reduction of 12.7% in the collector area and 10.2% reduction in the storage volume are observed.

Improvements related to the performance of a solar thermal system may be classified into categories: improvement of the collector performances, demand side management to reduce the mismatch between the supply and demand of thermal energy, improvements related to the storage tank, etc. System performance can be improved by installing more efficient solar collectors. Johannsen [1] has discussed about the improvements due to regenerating solar collectors. A multiple layer solar collector [2], evacuated and concentrating collectors [3] can also improve the performance of the overall system. Several investigations have also been reported to improve the performance of solar water heating systems employing standard collectors and storage tank. Investigations have been carried out to improve system performance by passing the fluid in multiple stages [4], maintaining an optimum variable collector flow rate [5], maintaining a reduced fixed collector flow rate [6], supplying water to the collector inlet from mains as well as from the storage [7], etc. Haghighat and Singh [8] suggested operating the system on a fixed temperature control that supplies water from the solar collectors to the storage tank at a fixed temperature using a controller. Such a system, being open loop, may require a larger collector area and storage size equal to the daily demand. A similar concept has been introduced by Rankin and Rousseau [9] using inline electric heating for residential water heating as an electric heater supplies water to the storage tank at a fixed temperature through a temperature and flow controller.
Storage related investigations have been focused on the characterization [10] and effects of stratification on the improved system performance [11–13]. Dgany and Sokolov [14] have determined the optimal transfer flow rate profile (i.e., the quantity of the hot water to be supplied to the load over a day) to enhance output and reduction in auxiliary cost. Hamdan et al. [15] recommended minimization of number of withdrawals and introduction of an automatic temperature controller in the storage tank for water conservation. Increasing number of tanks in series increases the expected life of collectors and storage, y. In solar water heating systems, the heat source is fluctuating therefore; for maintaining delivery at a fixed temperature, collector flow rate must be varied continuously. Such a control strategy is difficult to implement in practice.

In the present work, the appropriate makeup water replenishment profile is determined to minimize the requirement of the collector area and the storage volume to meet the load. It is observed that for the optimum water replenishment profile, the temperature of the water inside the storage tank remains constant. Based on this observation, a simple strategy for replenishing the makeup water is determined both analytically and numerically. This approximate method of water replenishment is simple to implement and near-optimal.

The analysis is performed and represented using the design space approach [19]. The design space is represented by tracing constant solar fraction lines on a collector area vs. storage volume diagram for a specified load. In this approach, all possible and feasible designs of a solar water heating system can be conveniently controlled, as in case of electric in line heating [9]. In solar water heating systems, the heat source is fluctuating therefore; for maintaining delivery at a fixed temperature, collector flow rate must be varied continuously. Such a control strategy is difficult to implement in practice.

Focus of most of the earlier investigations have been either on maintaining an optimum collector flow rate or maintaining stratification to improve system performance. The investigations also emphasized on supply of hot water at a fixed temperature from the heating source (a solar collector or an electric heater) to the storage tank. This can be effectively accomplished if the heating source can be conveniently controlled, as in case of electric in line heating [9]. In solar water heating systems, the heat source is fluctuating therefore; for maintaining delivery at a fixed temperature, collector flow rate must be varied continuously. Such a control strategy is difficult to implement in practice.

2. The mathematical model

Fig. 1 shows the schematic of a solar water heating system. Collector array is connected to the insulated storage tank. Solar energy
absorbed by collectors is transferred to the storage tank by circulating hot water through the collector tubes. Thermal demand is satisfied by supplying hot water from the storage tank. Makeup water may be added to the storage tank simultaneously or separately at different times and quantities.

Makeup water is supplied \((m_R)\) to the tank at temperature \(T_R\). Considering mass and energy balance of a well mixed storage tank, a mathematical model is built. Mass and energy balances across the storage are shown in Fig. 2. Mass leaves the storage for meeting the load at a rate \(m_{st}\) kg/s, while mass enters the storage during makeup water supply (Fig. 2a). At any instant of time the rate of change in the mass of the tank is given as

\[
\frac{dV_{st}}{dt} = m_c + m_R - m_{st} - m_t
\]  

(1)

Assuming that the collector operates under steady-state condition (i.e., the same quantity of water enters and leaves the storage for circulation in the collectors) the mass of the tank may be written as

\[
\frac{dV_{st}}{dt} = m_R - m_{st}
\]  

(2)

Eq. (2) may be approximately solved in terms of a difference between the initial and the final storage volumes over a small time period \(\Delta t\).

\[
V_{stf} = V_{sti} + \frac{1}{\rho} \left( m_R \Delta t - m_{st} \Delta t \right)
\]  

(3)

Care should be taken in evaluating \(m_R\Delta t\) and \(m_{st}\Delta t\) during periods when \(m_R\) or \(m_{st}\) undergoes a change. \(\Delta t\) should be sufficiently small and synchronous with changes in \(m_R\) and \(m_{st}\).

Change in the internal energy of the storage tank is the cumulative effect of energy added to the tank by solar heat and makeup water; and energy extracted by the load and storage losses (Fig. 2b). These energy interactions bring variation in storage temperature as well as volume.

\[
\rho C_p \frac{d(V_{st} T_{st})}{dt} = q_s + q_{stl} - q_{stl} - q_{stl}
\]  

(4)

Similar to Eqs. (2), (4) can be approximately solved to obtain the final storage temperature at the end of a time step.

\[
T_{stf} = \frac{1}{V_{stf}} \left( V_{sti} T_{sti} + \frac{\Delta t}{\rho C_p} (q_s - q_{stl} - q_{stl} + q_R) \right)
\]  

(5)

Solar useful heat gain rate, \(q_s\) may be calculated using Hottel–Whillier–Bliss equation [20].

\[
q_s = A_c \left( \frac{I_T}{T} \right) F_R \left( \frac{q_g}{C_0} \right) F_R \left( U L \left( \frac{T_{st}}{T_a} \right) \right)
\]  

(6)

‘+’ sign in the above equation implies, hot water from collector array enters the tank only when solar useful heat gain becomes positive. Solar flux falling on a tilted surface of the collector is determined [20] as

\[
I_T = (I_g/I_d) R_0 + I_d \left( 1 + \cos \beta \right) + I_s \left( 1 - \cos \beta \right) \rho_b
\]  

(7)

Storage heat losses, energy drawn from the storage tank to meet the load, and the energy supplied to the storage tank with makeup water are estimated using the following equations.

\[
q_{stl} = U_R A_R \left( T_{st} - T_a \right)
\]  

(8)

\[
q_{stl} = m_{st} C_p T_{sti}
\]  

(9)

\[
q_R = m_R C_p T_R
\]  

(10)
Substituting Eqs. (3), (9), and (10) in Eq. (5), the final temperature of the storage tank may be expressed as follows.

\[ T_{st} = \rho V_{st} T_{st} + \frac{\Delta T}{\rho V_{st}} [q_{sat} - q_{aux} - m_{st} C_p T_{st} + m_{st} C_p T_{st}] \]  

(11)

Whenever the storage temperature is higher than the desired load temperature, certain amount of hot water is drawn from the tank at a rate \( m_{st} \) and balance is taken from the cold water connection at a rate \( m_{x} \) to supply the demand at the desired load flow rate \( m_{L} \) and the desired temperature \( T_{L} \). Storage flow rate \( m_{st} \) and mixing water flow rate \( m_{x} \) are calculated by taking a mass and energy balance across the mixing junction.

\[ m_{st} = \frac{T_{st} - T_{R}}{T_{at} - T_{R}} m_{at} \]  

(12)

\[ m_{x} = \frac{T_{at} - T_{st}}{T_{at} - T_{R}} m_{at} \]  

(13)

In case of storage temperature dropping below the desired load temperature, hot water will be drawn at a desired load flow rate \( m_{at} \). An auxiliary heater, placed in series with the storage tank, is used to meet the load at the desired temperature. Auxiliary energy may be approximately calculated based on the average storage temperature.

\[ q_{aux} = m_{at} C_p (T_{at} - (T_{sat} + T_{st})/2) \Delta t \]  

(14)

The extent of demand met by solar energy is expressed in terms of solar fraction over a time horizon (a day, a month or a year). In the following expression, solar fraction is determined by summing up the energy interactions of the storage tank and auxiliary over a day.

\[ F = \frac{Q_{L} - \sum q_{aux}}{Q_{L}} \]  

(15)

For proper design of the system, transient behavior of temperature profile is neglected [19]. In this paper, it is assumed that the temperature profile reaches a steady-state condition. This implies that the net gain or loss of the stored thermal energy for the storage tank over a given time horizon of analysis is zero.

\[ \sum_{0}^{t} (\rho C_p \frac{d(V_{st} T_{st})}{dt}) dt = 0 \]  

(16)

### 3. Generation of the design space

The methodology described in the following paragraphs comprises mainly of generation of the design space [19]. Design space is the region bounded by constant solar fraction curves drawn on the collector area vs. storage volume diagram. Region bounded by constant solar fraction curves represents all possible designs of the system. The feature offers flexibility in solar system sizing. Employment of the design space approach reduces labor, expertise and expense involved in the design, optimization and parametric analysis of solar thermal systems. The methodology is simple, flexible and does not need any special computational setup. Any suitable objective function may be optimized to select a feasible design from the entire design space. Since all the feasible designs are known, even multi-objective design can easily be performed by the designer.

In the analysis of the proposed strategy, the storage temperature as well as volume changes over a day. A storage temperature profile and stored water volume profile exists for the configuration shown in Fig. 1. In order to capture storage parameter variations and seek the optimum system size, design space approach is employed in the proceeding analysis. The methodology of design space generation is illustrated through an example of domestic hot water system with immediate water replenishment [19]. The same approach is then extended to the analysis of solar water heating system with a makeup water profile.

In the illustrative example, storage volume remains constant over all time steps as the water is replenished immediately. For brevity, the design space is generated for a single day solar input and unity solar fraction. The various system parameters for this example are given in Table 1. Monthly mean values of hourly solar radiation [21] on April 15 are used in the example. Time step in the model calculations is chosen to be 1 h while time horizon of analysis is one day.

For a given collector, storage tank, solar insolation, and the thermal load characteristics, Eq. (11) uniquely predicts the temperature profile inside the storage tank for a given collector area \( A_c \) and storage volume \( V_{st} \). The surface area of the storage tank is assumed to be related to the storage volume by following relation, with equal height to diameter ratio.

\[ A_s = 5.54(V_{st})^{2/3} \]  

(17)

By varying the collector area and the storage volume, different system designs may be obtained. For unity solar fraction, the temperature of the storage tank during load must be greater than the desired load temperature.

\[ T_{st} > T_{L} \]  

(18)

Since, water is used as a working fluid; the storage tank temperature has to be less than the saturated temperature.

\[ T_{st} < T_{sat} (≈ 100 \degree C) \]  

(19)

An acceptable design with unity solar fraction must satisfy Eqs. (18) and (19). For a specified load, all combinations of collector area and storage volume that satisfy these conditions are searched. They are plotted on a collector area vs. storage volume diagram. Fig. 3 shows a storage temperature profile with a typical system size \( (A_c = 90 \text{ m}^2 \text{ and } V_{st} = 3.2 \text{ m}^3) \). In this example, the design constraints may be stated as follows,

(i) The tank temperature has to be greater than \( T_{L} = 60 \degree C \) during load and
(ii) It has to be always less than \( T_{sat} = 100 \degree C \) to avoid phase change of water.

\( A_c \) and \( V_{st} \) combinations are varied to generate the design space. The immediate water replenishment profile is shown in Fig. 4 which resembles hot water consumption pattern [22]. Resulting characteristic of collector area versus storage volume at unity solar fraction is plotted in Fig. 5. The constant solar fraction curve is drawn by accounting load temperature as well as limiting storage temperature constraint. Point ‘a’ in Fig. 5 indicates a design with minimum storage volume requirement while point ‘m1’ shows minimum collector area design for unity solar fraction.

Point ‘m1’ corresponds to a collector area of \( 67.2 \text{ m}^2 \) and a storage volume of \( V_{st} = 28.3 \text{ m}^3 \) required for a desired output.

<table>
<thead>
<tr>
<th>Location</th>
<th>Apartment building at Pune (18.53°North, 73.85°East), India</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>Domestic hot water load, 4500 LPD at 60 °C</td>
</tr>
<tr>
<td>Collectors</td>
<td>Flat Plate collectors (single cover and selective coated)</td>
</tr>
<tr>
<td></td>
<td>( F_E(\tau_a) = 0.675 ) and ( F_D U_L = 5.656 \text{ W/m}^2\text{K} )</td>
</tr>
<tr>
<td></td>
<td>South facing with tilt of 33.5°</td>
</tr>
<tr>
<td>Storage</td>
<td>Cylindrical, well mixed, always full, with ( (h/d) = 1 )</td>
</tr>
<tr>
<td></td>
<td>Mild steel, wall thickness 6 mm, density 7800 kg/m³</td>
</tr>
<tr>
<td></td>
<td>Insulation: Glass wool ( (k = 0.04 \text{ W/mK}) ) and 0.2 mm thick</td>
</tr>
</tbody>
</table>

Table 1: Input data for solar water heating system [20]
Any decrease in the collector area from ‘m1’ will fail to supply the desired load. At 2.5 m³, point ‘a’ serves as a lower limit to the storage volume corresponding to a collector size of 95 m². Any reduction in storage volume below ‘a’ will result in boiling of water inside the tank.

It may be noted that a vertical line crosses the limiting curve at two different points. This signifies that there exist a maximum and a minimum storage volume for a given collector area. For example, a constant collector area line at 95 m² intersects the limiting curve at ‘a’ and ‘b’. At 903 m³, point ‘b’ indicates a maximum limit of storage volume. Above point ‘b’, thermal losses from the storage tank will dominate and reduce solar fraction. The region bounded by the limiting curve includes all possible designs of the system and may be called as the design space for solar fraction unity. A detailed illustration on design space concept on annual basis and with different values of solar fractions has been provided by Kulkarni et al. [19].

The constant solar fraction curve in Fig. 5 corresponds to a predefined (unity) solar fraction and indicates load as well as limiting temperature constraints. On the similar lines, curves for predefined solar fraction less than unity can be generated. In that case load temperature constraint described by Eq. (19) is not considered. The model Eq. (11) is solved with limiting temperature constraint only for a given solar fraction.

The minimum and the maximum values of storage volumes are obtained with a condition that the average solar fraction over the time horizon should be greater than or equal to the predefined solar fraction. Curves with solar fraction less than unity can thus, be plotted to seek the entire design space.

4. Determination of water replenishment profile

In this section, optimum makeup water profile that requires a minimum collector area is explored first. Taking a guideline from the optimum profile, an approximate water replenishment profile that has the same effect on the system performance is proposed subsequently.

4.1. Optimum water replenishment profile through numerical optimization

Attention is now focused on the search of optimum makeup water profile that will yield the minimum collector area. A mathematical problem is formulated with the objective of minimizing the collector area. In this formulation, hourly makeup water quantity is a variable. The optimum solution is obtained using an optimization solver. Point ‘m*’ in Fig. 5 represents the optimum solution. The solution indicates a minimum collector area of 62.6 m² and corresponding storage volume is 6.12 m³. A reduction of 6.8% in collector area may be obtained by varying cold make up water replenishment profile. Storage temperature profile corresponding to the optimum design is shown in Fig. 6. The temperature profile with the optimum design is more or less uniform over a day. It may be noted that an extra constraint of non-empty storage is added during optimization.
To generate the entire design space using numerical optimization, similar optimization procedure may be repeated. Generation of such a design space becomes numerically intensive. To overcome this difficulty, a simple and approximate procedure has been developed. The proposed approximate procedure relies on the fact that the temperature profile of the water inside the storage tank remains constant throughout the day.

4.2. Approximate water replenishment profile

Makeup water profile obtained with the optimum design depends on the quantity of hot water load, load temperature, and solar radiation intensity. It is practically difficult to vary makeup water profile with changes in the seasonal variables. A strategy for makeup water profile is needed which will automatically take care of the seasonal variables and still yield the optimum design and desired performance. A makeup water profile that attempts to achieve uniform storage temperature over a day is approximated and analyzed in the following paragraphs.

For simplicity, it is assumed that the temperature of replenished water is equal to the ambient and is constant throughout the day. For the approximate profile, we assume a constant storage temperature over a time step.

\[ T_{st} = T_{si} \quad (20) \]

Substituting Eq. (20) in Eq. (11), the following equation is obtained.

\[ T_{st} = \frac{\rho V_{st} T_{st} + \Delta \frac{\rho \Delta V_{st} - m_{st} C_p T_{st}}{C_p (T_{st} - T_R)}}{\rho V_{st} + (m_{st} \Delta T - m_{st} \Delta T)} \quad (21) \]

Simplification yields an expression for instantaneous makeup water flow rate needed to keep storage temperature constant.

\[ m_{st} = \frac{(q_s - q_{sf})}{C_p (T_{st} - T_R)} \quad (22) \]

Expressing makeup water quantity needed in an hour in integral form:

\[ \int_0^1 m_{st} dt = \frac{\int_0^1 (q_s - q_{sf}) dt}{C_p (T_{st} - T_R)} \quad (23) \]

The quantity in integral sign on the right hand side of Eq. (23) signifies net storage heat gain or loss in one hour. Positive values of mass flow rates demand that makeup water can only be supplied when the storage heat gain is positive i.e. during sunshine hours. It is assumed that the storage losses during no sunshine hours are negligible. The assumption is done only for calculation of makeup water supply quantity in a time step. Storage losses are accounted for determination of storage temperatures over the time step and system performance over the time horizon. The condition is indicated with a * sign in Eq. (23).

To prevent storage tank overflow or starving, mass of water to be replenished daily must be equal to the mass withdrawn by load.

\[ \sum_{j=1}^{24} \left( \int_0^1 m_{st} dt \right)_j = \sum_{j=1}^{24} \left( \int_0^1 m_{st} dt \right)_j \quad (24) \]

Expressing hourly makeup water requirement in terms of a fraction \((K)\) of total mass required to be made up over a day.

\[ \int_0^1 m_{st} dt = K \sum_{j=1}^{24} \left( \int_0^1 m_{st} dt \right)_j \quad (25) \]

Substituting Eq. (23) in Eq. (25), we obtain:

\[ \frac{\int_0^1 (q_s - q_{sf}) dt}{C_p (T_{st} - T_R)} = K \sum_{j=1}^{24} \left( \int_0^1 m_{st} dt \right)_j \quad (26) \]

Similarly, net storage heat gain in an hour is expressed in terms of a fraction \((J)\) of total net storage heat gain over a day.

\[ \int_0^1 (q_s - q_{sf}) dt = J \sum_{j=1}^{24} \left( \int_0^1 (q_s - q_{sf}) dt \right)_j \quad (27) \]

Combining Eqs. (26) and (27), following equation may be obtained.

\[ \sum_{j=1}^{24} \left( \int_0^1 (q_s - q_{sf}) dt \right)_j = \sum_{j=1}^{24} \left( \int_0^1 (q_s - q_{sf}) dt \right)_j \quad (28) \]

Recalling Eq. (16), there is no change in the internal energy of the storage over a day, energy interactions taking place in the storage may be expressed as

\[ 0 = \sum_{j=1}^{24} \left( \int_0^1 (q_s - q_{sf}) dt \right)_j - \sum_{j=1}^{24} \left( \int_0^1 (q_s - q_{sf}) dt \right)_j + \sum_{j=1}^{24} \left( \int_0^1 (q_{sf}) dt \right)_j \quad (29) \]

Neglecting storage losses during no sunshine hours and substituting the expressions of \(q_{sf}\) and \(q_s\), Eq. (29) may be simplified to the following equation with simple algebraic manipulations.

\[ \sum_{j=1}^{24} \left( \int_0^1 (m_{st}) dt \right)_j \leq K \sum_{j=1}^{24} \left( \int_0^1 (m_{st}) dt \right)_j \quad (30) \]

which implies that

\[ K \leq J \quad (31) \]

Eq. (31) shows that the fraction of makeup water quantity should at most be equal to the fraction of net storage heat gain.

In the above analysis storage losses during no sunshine hours are neglected. A repercussion of the assumption is that storage temperature remains constant as long as solar heat gain is positive. During no sunshine hours, the same drops slightly due to the storage losses.

An expression for makeup water quantity required to be supplied to maintain a constant storage temperature may be obtained as follows:

\[ \int_0^1 m_{st} dt = \frac{\int_0^1 (q_s - q_{sf}) dt}{C_p (T_{st} - T_R)} \quad (32) \]

Eq. (32) reflects that solar energy utilization will be effective with the maximum rate of replenishment matching the net storage heat gain. The replenishment profile of Eq. (32) may be applied to determine the system size near-optimally as described in the following sub-section. It may be noted that maintenance of replenishment profile of Eq. (32) can be practically achieved through temperature and liquid level sensors. The makeup water can be added so as to keep temperature constant at present level and liquid level above minimum.

4.3. Single day analysis with unity solar fraction

The benefit in maintaining a makeup water profile that maintains a constant storage temperature is now studied and compared with immediate replenishment. Effect of the same on the design parameters is also demonstrated in this sub-section. Two profiles of makeup water supply are compared. Both the profiles will supply same load profile with unity solar fraction.
1. Immediate replenishment profile (P1): The makeup water profile is shown in Fig. 4. Makeup water quantities are expressed in terms of percentage of total daily requirement of 4500 L (Table 1). Makeup water is added simultaneously with the draw off, in equal quantities. The profile resembles ISO hot water consumption pattern [22].

2. Profile 2 (P2): A water replenishment profile shown in Fig. 7 described by Eq. (32) that attempts to maintain storage temperature constant. This particular profile is obtained at a collector area of 70 m² and makeup water quantities expressed in terms of percentage of daily total requirement. The same resembles the profile of solar useful heat gain over a day. It may be noted that to generate the entire design space the collector area may be varied.

In the analysis of profile P2, initial storage volume at the beginning of the day \( V_{inib} \) is initially assumed. The same is then used as a variable in optimization. A minimum value of \( V_{inib} \) is searched at different collector areas and at unity solar fraction. Initial storage volume \( V_{in} \) for a certain time step will be equal to the final volume, \( V_{arf} \) of the preceding time step. Final volume in a time step is determined using Eq. (3). Design value of the storage tank is the maximum stored water volume observed during the day. Storage loss in a time step \( q_{uls} \) is calculated on the basis of initial storage volume \( V_{in} \). In this model a constant buffer of 25 L for the storage tank is assumed. This buffer volume is assumed to incorporate uncertainty of solar insolation and enhance reliability of the system. It may be noted that the initial assumption of no storage loss during no sunshine hours is relaxed during numerical computation.

Fig. 8 shows the storage temperature profile at the minimum collector area design obtained with the makeup water profile P2. The temperature profile obtained with profile P2 is almost uniform and closely confirms that obtained with the optimum design shown in Fig. 6.

On the similar lines described in Section 3, the design space with profiles P2 is identified and shown in Fig. 9. Fig. 9 also shows the optimum design point \( m^* \) and design space with profile P1. Point \( m_1 \) represents minimum collector area design with immediate replenishment P1, while \( m_2 \) represents the same with profile P2. The optimum designs with minimum collector area, obtained using different procedures are compared in Table 2. It may be observed that the system size obtained using P2 is closer to the system size obtained using numerical optimization.

For immediate replenishment profile (P1), if one proceeds down the curve from \( m_1 \), there is a decrease in storage volume with increase in collector area up to a certain point ‘a’. This is the minimum storage volume design for profile P1. Beyond point ‘a’ minimum storage volume requirement increases with collector area so as to maintain a maximum storage temperature within a limit of 100 °C. In this case, the quantity and schedule of water replenishment depends on the hot water withdrawal. With higher collector areas the storage volume has to be increased to keep tank temperature within the limit. This is not the case with profiles P2. Makeup water replenishment quantity depends on the net storage heat gain. With increase in collector area, makeup water quantity and consequently, mass in storage tank is adjusted to keep a check on storage temperature. As a result the storage volume requirement is less than in P1. Lower storage volume limit at all collector areas with profile P2 is observed to be more or less constant (Fig. 9).

Fig. 7. Makeup water profile P2 in terms of percentage of daily load at \( A_c = 70 \text{ m}^2 \).

Fig. 8. Storage temperature profile at minimum collector area design obtained with profile P2.

Fig. 9. Design space for \( F = 1 \) with different makeup water profiles and optimum design point ‘m*’.

<table>
<thead>
<tr>
<th>Profile</th>
<th>Collector area ( (\text{m}^2) )</th>
<th>Storage volume ( (\text{m}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical optimum profile ( (m_1) )</td>
<td>62.6</td>
<td>6.14</td>
</tr>
<tr>
<td>Immediate replenishment profile P1 ( (m_2) )</td>
<td>67.2</td>
<td>28.3</td>
</tr>
<tr>
<td>Approximated makeup water profile P2 ( (m_2) )</td>
<td>64.1</td>
<td>6.73</td>
</tr>
</tbody>
</table>

Table 2 Comparison of system configurations at the minimum collector area designs for unity solar fraction

By maintaining a makeup water profile P2, reduction in the collector area as compared to the immediate replenishment at unity solar fraction is 5%, while reduction in the storage volume is 76%. 

...
Profile P2 shows a benefit in its employment. The remarkable reduction in storage volume may initiate enhanced utilization of storage capabilities in solar systems.

Fig. 10 shows a comparison of storage temperature profiles over the day for P1 and P2 for a collector area of 70 m$^2$. In case of P1, unavailability of solar energy in the morning hours followed by hot water withdrawal causes storage temperature to drop to a minimum of 58.5 °C at 9 a.m. As solar radiation is available, storage temperature rises, attaining 69 °C at 3 p.m. followed by a steady drop due to draw off and losses. Daily fluctuation in the storage temperature is observed to be 16% with P1. P2 is characterized by a higher storage temperature at the beginning of the day. Temperature profile is almost uniform with P2. Hot water withdrawal between 6 and 8 a.m. drops storage temperature to a minimum of 62 °C. The drop is due to addition of makeup water to a small mass of 70 L left in the tank at 9 a.m after withdrawals. The proportion of addition is too high (150 L) to keep the storage temperature uniform. Once the storage mass and energy input increases the temperature increases and attains a peak of 68 °C. The daily fluctuation in the storage temperature with profile P2 is 8% which is less than the immediate replenishment. Profile P2, though approximated proves to be superior by demonstrating a less fluctuation in the storage temperature.

Fig. 11 shows the variation of the stored water volume over a day for the profile P2 and the stored water volume obtained using numerical optimization procedure at the minimum collector area. There is a similarity in the nature of the two storage volume profiles with a slight variation in quantities. While searching for a system design with the minimum storage volume, it was observed that the minimum storage tank size is predominantly influenced by the mass balance constraint. The load and the maximum allowable temperature inside the storage tank do not play a significant role in determining the system design with the minimum storage volume. The minimum storage volume almost remains constant in case of P2 (Fig. 9).

5. System optimization

Generation of the design space considering the annual performance and the economic optimization of the system are addressed in this section based on the approximate water replenishment profile proposed in the previous section.

5.1. Annual performance

In evaluating the annual performance, mathematical model described earlier is employed over a time horizon of one year. Time step of analysis is 1 h. Monthly average of hourly radiation data is utilized for calculations [21]. Entire design space for different solar fractions is generated on the similar lines discussed in the previous section. Results obtained using the immediate replenishment profile (P1), are compared with those obtained by applying the approximate water replenishment profile (P2), proposed in the previous section.

Fig. 12 shows the entire design space with P1 while Fig. 13 shows the same with profile P2. Referring to Fig. 12, generation of the design space involves consideration of the load temperature and the maximum storage temperature constraint. All constant solar fraction curves in Fig. 12 show two limiting designs. One represents a minimum collector area design depicted by a1, b1, c1, d1, e1 and second, minimum storage volume design shown by a2, b2, c2, d2, e2. Pareto optimality exists between these designs. Application of suitable economic criteria to the Pareto region yields an optimum design.

Fig. 13 shows entire design space with a makeup water profile P2. For generating the design space in Fig. 13, additional constraint of mass balance must be taken into account. The constraint involves provision of a minimum quantity of water in the storage tank so that the tank does not become empty. This constraint is predominant over the load and storage temperature constraints. Based on this criterion, minimum storage volume is searched that is observed to be almost the same for different solar fractions. It may be noted that for low solar fraction, system configuration corresponds to the minimum collector area and to the minimum storage volume almost coincides. Therefore, for low solar fraction
5.2. Economic optimization

The economic criterion adopted for optimization is annualized life cycle cost of the system. The objective function is to minimize the total annual cost. However, it may be noted that different economic criteria may be chosen for the overall optimization of the entire system. The proposed methodology is not restricted to the choice of the objective function. Total annual system cost can be expressed as

\[ TAC = (C_c + C_a V_{st}^{1/3}) CRF_{sys} + C_{OM} + (C_R) CRF_{aux} + (1 - F) Q_L - C_F \]

\[ \frac{(CV)_{P1} + (CV)_{P2}}{(CV)_{P1}} \] (33)

The capital recovery factor (CRF) is calculated based on the following expression:

\[ CRF = \frac{r(1 + r)^n}{(1 + r)^n - 1} \] (34)

Parameters utilized for economic optimization are given in Table 3. Operating and maintenance cost of 2% of the capital cost has been assumed. Auxiliary employs a liquefied petroleum gas (LPG) heater. Calorific value of LPG is assumed to be 50.3 MJ/kg with a burner efficiency of 90%. The auxiliary heater rating (R) is obtained on the basis of the maximum rate of thermal energy required over the entire time horizon. Total annualized cost is evaluated in case of makeup water profiles P1 and P2. Variations of total annualized cost with solar fraction are plotted in Fig. 14. With increase in solar fraction, capital cost increases, while at the same time auxiliary cost reduces. A trade off between the capital and the operating cost results into an optimum design that yields the maximum economic benefit. With P1 and P2 optimum designs are observed at a solar fraction 0.8. The results are shown in Table 4. If makeup water profile based on the net storage heat gain is implemented, a substantial benefit of 13.7% in the system cost at the same performance can be achieved. A benefit in system size is also possible with 12.7% reduction in collector area and 10.2% reduction in storage volume.

### 6. Conclusions

In a typical solar water heating system, cold water is replenished into the storage tank as soon as the load is served. Due to the mixing of the cold makeup water with the hot water inside the storage tank, significant amount of entropy is generated. It is possible to determine the water replenishment profile (i.e., the quantity of the cold makeup water to be supplied to the storage tank over a day) to reduce the entropy generation and this leads to a significant improvement in the system configuration. Numerical optimization has been performed to optimize the system configuration with varying water replenishment profile. It has been observed that the storage temperature profile remains almost constant for the optimum water replenishment profile. Based on this observation, a simplified and approximate water replenishment profile has been proposed in this paper. The proposed profile involves hourly replenishment of cold makeup water in proportion with the fraction of hourly net storage heat gain. System configuration obtained with the proposed profile closely matches with the one obtained using numerical optimization. Based on the proposed water replenishment profile, annual performance of the system has been studied and the same has been represented as a design space. Design space obtained using the proposed methodology has also been compared against the design space obtained using the immediate water replenishment. It is observed that the annualized system cost can be reduced by 13.7%. For the cost-optimal system configuration, a reduction of 12.7% in the collector area and 10.2% reduction in the storage volume are observed.

### Table 3

<table>
<thead>
<tr>
<th>Economic parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual discount rate, r</td>
<td>10.75%</td>
</tr>
<tr>
<td>Life of collectors and storage, n years</td>
<td>15</td>
</tr>
<tr>
<td>Life of auxiliary, n aux years</td>
<td>10</td>
</tr>
<tr>
<td>Collector cost coefficient, C_c US$/m^2</td>
<td>106.8</td>
</tr>
<tr>
<td>Storage tank cost coefficient, C_s US$/m^3</td>
<td>466.4</td>
</tr>
<tr>
<td>Tank insulation price, C_{ins} US$/m^2</td>
<td>2.67 (for a slab thickness of 25.4 mm)</td>
</tr>
<tr>
<td>Cost coefficient for LPG water heaters, C_L US$/W</td>
<td>0.055</td>
</tr>
<tr>
<td>Fuel price, CF US$/kg</td>
<td>0.47</td>
</tr>
</tbody>
</table>

### Table 4

Comparison of economically optimum designs

<table>
<thead>
<tr>
<th>Profile</th>
<th>Solar fraction</th>
<th>Collector area (m²)</th>
<th>Storage volume (m³)</th>
<th>Annualized system cost, US$/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immediate replenishment profile P1</td>
<td>0.8</td>
<td>55</td>
<td>3.12</td>
<td>1631</td>
</tr>
<tr>
<td>Approximated makeup water profile P2</td>
<td>0.8</td>
<td>48</td>
<td>2.8</td>
<td>1407</td>
</tr>
</tbody>
</table>

Fig. 13. Design space based on the annual performance with profile P2.

Fig. 14. Variation of total annual costs with solar fraction.
The proposed methodology retains all the benefits of the design space approach. It is possible to apply the proposed methodology to incorporate design constraints such as limitation in collector area, and limitation of storage volume. The proposed methodology is particularly important and advantageous for large commercial and industrial solar water heating systems. It may be noted that the applicability of the design space approach is not restricted to the type of the collector, nature of availability of solar radiation data, system configuration and working fluid used in the system [23]. For different collector, different characteristic equation, other than the linear Hottel–Whillier–Bliss equation, may be used and the proposed methodology is still applicable.

References