A Quadratic Inequality Approach for Design of Robust Controller for a Parametric Uncertain Jet Engine

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Abstract - The control logic of a modern jet engine is comprised of many control loops and the control system must account for the uncertainties in the model with which it was designed. This paper presents a quadratic inequality based methodology for designing a robust controller of a parametric uncertain jet engine. The inequalities are used for generation of controller bounds to achieve robust stability and tracking specifications prior to loop shaping. The design is done in frequency domain and the evolved robust controller controls the system that does not have a distinct set of poles and zeros but a range over which each of the poles and zeros might lie. The methodology has multifarious advantages as it can be easily implemented, does not require template generation and provides the designer a good insight of the QFT bounds at any given design frequency and controller phase. The approach is demonstrated for the design of speed control loop of a parametric uncertain twin spool jet engine.

Keywords: Robust control; Jet engine control; Quantitative feedback theory (QFT), uncertain system.

NOMENCLATURE

\[ G(s) \] : Uncertain plant
\[ K(s) \] : Compensator
\[ F(s) \] : Prefilter
\[ NL \] : Low pressure Compressor Spool speed
\[ NH \] : High pressure Compressor Spool speed
\[ Wf \] : Fuel flow
\[ H/M \] : Hydromechanical
\[ QFT \] : Quantitative Feedback Theory

I. INTRODUCTION

The multi-mission requirements of today's commercial and fighter aircraft have dictated a significant increase in propulsion system capability. This has resulted in a continuing trend towards increased gas turbine engine complexity and a corresponding increase in the complexity of the control system [1]. Control requirements applied to gas turbine engines consist of ensuring safe, stable engine operation. Specific engine performance rating points are generally defined as basic steady state design goals for the control. A review of the basic theory of jet engine operation and on control designs which are currently in commercial and military use is dealt by Spang and Brown [2].

The control logic of the modern Full Authority Digital Engine Control (FADEC) is comprised of many control loops, each of which has a specific purpose. Typical control loops include (but are not limited to) a high or low rotor speed governor, an acceleration and deceleration loop, and various limiting loops for temperature, speed and fuel flow. In a typical compressor speed control, the speed demand schedule establishes the desired compressor speed as a function of inlet temperature and throttle position. Compressor speed error is determined from the difference in the desired and the actual speed. A variant of proportional control uses the derivative of rotor speed (Ndot or rotor acceleration) to control engine acceleration and deceleration as a function of inlet temperature.

All existing systems are subject to various disturbances and uncertainties. Mathematically, we can only approximate an existing system with a transfer function depending upon the information available about a system and the observations over a certain period of time. The difference between the performance of the actual system and model gives the estimate of uncertainties in the actual system [3]. These uncertainties can be represented as variations in coefficients of transfer functions in frequency domain and form interval systems.

In the bound generation step of H"{o}rmwitz's QFT design procedure [4], the plant template is used to
translate the given robustness specifications into domains in the Nichols chart, where the controller gain-phase values are allowed to lie. These domains define what are commonly known as QFT controller bounds. Various approaches to the bound generation problem are available [5]. Recently, more efficient numerical algorithms based on quadratic constraints have been proposed to automate this step ([6], [7]).

In the present work, we propose a quadratic inequality based approach for bound generation of parametric uncertain jet engine power plant. The inequalities have been derived from the fundamental requirement of the robust tracking and stability specifications given in frequency domain. Controller and prefilter design through loop shaping is then carried out on the generated bounds. This approach provides the designer with one more approach for obtaining robust controller meeting desired performance specifications. The design is done in frequency domain and the evolved robust controller controls the system that does not have a distinct set of poles and zeros but a range over which each of the poles and zeros might lie. The methodology can be easily implemented, does not require template generation and provides the designer a good insight of the QFT bounds at any given design frequency and controller phase.

We demonstrate the applicability of our results by considering a specific example of speed control of an experimental twin spool jet engine being developed in India.

2. JET ENGINE SPEED CONTROL

Initially turbofan engines were controlled exclusively by controlling core speed. This provided a simpler control which could satisfy engine constraints and be implemented using a hydro mechanical controller. With electronic controls, more accurate control of thrust can be obtained through control of fan speed while satisfying the same set of engine constraints. It is generally not possible to control core and fan speed independently since they are tightly coupled. A fan speed control can be subdivided into a steady state control and a control for transient operation. The steady state control maintains engine operation along its steady state operating line. The transient control is necessary to accelerate and decelerate the engine while meeting stall, flameout and critical temperature limits.

Figure 1 shows the block diagram of the low-pressure compressor spool speed control of a twin spool gas turbine with a conventional controller. Acceleration of high-pressure compressor spool forms the inner loop. Direct control of acceleration, rather than speed, allows tighter control of engine acceleration thereby improving transient response and reducing mechanical stress.

3. QUADRATIC INEQUALITY APPROACH

Consider a single-input-output linear time invariant plant G(s) and controller K(s) embedded in a two-degree-of-freedom structure. Suppose there is uncertainty in some or all of the plant parameters so that we have a plant family \( G(s) \). If K(s) is such that it meets the performance requirements for the entire \( G(s) \), then K(s) is said to robustly control \( G(s) \).

3.1 Robust Tracking Specification

The QFT robust tracking specification follows from the requirement that the plant output should follow a given desired output (as in a servo system) where, due to the plant uncertainty, the desired output is bounded between upper and lower time functions. In the frequency domain, this requirement can be specified as follows. At each frequency \( \omega \), for any plant in the plant family the magnitude of the closed loop transfer function \( T(s) \) from the set point to the output is bounded by

\[
20 \log_{10} (\alpha (\omega)) \leq 20 \log_{10} |T(j \omega)| \leq 20 \log_{10} (\beta (\omega))
\]

(1)

where \( \alpha(\omega) \) and \( \beta(\omega) \) are positive real valued functions of \( \omega \) (for details of modelling the functions \( \alpha(\omega) \) and \( \beta(\omega) \) from given lower and upper time functions, see [4]).

For any plant family elements \( G_p(s) \) and \( G_u(s) \) we can rewrite (1) as

\[
20 \log_{10} |T_p(j \omega)| - 20 \log_{10} |T_u(j \omega)| \leq 20 \log_{10} (\beta (\omega)) - 20 \log_{10} (\alpha (\omega))
\]

(2)

or (henceforth we shall drop the arguments)

\[
\frac{|KG_p(1 + KG_u)|}{|KG_p(1 + KG_u)|} \leq \frac{\alpha}{\beta}
\]

(3)

Let \( K(\omega) = k(\omega)e^{j\alpha(\omega)} \) and
\( G(\omega) = g(\omega)e^{j\psi(\omega)} \)

(4)

where \( k \) & \( \delta \) are gain & phase of controller at \( \omega \), and \( g \) & \( \psi \) are gain and phase of plant at \( \omega \).

Substituting (4) in (3) and simplifying we can write the QFT robust tracking specification as...
Therefore, to achieve the QFT robust tracking specifications, we need to find a controller $K(s)$ such that at each frequency $\omega$, the inequality (5) is satisfied for all pairs of plant elements $G_i(s)$ and $G_k(s)$ in the plant family $\{G(s)\}$.

3.2 Robust Stability Specification

The gain/phase margin specification is given by

$$\left| \frac{K(j\omega)G(j\omega)}{1 + K(j\omega)G(j\omega)} \right| \leq \sigma_m$$

where $\sigma_m$ is constant with respect to frequency. Substituting (4) (6) and simplifying we get

$$k^2g^2(\sigma_m^2 - 1) + 2kg\cos(\theta + \psi) + (\sigma_m^2 - \kappa_1^2) \geq 0$$

This is quadratic inequality in $k$. Based on the roots of the equation, the stability bounds can be calculated at a given fixed frequency. For a given $\sigma_m$, $g$, & $\psi$ we have to find the combination of controller gain and phase which satisfy the inequality (7) i.e. stability constraints.

3.3 Loop Shaping

The intersection of tracking and stability bounds obtained by quadratic inequalities form magnitude constraints on a nominal open-loop function. A nominal open loop function is then designed by loop shaping to simultaneously satisfy its constraints as well as to achieve nominal closed loop stability. In a two degree-of-freedom design, a pre-filter will be designed after the loop is closed (i.e., after the controller has been designed) [4]. Design of filter guarantees that the variation in $|T(j\omega)|$ is less than or equal to that allowed.

4. THE CONTROL PROBLEM AND DESIGN SPECIFICATION

The designed controller must fulfill two objectives: fan speed keeping and fan speed changing. In the first case, the control objective is to maintain the fan speed. In the second case, the aim is to implement the change of fan speed without overshoot and in the shortest time possible. In both situations, the operability of the system must be independent of the uncertainties in the dynamics of the engine model.

The mathematical model of the fuel electro hydromechanical system between the fuel flow $W_f(s)$ and the drive current to the servo valve $I(s)$ is approximated by the transfer function:

$$\Delta W_f(s) = \frac{b_1}{\Delta I(s) s(a_1 s + 1)}$$

Similarly the engine dynamics between the LP and HP spool speed and fuel flow is approximated by the transfer function

$$\Delta N_L(s) = \frac{b_2}{\Delta W_f(s) s(a_2 s + 1)}$$

$$\Delta N_H(s) = \frac{b_3}{\Delta W_f(s) s(a_3 s + 1)}$$

$b_1$ and $a_1$ are different constant parameters at various engine speeds.

Considering the model uncertainties at various flight operating points and integrating it with variation of $b_2$ and $a_2$ parameters owing to non-linearity, interval bounds on these parameters are specified. Thus despite the fact that the model is non-linear, the QFT model for linear SISO systems with parametric uncertainty is used. The design of the controller includes a cascade compensator, $K(s)$ and a prefilter, $F(s)$ (both LTI) in order to reduce the variations in the output of the system caused by the uncertainties in plant parameters.

The system must fulfill robust stability and robust tracking specifications. For the robust stability margins, the phase margin angle should be at least $45^\circ$ and the gain margin $6$ dB. Thus the robust stability specification is defined by:

$$\left| \frac{K(j\omega) G(j\omega)}{1 + K(j\omega) G(j\omega)} \right| \leq \delta = 2.3$$

The robust tracking, must be defined within an acceptable range of variation. This is generally defined in the time domain but is normally transferred to the frequency domain, being expressed by

$$T_L(j\omega) \leq T(R)(j\omega) \leq T_U(j\omega)$$

where $T_R(s)$ represents the closed loop transfer function and $T_L(s)$ & $T_U(s)$ the equivalent transfer functions of the lower and upper tracking bounds.

5. JET ENGINE APPLICATION

The 2 DOF controller design has been carried out at one flight operating point for following parametric
uncertain twin spool turbofan engine under development.

$\{b_1 \in [2.5, 3.3] ; b_2 \in [7.5, 8] ;
 b_3 \in [10, 15] ; a_1 \in [0.01, 0.03] ;
 a_2 \in [0.4, 0.5] ; a_3 \in [0.27, 0.33] ; \}

Acceptable range of variation of rise time for acceleration and fan speed loop is specified as 0.26 to 0.38 sec and 0.7 to 1.05 sec respectively. Following set of frequencies for the design has been used.

$\Omega = \{0.1, 1, 4, 7, 9, 10, 12, 13, 20, 50, 100 \}$

The inner acceleration loop is first designed and subsequently outer speed control loop is designed. For the given performance specifications, the robust stability and robust tracking bounds are computed using the quadratic inequalities. The intersection of all the bounds at various frequencies is obtained. For the design of the cascade compensator $K(s)$, the Nichols' chart is used, adjusting the nominal open-loop transfer function $L_0 = G_0K$ ($G_0$ is the nominal plant) in such a way that no bounds are violated. Figure 2 and 3 shows the controller design carried out for inner acceleration control loop and outer speed control loop. Corresponding controller obtained are:

$K_1(s) = \frac{20s^2 + 116s + 80}{s(5.425e-7s^2 + 6.517e-7s^2 + 7.821s + 1)}$

$K_2(s) = \frac{3.036e+04s^7 + 6.517e-02s^2 + 7.821s + 1)}{s(2.038e+07s^2 + 7.713e+05s^2 + 0.032s^2 + 1.44e+03)}$

With this controller, the robust stability specification is fulfilled but not the robust tracking specification. By adjusting the corresponding prefilters

$F_1(s) = \frac{1}{(8.772e-3s^2 + 0.2193s + 1)}$

$F_2(s) = \frac{1}{(3.036e-4s^2 + 4.863e-2s^2 + 0.4713s + 1.02)}$

desired frequency response of the speed control loop, as shown in figure 4, is obtained such that it is maintained within the limits imposed in the design. Figure 5 shows the closed loop response of outer speed control loop for step change of low pressure compressor spool speed for the uncertain plant set.

6. CONCLUSION

In this paper, we propose a new quadratic inequality based approach for designing a robust controller of a parametric uncertain jet engine. It provides the designer one more way of designing robust controller for various control loops. This frequency based design methodology can be easily implemented, does not require template generation and provides the designer a good insight of the QFT bounds at any given design frequency and controller phase. Any redesign, if required, can be carried out very fast and this technique is particularly more useful for robust controller design of jet engine under development. With present day sophisticated electronic controls, the designed controller can be easily implemented. Controller design has been successfully accomplished at one operating point of an aero gas turbine engine under development. Research is under progress to extend this technique of robust control to design a robust controller for other loops of a typical aero gas turbine engine.

REFERENCES


Figure 1: Block diagram of the low-pressure compressor speed control of twin spool jet engine.

Figure 2: Quadratic Inequality Bounds and controller design for the nominal plant of inner acceleration control loop.

Figure 3: Quadratic Inequality Bounds and controller design for the nominal plant of outer speed control loop.

Figure 4: Frequency response of outer speed control loop with designed controller & prefilter for the uncertain plant set.

Figure 5: Closed loop response of outer speed control loop for unit step of LP compressor spool speed for the uncertain plant.