Spatial Control of a Large PHWR by Decentralized Periodic Output Feedback and Model Reduction Techniques

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Abstract—Nuclear reactors of small and medium size are generally described by point kinetics model, however, this model is not valid in case of a large reactor, because in that flux shape undergoes appreciable variation with time. The behaviour of large reactor core can be explained with reasonable accuracy by spatial model like nodal model, which considers the reactor to be divided into number of regions or nodes. The thermal feedbacks which introduce nonlinearity into the problem, should be considered for realistic modeling. The spatial model of 540-MWe PHWR developed in [4] is augmented with the dynamics of coolant and fuel temperatures and a 72nd-order model is obtained. As working with such a large model is difficult from the point of view of numerical computations, the new model having 14 inputs and 14 outputs, is suitably reduced by aggregation technique to obtain a 26th-order reduced model, which is more suitable to handle. The design of spatial controller by a state feedback based on reduced model needs the availability of all the states of the system for feedback purpose. As all the states of the reactor are not accessible for measurement, one has to resort to output feedback. Also, as the stability is not guaranteed by static output feedback, here the spatial controller is designed by periodic output feedback which is static in nature and at the same time guarantees complete closed-loop pole assignability. The various zones in a large reactor are coupled and a change in the control input to any zone causes respective change in the neutron flux of the other neighbouring zones, which may not be desirable. Therefore, a decentralized controller would serve as a better option, as it ensures that the input to any zone affects corresponding zone only and other zones are not affected by it. The above idea of periodic output feedback controller design yields a gain matrix with large magnitude, which amplifies measurement noise and is difficult to implement practically. Hence, it is desirable to keep the gain low. This objective is suitably expressed as LMI problem and putting appropriate design constraints, a better gain is obtained. The nonlinear model of the 540-MWe PHWR with the controller as above is tested for the reactivity transient simulation and the results of such a simulation are presented.

Index Terms—Aggregation, decentralized systems, discrete systems, large scale systems, periodic output feedback, pole assignment, reactivity, reduced-order model, spatial control.

I. INTRODUCTION

LARGE nuclear reactors are fast assuming considerable share in the overall electrical power generated in many grids. The foremost step in control, analysis and design of a large reactor is the model development. A reasonably accurate mathematical model for describing the behaviour of large cores can be obtained with nodal methods, in which the reactor core is considered to be divided into a number of regions or nodes and the material composition and flux are treated to be uniform [1]–[3] in these regions.

For a realistic modeling, the thermal feedbacks which introduce nonlinearity into the problem, are very important and should be considered. The present work attempts to study the reactivity feedbacks related to the fuel and the primary coolant temperatures for a 540-MWe pressurized heavy water reactor (PHWR), whose spatial model has already been obtained [4]. This is a 14–zone reactor model with 5 state equations for each zone making it a 70th-order system. With the introduction of the state variables corresponding to fuel and the coolant temperatures the system order increases to 72. As it is difficult to work with such large order system for controller design the same is reduced to a 26th-order system by aggregation technique. This reduced-order model is more suitable to handle. Also the design of a spatial controller for the above reactor using state feedback needs the availability of all the states of the system for feedback purpose. Hence, a spatial control design based on output feedback is preferable. The static output feedback problem is one of the most investigated problems in control theory. However, complete pole assignment and guaranteed closed-loop stability is still not obtained by using static output feedback. Another approach to pole placement problem is to consider the potential of time-varying periodic output feedback. It was shown by Chammas and Leondes [5] that a controllable and observable plant was discrete time pole assignable by periodically time-varying piecewise constant output feedback. Such a control law can stabilize a much larger class of systems than static output feedback does.

Due to the weak but significant coupling between various zones in a large reactor, a change in the control input to a zone would also cause a respective change in the neutron flux of the other zones, which may not be desirable. To overcome this, a decentralized controller which ensures that input to any zone affects the corresponding zone only and other zones are not affected by it, may be designed. This would render better control of the system. Thus, a decentralized periodic output feedback controller is designed for the nuclear reactor.

The rest of the paper is organized as follows. The 540-MWe PHWR is described in the next section, which is followed by the discussion on its modeling and linearization. Section III discusses the model reduction and aggregation. Section IV gives the periodic output feedback design for the decentralized controller, which is followed by the simulation results in Section V.
II. THE 540 MWE PHWR

The 540 MWe PHWR is a horizontal pressure tube type heavy water reactor using natural Uranium Oxide fuel and heavy water as moderator and coolant. It is rated for thermal power output of 1800 MW and electrical power output of 540 MW. Its dimensions (800 cm diameter and 600 cm length) [1], [2], [4] are very large compared to the neutron migration length, requiring spatially distributed reactivity devices and flux sensing mechanisms. For controllability and observability of flux distribution in the core, it is considered to be divided into 14 zones. There is a Liquid Zone Controller (LZC) compartment in each zone in which water level can be varied for reactivity control. The power distribution and total power can be estimated on the basis of signals obtained from a number of in-core neutron detectors distributed appropriately in the core and also from the measurement of thermal power and thereby these can be controlled.

A. Nodal Core Model of the 540 Mwe Phwr

The 14 zones, in which the reactor is considered to be divided, are treated as small cores coupled through neutron diffusion. Considering the mechanisms of neutron production and absorption in each zone and leakage of neutrons among different zones, the rate of change of power in a zone can be given by [6]

\[
\frac{dP_i}{dt} = \left(\frac{\rho_{\text{exi}} + \rho_{\text{fi}} + \rho_{\text{c}i}}{I} - \beta - \frac{\sigma_{x'i}}{\Sigma_{a'i}} P_i \right)
+ \sum_{k=1}^{m_d} (\lambda_k C_{ik})
+ \frac{1}{I} \sum_{j=1}^{N} (\alpha_{ij} P_j - \alpha_{ij} P_i)
\]

\[\sigma_{x'i} = \frac{\sigma_{x'i}}{E_{\text{eff}} \Sigma_{f'i} V_i} \quad (i = 1, 2, \ldots, N) \] (2)

where subscripts i and j have been used to denote zones; N is the number of zones in the reactor and \(m_d\) is the number of delayed neutron precursors; \(P\) is the power level; \(\rho\) indicates the reactivity such that \(\rho_{\text{exi}}, \rho_{\text{fi}}, \rho_{\text{c}i}\) are the components of reactivity related to external control mechanism, feedback due to fuel temperature and feedback due to primary coolant temperature respectively for the ith zone; \(C_{ik}\) is the kth group delayed neutron precursor concentration for the ith zone; \(\lambda_k\) denotes the kth delayed neutron decay constant for kth group of delayed neutron precursors; \(X\) is the xenon concentration; \(\Sigma_{a'}\) and \(\Sigma_{f'}\) represent thermal neutron absorption and fission cross sections respectively; \(I\) is prompt neutron life time; \(E_{\text{eff}}\) is energy liberated per fission; \(V_i\) denotes volume and \(\sigma_{x'}\) denotes microscopic thermal neutron absorption cross sections of xenon.

The coupling coefficients, \(\alpha_{ij}\), are dependent upon the geometry, material composition and the characteristic distance between the zones. Mathematically, they can be expressed as

\[
\alpha_{ij} = \frac{D d_{ij} \Psi_{ij}}{d_{ij} V_i'} ; \quad i \neq j \quad (3)
\]

where \(D\) is diffusion coefficient, \(\nu\) is thermal neutron speed, and \(\Psi_{ij}\) and \(d_{ij}\) are, respectively, the area of interface and the distance between \(i\)th and \(j\)th zones. The coupling coefficient between a zone and a non-neighbouring zone is assumed to be zero. Because there is no sharp boundary between zones, the parameters \(\psi_{ij}, d_{ij},\) and \(V_i'\) are somewhat uncertain. For improving the model accuracy it is sometimes necessary to employ semi-analytical methods to tune the values of coupling coefficients. Nevertheless, it is assumed here that the coupling coefficients defined by (3) are sufficiently accurate.

Delayed neutron precursors are produced from fission and they are lost due to radioactive decay. Hence, the rates of change of delayed neutron precursors’ concentrations in different zones are given by

\[
\frac{dC_{ik}}{dt} = \frac{\beta_k}{T} P_i - \lambda_k C_{ik}, \quad i = 1, 2, \ldots, N, \quad k = 1, 2, \ldots, m_d. \] (4)

For simplicity, only one group of delayed neutrons is considered, i.e., \(m_d = 1\). Similarly, in the \(i\)th zone the iodine and xenon concentrations vary as represented by the equations

\[
\frac{dI_i}{dt} = \gamma_{i} \Sigma_{f'i} P_i - \lambda_I I_i \] (5)

\[
\frac{dX_i}{dt} = \gamma_{x} \Sigma_{f'i} P_i + \lambda_I I_i - (\lambda_x + \sigma_{x'i}) X_i \] (6)

respectively, where \(I\) is the iodine concentration, \(\gamma_{x}\) and \(\gamma_{I}\) denote xenon and iodine yield per fission, and \(\lambda_{x}\) and \(\lambda_{I}\) denote xenon and iodine decay constants.

The variation of fuel and coolant temperatures[7], may be modeled as

\[
\frac{dT_f}{dt} = k_a P_g - k_a(T_f - T_c), \quad (7)
\]

\[
\frac{dT_c}{dt} = k_c(T_f - T_c) - k_d(T_c - T_1). \quad (8)
\]

\(T_f\) is the temperature at any point within the fuel volume and it is in general a function of all spatial co-ordinates as well as time, \(T_c\) is the coolant outlet temperature and \(T_1\) is the coolant inlet temperature. \(P_g\) is the global power, which is the sum of all the zonal powers. The constants \(k_{a}, \kappa_{an}, \kappa_{nc}, \kappa_{nd}\) depend on the thermal capacity and conductivity of the fuel and coolant, whose values are obtained from the reactor parameters.

The following approximate equation can be used to describe the variation of water level as a function of input signal to control valves [4]:

\[
\frac{dh_i}{dt} = -m_i q_i \] (9)

where \(h_i\) is the instantaneous water level in \(i\)th zone control compartment (ZCC), \(m_i\) is a constant, and \(q_i\) is the voltage signal given to the control valve of \(i\)th zone.

The reactivity in a zone varies with change of water level of ZCC as well as variation in the fuel and coolant temperature and variation in xenon concentration. The effect of xenon has already been taken into account in (1). To account for reactivity changes associated with fuel and coolant temperature changes, it is assumed that the power distribution does not change appreciably during normal control related transients and the reactivity...
variations are approximately linear over a considerable range and so it can be written as [8]
\[ \rho_{fi} = \mu_{fi}(T_f - T_{f0}) = \mu_{fi}\delta T_f \] (10)
and
\[ \rho_{ci} = \mu_{ci}(T_c - T_{c0}) = \mu_{ci}\delta T_c \] (11)
where \( \mu_{fi} \) and \( \mu_{ci} \) are the coefficients of reactivity for fuel and coolant, respectively. \( T_{f0} \) and \( T_{c0} \) indicate the steady-state values of the fuel and coolant temperatures, respectively.

The component of reactivity due to LZC, \( \rho_{cti} \), in a zone is directly proportional to the water level in the ZCC of that zone given by
\[ \rho_{cti} = -K'_i(h_i - h_{i0}), \quad (i = 1, 2, \ldots, N). \] (12)

The salient physical data of the reactor are given in the Tables I and II.

B. Linearization
Considering a small change in power, precursor’s concentration, iodine, xenon concentration, water levels of the zone control compartments (ZCC), fuel and coolant temperatures, a set of linear equations would be obtained for describing the behaviour of the reactor in the vicinity of the steady-state operating point as follows:

\[ \frac{d}{dt} \left( \frac{\delta P_i}{P_{i0}} \right) = -\frac{1}{l} \left( \beta + \sum_{j=1}^{N} \alpha_{ij} \frac{P_{j0}}{P_{j0}} \right) \delta P_i + \frac{1}{l} \sum_{j=1}^{N} \alpha_{ij} \frac{P_{j0}}{P_{j0}} \delta P_j + \beta \frac{\delta C_{ik}}{l} \sum_{j=1}^{N} \frac{\delta X_{i}}{X_{i0}} \delta X_j \] (13)

\[ \frac{d}{dt} \frac{\delta C_{ik}}{C_{ik0}} = \lambda_k \frac{\delta P_i}{P_{i0}} - \lambda_k \frac{\delta C_{ik}}{C_{ik0}} \] (14)

\[ \frac{d}{dt} \frac{\delta I_i}{I_{i0}} = \lambda_{I} \frac{\delta P_i}{P_{i0}} - \lambda_{I} \frac{\delta I_i}{I_{i0}} \] (15)

\[ \frac{d}{dt} \frac{\delta X_i}{X_{i0}} = \left( \lambda_x - \lambda_{I} \frac{I_{i0}}{X_{i0}} \right) \frac{\delta P_i}{P_{i0}} + \lambda_{I} \frac{I_{i0}}{X_{i0}} \frac{\delta I_i}{I_{i0}} - \left( \lambda_c + \sigma_{x} \right) \frac{\delta X_i}{X_{i0}} \] (16)

\[ \frac{d}{dt} \delta h_i = -m_i \delta q_i \] (17)

\[ \frac{d}{dt} \left( \delta T_f \right) = k_{d0} \sum_{i=1}^{N} \delta P_i - k_d \delta T_f + k_d \delta T_c \] (18)

\[ \frac{d}{dt} \left( \delta T_c \right) = k_d \delta T_f - (k_c + k_d) \delta T_c. \] (19)

The state, control and output vectors can be defined as follows:
\[ z = \left[ z_{f1}^T \ z_{c1}^T \ z_{1}^H \ z_{P}^T \ z_{T_f}^T \ z_{T_c}^T \right]^T \] (20)
\[ u = [\delta q_1 \ \delta q_2 \ \delta q_3 \ \ldots \ \delta q_N]^T \] (21)
\[ y = z_P \] (22)

<table>
<thead>
<tr>
<th>Zone No.</th>
<th>Power (MW)</th>
<th>Volume (m³)</th>
</tr>
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<tbody>
<tr>
<td>1,6,8,13</td>
<td>132.75</td>
<td>14.7</td>
</tr>
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</tr>
<tr>
<td>3,10</td>
<td>123.30</td>
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</tr>
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**TABLE I**

**STEADY-STATE ZONE POWER LEVELS AND VOLUMES**

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**TABLE II**

**PHYSICAL DATA FOR THE 540 MWe PHWR FOR ALL ZONES**

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where
\[ z_t = \left[ \frac{\delta I_1}{I_{i0}} \ \frac{\delta I_2}{I_{i0}} \ \frac{\delta I_3}{I_{i0}} \ \ldots \ \frac{\delta I_N}{I_{i0}} \right]^T \] (23)

\[ z_X = \left[ \frac{\delta X_1}{X_{i0}} \ \frac{\delta X_2}{X_{i0}} \ \frac{\delta X_3}{X_{i0}} \ \ldots \ \frac{\delta X_N}{X_{i0}} \right]^T \] (24)

\[ z_C = \left[ \frac{\delta C_1}{C_{i0}} \ \frac{\delta C_2}{C_{i0}} \ \frac{\delta C_3}{C_{i0}} \ \ldots \ \frac{\delta C_N}{C_{i0}} \right]^T \] (25)

\[ z_H = \left[ \delta h_1 \ \delta h_2 \ \delta h_3 \ \ldots \ \delta h_N \right]^T \] (26)

\[ z_P = \left[ \frac{\delta P_1}{P_{i0}} \ \frac{\delta P_2}{P_{i0}} \ \frac{\delta P_3}{P_{i0}} \ \ldots \ \frac{\delta P_N}{P_{i0}} \right]^T \] (27)

\[ z_{T_f} = \delta T_f \] (28)

\[ z_{T_c} = \delta T_c. \] (29)

Now the system of state (13)–(19) is expressed in standard state variable representation as
\[ \dot{z} = Az + Bu \] (30)
\[ y = Mz. \] (31)

There are 5 state variables, i.e., Power, Precursors’ Concentration, Iodine, Xenon, and LZC water levels, characterizing the behaviour of each zone and 2 variables for the temperature changes resulting into 5N + 2 state variables. Thus, there would be 72 state variables, 14 inputs and 14 outputs in this system. The elements of the system and input matrices in (30)
and (31) depend upon the physical parameters as well as on the steady-state zonal power levels which in turn depend upon the global power level $P_{g}$. Hence, the system characteristics would be different at different power levels of operation. The system expressed by (30) and (31) is proven to be completely control- and observable in [9].

III. MODEL REDUCTION

Model reduction results in simpler mathematical model of a complex system. The reduction should be done in such a way that the important dynamic characteristics of the process are preserved while those less dominant are ignored. This reduces the complexities in control design. There are various techniques available for model reduction. One of them is Aggregation method [10], which has the following approach.

A linear time invariant (LTI) system with $n$ states, $m$ inputs and $p$ outputs has the form as in (30) and (31), where $A$ is $n \times n$ system matrix, $B$ is $n \times m$ input matrix and $M$ is $p \times n$ output matrix.

The above system discretized with sampling rate of $(1)/(\tau)$ is given as

$$z(k+1) = \Phi_{\tau} z(k) + \Gamma_{\tau} u(k)$$  \hspace{1cm} (32)

$$\gamma(k) = Mz(k)$$  \hspace{1cm} (33)

where $\Phi_{\tau} = e^{A\tau}$ and $\Gamma_{\tau} = \int_{0}^{\tau} e^{As}Bds$. The dual system[11] for system in (32) and (33) given by

$$\bar{z}(k+1) = \Phi_{\tau} \bar{z}(k) + M^{T} \bar{u}(k),$$  \hspace{1cm} (34)

$$\bar{\gamma}(k) = \Gamma_{\tau}^{T} \bar{z}(k)$$  \hspace{1cm} (35)

can be diagonalized by using a transformation matrix $T$ as

$$\bar{z}(k+1) = \Lambda \bar{z}(k) + \Pi \bar{u}(k)$$  \hspace{1cm} (36)

where

$$\tilde{z}(k) = T\bar{z}(k)$$  \hspace{1cm} (37)

$$\tilde{\gamma}(k) = \varphi \bar{z}(k)$$  \hspace{1cm} (38)

$$\Lambda = T \times \Phi_{\tau} \times T^{-1}$$  \hspace{1cm} (39)

$$\Pi = T \times M^{T}$$  \hspace{1cm} (40)

$$\varphi = \Gamma_{\tau}^{T} \times T^{-1}.$$  \hspace{1cm} (41)

However, if the system has repeated eigenvalues, the method will be still applicable after transforming the system into Jordan Canonical form.
The system given in (36) and (37) can be represented such that it is decomposed into two parts; one containing all unstable eigenvalues and the other containing all stable ones, i.e.,

\[
\begin{align*}
\dot{z}_1(k+1) &= \Lambda_1 \dot{z}_1(k) + \Pi_1 \bar{u}(k) \quad (41) \\
\dot{z}_2(k+1) &= \Lambda_2 \dot{z}_2(k) + \Pi_2 \bar{u}(k) \quad (42)
\end{align*}
\]

where the system matrix \( \Lambda_1 \) is for states \( \dot{z}_1 \) which are unstable and has order say \( r \). \( \Lambda_2 \), having order \( (n-r) \) includes all stable eigenvalues. Now, the system represented by the (41) is the reduced-order system for (36) and can be used for controller design.

Let \( \bar{u}(k) = K_1 \dot{z}_1(k) \) be a stabilizing control for system in (41). States \( \dot{z}_1 \) are related to \( \dot{z} \) by an aggregation matrix \( [10] \), \( C_\alpha \), i.e.,

\[
\dot{z}_1 = C_\alpha \dot{z}
\]

where \( C_\alpha = [I_r \ 0] \), and \( I_r \) an \( r \)-identity matrix.

Now, the control for the system in (34) can be constituted using aggregation matrix given in (43)

\[
\bar{u}(k) = K_1 C_\alpha T \zeta(k) \quad (44)
\]

The closed-loop eigenvalues of the system in (34) with the control in (44) are the disjoint sum of the eigenvalues of \( (\Lambda_1 + \Pi_1 K_1) \) and the eigenvalues which are not retained in the reduced model (41). As \( (\Lambda_1 + \Pi_1 K_1) \) is stable by design and the eigenvalues which are not retained in the reduced model are stable, control in (44) will always result in a stable closed-loop system [12].

Thus, \( (\Phi^T_r + M^T R_{11}) \) is stable, which implies \( (\Phi^T_r + M^T R_{11})^T \) is stable and further implies that

\[
(\Phi_r + R_{11}^T M)
\]

is stable.

IV. PERIODIC OUTPUT FEEDBACK DESIGN

The problem of pole assignment by piecewise constant output feedback was studied by Chammas and Leonides [5] for LTI systems with infrequent observation. They showed that, by use of periodically time-varying piecewise constant output feedback gain, the poles of a discrete-time control system could be assigned arbitrarily (with the natural restriction that they be located symmetrically with respect to the real axis) [13]–[15].

In [13], they have also suggested fast output sampling method for pole assignment. Design of a controller based on fast output sampling technique for a large PHWR by converting it into block diagonal form and then decomposing into a fast subsystem and a slow subsystem, has been demonstrated [16]. State feedback controls are designed separately for the slow subsystem and the fast subsystem and then a composite state feedback control is obtained. In fast output sampling, the control is generated as a linear combination of the past output samples collected in one sampling interval. Here, input sampling time is larger compared to the output sampling time, whereas in this paper, periodic output feedback technique is applied to design the controller, in which the input is changed several times in one output sampling interval. This is explained in the following.
For the discrete time invariant system given by (32) and (33), the output can be measured at the time instant \( t = k\tau \), \( k = 0, 1, \ldots \). The output sampling interval can be divided into \( N_c \) subintervals of length \( \Delta = \tau / N_c \), and the hold function is assumed to be constant on these subintervals. Thus, the control law with periodic output feedback [15] is

\[
\begin{align*}
    u(t) &= K_I z(k\tau) \\
    \Delta_k < t &\leq \Delta_{k+1}, \quad K_{I+1} = K_I
\end{align*}
\]  

(46)  

(47)

for \( l = 0, 1, \ldots, N_c - 1 \). Now, consider the system obtained by sampling the system in (32) and (33) at sampling interval \( \Delta \), i.e.,

\[
\begin{align*}
    z(k + 1) &= \Phi z(k) + \Gamma u(k) \\
    y(k\tau) &= M z(k\tau)
\end{align*}
\]  

(48)  

(49)

where \( \Phi = e^{A\Delta} \) and \( \Gamma = \int_0^\Delta e^{As} B ds \). A useful property of the control law in (46) is given by the following lemma [5]. Assume \( \Phi^T \Gamma \), is observable and \( \Phi, \Gamma \) is controllable with controllability index \( \nu \) such that \( N_c \geq \nu \), then it is possible to choose a gain sequence \( K_t \) such that the closed-loop system, sampled over \( \tau \), takes desired self-conjugate set of eigenvalues. Define

\[
K = [K_0 \ K_1 \ldots \ K_{N_c-1}]^T
\]  

(50)

then a state space representation for the system sampled over \( \tau \) is

\[
\begin{align*}
    z(k\tau + \tau) &= \Phi z(k\tau) + \Gamma u(k\tau), \\
    y(k\tau) &= M z(k\tau)
\end{align*}
\]  

(52)  

(53)

\[
\Gamma = [\Phi^{N_c-1}\Gamma, \Phi^{N_c-2}\Gamma, \ldots, \Gamma].
\]

Applying periodic output feedback for the system in (46), the closed-loop system becomes

\[
\begin{align*}
    z(k\tau + \tau) &= (\Phi^{N_c} + \Gamma K M) z(k\tau).
\end{align*}
\]  

(54)

Now, an output injection matrix \( G \) can be obtained such that

\[
\rho(\Phi^{N_c} + GM) < 1
\]  

(55)

where \( \rho(\cdot) \) denotes the spectral radius.

Comparing (45) and (55) it is obvious that \( G = K_I^T \). Now, it is observed by comparing (54) and (55) that a periodic output feedback gain which realizes the output injection gain \( G \) can be obtained by solving

\[
\Gamma K = G.
\]  

(56)
Fig. 3. Transient variation of power, Iodine, Xenon and Precursors’ Concentration of zone-1 after the reactivity disturbance.

Fig. 4. Transient variation of axial, top to bottom and side to side tilts after the reactivity disturbance.
A. Decentralized Gain Design

When the above idea of periodic output feedback controller design is realized in practice, there is a problem such that the gain matrix $K$ may have elements with large magnitude. They amplify measurement noise and a difficulty is experienced while implementation of the control. Hence, it is desirable to keep these values low. This objective can be expressed by an upper bound $\theta_1$ on the norm of the gain matrix $K$, such that

$$\|K\| < \theta_1,$$

(57)

When trying to deal with this problem, it turns out to be better not to insist on an exact solution of the design (56), but to allow a small deviation and use an approximation $\Gamma K \approx G$, which hardly affects the desired closed-loop dynamics but may have considerable effect on the problem described above. Thus, instead of looking for an exact solution for the (56), the following inequality is solved along with (57) with appropriately chosen values of the design constraints, $\theta_1$ and $\theta_2$.

$$\|\Gamma K - G\| < \theta_2,$$

(58)

These inequalities may be solved using the LMI Toolbox of Matlab [17] for obtaining $K$ and they can be formulated as follows:

$$\begin{bmatrix}
-\theta_2^2 I & K \\
K^T & -I
\end{bmatrix} < 0$$

$$\begin{bmatrix}
-\theta_2^2 I & (\Gamma K - G)^T \\
(\Gamma K - G) & -I
\end{bmatrix} < 0.$$

(59)

Further, the choice of a decoupled or decentralized gains will lead to a simpler realization [18] of the practical controller, which ensures that input to any zone affects the corresponding zone only and other zones are not affected by it. The decentralized gains are obtained by structuring the $K$ matrix such that its respective elements interacting with the 14 outputs are placed diagonally and making the off-diagonal elements zero. The decentralized periodic output feedback gain matrix $K$ is organized as follows:

$$K = [K_0^T \ K_1^T \ \cdots \ K_{N-1}^T]^T$$

where

$$K_0 = \text{diag}[K_{01} \ K_{02} \ \cdots \ K_{014}].$$

Similarly, $K_1, K_2, K_3, \ldots, K_{N-1}$ are diagonal matrices of size $14 \times 14$ having diagonal elements.

V. SIMULATION RESULTS

In the preceding sections the model of the 540-MWe PHWR was presented, which has a high order of 72 and a complex, nonlinear nature. The linear model of the reactor given by the set of (13)–(19) and represented in standard state variable form by (30)–(31) is discretised with a sampling period of 0.05 s. The eigenvalues of the open-loop system are as given in Tables III–V. It is difficult to handle such a large system numerically for controller design and, hence, a reduced model will be preferred in such a situation. Here, one can notice that 26 eigenvalues (serial number 47 to 72) are out of the unit circle indicating unstable modes and should be included in the reduced-order model. Hence, the model reduction procedure as explained in Section III has been carried out with $r = 26$ to obtain a 26th-order reduced model.

Now, the method described in Section IV is applied for designing a periodic output feedback for the 14-zone nodal model.
of the 540-MWe PHWR. Variation of xenon and iodine concentrations takes place at much slower rate than the precursor concentration does. Hence, the choice of the sampling time is limited by only the time constant characterizing the dynamics of delayed neutron precursors. The largest unstable eigenvalue of the open-loop continuous time system matrix is \(8.3667 \times 10^{-3}\), which indicates that a sampling time \(\tau < (1)/(8.3667 \times 10^{-3})\) or 119 s can be chosen. It has been observed that for several values of \(\tau\) in the range of 0.01 upto 119 s a stable closed-loop response can be obtained with the proposed controller. However, it is desirable for practical reactor control to work with smaller value of \(\tau\), because in small time the reactor power can undergo a considerable change. Thus, \(\tau = 0.05\) s is chosen, which is not very small for the implementation of the algorithm. There are 14 inputs in the system. For the existence of the periodic output feedback gain, the number of gain matrix changes should be greater than the controllability index of the system. So in this case number of gain changes, \(N_c\) is chosen as 7. For \(N_c = 7\) and \(\tau = 0.05\) s and following the procedure given in Section IV, the output injection gain matrix \(G\) is obtained as explained in (55)–(56). The periodic output feedback gain \(K\) is given by (56). However, the gains obtained by exact solution of (56) are very large and as such they would cause large variation of input during an output sampling interval. The optimization procedure described in (58)–(59) is used to obtain \(K\) that would minimize the variation of input during an output sampling interval. Here, selecting the values for the constraints as \(\theta_1 = 0.1\) and \(\theta_2 = 0.1\), the periodic output feedback gain \(K\) is obtained. The numerical values of the diagonal entries of the \(K\) matrix range between \(-2.5080 \times 10^{-3}\) to \(-5.5253\).

The eigenvalues of the closed-loop system in (54) are given in Tables VI–VIII. At \(t = 0\), the reactor is assumed to be operating at 1800 MW, with the zonal power distribution as given in Table I. A 70 hour simulation of the closed-loop system is carried out by disturbing the zone—1 reactivity with a standard disturbance such that the reactivity is decreased with a rate of 0.25 mk/s for 10 s and then increased with the same rate for 10 s as shown in Fig. 1(a). The effect of such type of external disturbance is observed as in Fig.1(b) and (c) on the zonal and global powers as well as for Xenon, Iodine and Precursors concentrations of different zones as in Figs. 2 and 3. It is seen from Fig. 1(c) that the global power settles within \(\pm 2\%\) of its steady-state value in about 200 s. The zonal power levels after the reactivity disturbance settle to within \(\pm 2\%\) of their respective steady-state values in 200 s, while the time taken for settlement xenon and iodine concentrations to their respective steady-state values is about 15 h and 35 h, respectively.

### Table VI

<table>
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<tr>
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<tbody>
<tr>
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</tr>
<tr>
<td>2</td>
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<td>15</td>
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### Table VII

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<tr>
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<td>31</td>
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<td>32, 33</td>
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### Table VIII

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<tr>
<td>34</td>
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<td>42</td>
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<td>43-46</td>
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<td>47-72</td>
<td>9.9999 \times 10^{-1}</td>
</tr>
</tbody>
</table>
axial, top to bottom and side to side tilts are also observed as shown in the Fig. 4, whereas the variations of fuel and coolant temperatures is observed as shown in the Fig. 5. Such a response is considered satisfactory for the operation of 540-MWe PHWR.

VI. CONCLUSION

The spatial model for the 540-MWe PHWR as obtained above is necessarily an extension of the models discussed in [1] and [4]. By inclusion of the fuel and coolant temperatures into the nuclear reactor model, the temperature dependence of reactivity can be better understood. The periodic output feedback method can be effectively employed to obtain the controller gains for such a large system with ill-conditioned nature. The decentralized control proves to be simpler for practical implementation. The transient variations arising due to external reactivity disturbance can give better understanding of the system. The dynamics of primary coolant loop temperatures can be better studied by this model and thus, it poses advantages over the earlier models.

REFERENCES