Model for oxygen transfer in rotating biological contactor

Vijay Kubsad, Sanjeev Chaudhari*, S.K. Gupta

Centre for Environmental Science and Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

Abstract

Rotating biological contactor is being widely used for wastewater treatment but there is an apparent lack of knowledge about the rate at which oxygen transfer occurs, in physical and biological system. In this study the transfer of oxygen from air to water by a rotating disc air–liquid contactor in physical system is investigated. The oxygen transfer model suggested by Kim and Molof, Water Sci. Technol. 14 (1982) 569, was modified and the developed model is termed as modified Kim and Molof model. The model was calibrated by using available data in literature and validated by experiments conducted in this study. The effect of significant physical parameters was integrated into a single term and is termed as volume renewal number. The modified Kim and Molof model was compared with the other available models. The coefficient of determination ($R^2$) for the modified Kim and Molof model obtained is 0.95 which is much higher than in the other available models. Thereby the model is expected to estimate oxygen transfer more accurately. Further, a simplified linear model between $K_La$ and the volume renewal number is proposed. Both modified Kim and Molof and linear model estimate the overall oxygen transfer coefficient ($K_La$) accurately.

Keywords: Oxygen transfer; RBC; Volume renewal number; Physical factors; Scale-up

1. Introduction

The rotating biological contactor is a fixed biomass system comprising rotating discs. Biofilm gradually forms on the disc surface. The constant rotation of the disc causes mixing of the liquid. Also, the rotating disc surface alternately comes into contact between air and wastewater and thus acts as an aeration device for wastewater treatment. However, the practical use of rotating biological contactor (RBC) was introduced, on the basis that the dissolved oxygen (DO) in the reactor did not have significance on treatment efficiency because adequate amount of oxygen could be supplied during the air exposure cycle. Thereby most of the mathematical models have been developed considering the biological step to be the rate limiting step (Kornegay and Andrews, 1968; Clark et al., 1978). The limitation of this type of model lies in the assumption that the only limiting factor for microbial growth is substrate concentration itself. These models are therefore not suitable when RBC is operated at a high organic loading rate or when dissolved oxygen is lower than 1–2 mg/L. It is therefore important to quantify the oxygen transfer in RBC.

One of the critical points in the use of a RBC is the estimation of aeration/oxygenation capacity during treatment. Various authors have done experiments and provided physical oxygen transfer data (Kim and Molof, 1982; Bintanja et al., 1975; Zeevalkink et al., 1979). Also, few researchers have attempted to develop empirical/mathematical relationship for estimation of
oxygen transfer in a rotating biological contactor. Yamane and Yoshida (1972), used a theoretical approach to solve the differential equation of oxygen diffusion through the liquid film covering a disc. However, the presence of attached biomass on disc surface causes a significant increase in the amount of oxygen transport. Kim and Molof (1982), suggested that the possible mechanism for the enhancement is due to direct oxygen absorption by the microorganism during the exposure of the discs, whereas, Zeevakink et al. (1979), proposed based on simple linear relationship between oxygenation by surface renewal of the liquid film on the RBC disc at high revolutions per minute (RPM) is determinant (penetration theory (Higbie model) is applicable) and also at low RPM when tank surface area significantly contributes toward oxygenation. The suggested model is also expected to be more appropriate and valid in the case of RBC geometry different from the ones tested in literature. Further a simple model is also proposed based on simple linear relationship between $K_{La}$ and the volume renewal number ($N_v$).

### 1.1. Available models for physical oxygen transfer

A simple empirical model was proposed by Friedman et al. (1979), by only considering the rotational velocity of the disc and presented the following relationship:

$$\ln K_L = 1.31 \ln \omega + 14.78,$$

where the unit of $K_L$ was $10^{-6}$ m/s and that of $\omega$ was RPM.

Ouano (1978), used dimensional analysis to estimate oxygen transfer in RBC and correlated the overall liquid phase oxygen mass transfer coefficient ($K_L$) and Reynolds number by dimensional analysis and suggested the following equation:

$$K_L = \frac{V}{A_1} \frac{A_{1,0}}{D_L} \left( \frac{\phi \left( \frac{\delta}{\mu} \omega' \rho \right)}{\rho} \right)^{1/2},$$

where $K_L$ is the oxygen mass transfer coefficient, $K$ is the general proportionality constant, $V$ is the effective reactor volume, $A$ is the total area of gas–liquid

### Nomenclature

- $A$: exposed surface area (L$^2$)
- $A_i$: surface area of reactor/tank (L$^2$)
- $a$, $b$, $K$: constant
- $\phi$: diameter of disc (L)
- $\phi_i$: wetted diameter of disc (L)
- $e$: distance from the disc rim to inner lining of reactor (L)
- $C$: dissolved oxygen concentration (ML$^{-3}$)
- $C_e$: oxygen concentration at equilibrium (ML$^{-3}$)
- $D_L$: diffusivity of oxygen in water (L$^2$ T$^{-1}$)
- $K_L$: overall liquid phase mass transfer coefficient (L T$^{-1}$)
- $K_{La}$: volumetric oxygen transfer coefficient (T$^{-1}$)
- $Q_f$: liquid film flow rate (L$^3$ T$^{-1}$)
- $Re$: Reynolds number, dimensionless
- $YI$: dimensionless immersion depth ($= (\Phi - \Phi_0)/\Phi$) (−)
- $S$: half space between the discs (L$^{-1}$)
- $t$: time (T)
- $t_d$: dimensionless film thickness ($= \sqrt{d/D_{It}}$) (−)
- $t_{R}$: average contact time per rotation (T)
- $V$: water volume in trough (L$^3$)
- $V_c$: vertical component of circuirferential velocity where the disc emerges from the water (L T$^{-1}$)
- $V_p$: peripheral velocity of disc (L T$^{-1}$)
- $\omega$: rotational speed (RPM)
- $\omega'$: $\omega/60$
- $\delta$: thickness of water film (L)
- $\rho$: density of liquid (ML$^{-3}$)
- $\mu$: absolute viscosity of liquid (ML T$^{-1}$)

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interface, and $\rho$ is density of the liquid, $D_L$ is the molecular diffusion coefficient, $\mu$ is the absolute viscosity of liquid, $A_p$ is the projected area of the disc and $\phi_0$ is the equilibrium concentration of the gas in solution. Noting that $d_t$ can be written as change in concentration with time,

\[ \frac{dC}{dt} = K_L(A/V)(C_s - C), \]

and $C_s$ is the saturation concentration of DO, $C$ is the dissolved oxygen concentration in liquid and $K_L$ is the overall liquid phase mass transfer coefficient.

K and Molof (1982), assumed that oxygen transfer only occurred from liquid film entrained onto the RBC disk, and used Eq. (5) suggested by Zeevalkink et al. (1978) to compute liquid film thickness ($\delta$), which was further used to compute volume renewable number. The volume renewal number computed according to Kim and Molof model is termed as $KN_v$. They also studied some of the physical factors affecting RBC oxygen transfer into the mixed liquor using three differently sized laboratory scale RBC units. The overall oxygen transfer coefficient $K_L$ was calculated from the laboratory data. The physical parameters studied included spacing between the discs, size of the discs, rotational velocity, peripheral velocity and number of discs per stage. The study correlated the overall oxygen transfer coefficient $K_L$ with volume renewable number ($KN_v$), which has been dealt later in the paper. Rittmann et al. (1983) also studied the oxygen transfer in absence of biomass. At high disc speed (RPM) oxygen absorption by the liquid film was dominant and at low speed this mechanism was insignificant in comparison to the aeration that occurred at the air/reactor liquid interface. Aeration through the liquid film thus seemed to be the dominant mechanism for biochemical oxygen transfer at rotational speeds investigated (3–25 RPM).

2. Oxygen transfer model

2.1. Physical oxygen transfer and basic relationships

In wastewater treatment, the rate of gas transfer is proportional to the difference between the existing and the equilibrium concentration of the gas in solution. This relationship can be expressed as:

\[ \frac{dC}{dt} = K_L(A/C_s - C), \]

Noting that $dC/dt = V \frac{dC}{dt}$, above equation can be written as

\[ \frac{dC}{dt} = K_L(A/V)(C_s - C), \]

where $A$ is the exposed area, $V$ is the volume of the reactor, $C_s$ is the saturation concentration of DO, $C$ is the dissolved oxygen concentration in liquid and $K_L$ is the overall liquid phase mass transfer coefficient.

In most cases the interfacial area of contact, $A$, is difficult to determine. To circumvent this problem, a second constant, $K_L$, is introduced. This constant has a value equal to the product of $K_L$ and $(A/V)$. Eq. (7) can be written as change in concentration with time, $dC/dt = K_L a(C_s - C)$.

In a RBC, physical oxygen transfer occurs through the liquid film flowing over the disc surface and from the air-reactor liquid interface in the reactor (trough). Kim and Molof (1982), assumed that oxygen transfer only occurred from liquid film entrained onto the RBC disk, and used Eq. (5) suggested by Zeevalkink et al. (1978) to compute liquid film thickness ($\delta$), which was further used to compute volume renewable number. The volume renewal number computed according to Kim and Molof model is termed as $KN_v$. They also studied some of the physical factors affecting RBC oxygen transfer into the mixed liquor using three differently sized laboratory scale RBC units. The overall oxygen transfer coefficient $K_L$ was calculated from the laboratory data. The physical parameters studied included spacing between the discs, size of the discs, rotational velocity, peripheral velocity and number of discs per stage. The study correlated the overall oxygen transfer coefficient $K_L$ with volume renewable number ($KN_v$), which has been dealt later in the paper. Rittmann et al. (1983) also studied the oxygen transfer in absence of biomass. At high disc speed (RPM) oxygen absorption by the liquid film was dominant and at low speed this mechanism was insignificant in comparison to the aeration that occurred at the air/reactor liquid interface. Aeration through the liquid film thus seemed to be the dominant mechanism for biochemical oxygen transfer at rotational speeds investigated (3–25 RPM).
and Molof (1982) assumed that oxygen transfer occurs only through the liquid film flowing on the disc during the air-exposed cycle and the physical variables of RBC were incorporated in the term volume renewal number. The liquid film thickness on the disc was computed by Zeevalkink et al. (1978) model. The volume renewal number was defined as the ratio of liquid film flow rate and the effective reactor volume (only the liquid volume between the discs was considered and the liquid between the trough and the discs was neglected).

\[(K_{Nv}) = Q_f / V_E,\]

where \(K_{Nv}\) is the volume renewal number according to Kim and Molof (1982), \(Q_f\) is the liquid film flow rate, which is the total film flow volume per unit time, \(V_E\) is the effective reactor volume, that is volume of liquid between the RBC discs.

The renewal number relationship suggested by Kim and Molof (1982), is applicable for clean flat discs with \(e/R = 0.042\) and \(H/t_k = 0.15\), where \(e\) is the distance from the disc rim to the inner lining of the reactor, \(R\) is the radius of disc, \(H\) is the distance from the disc center to the liquid free surface and \(t_k\) is the contact time per rotation. \(K_{Nv}\) was computed as mentioned below

\[K_{Nv} = 0.0011(K_{Nv})^{0.732}, \tag{9}\]

where \(S\) is the distance between the discs/2.

They assumed the relationship between \(K_{La}\) and \(K_{Nv}\) as

\[K_{La} = a(K_{Nv})^b, \tag{10}\]

The coefficients \('a'\) and \('b'\) were obtained from the log-log plot of \(K_{La}\) and \(K_{Nv}\) and the mathematical relationship between \(K_{La}\) and \(K_{Nv}\) has been given as

\[K_{La} = 0.0011(K_{Nv})^{0.732}. \tag{11}\]

It is important to note that the relationship between \(K_{La}\) and \(K_{Nv}\) would be linear if oxygen transfer is assumed to occur only through the liquid film on the disc. But the relationship assumed in Eq. (10) would tend to increase the \(K_{La}\) value for low \(K_{Nv}\) values and thereby account for the oxygen transfer from the liquid surface in the tank.

In general, the surface renewal theory for oxygen transfer has been widely accepted. In a RBC the liquid film formed on the disc during the air-exposure cycle would renew its surface due to the forces, such as gravitational and centrifugal acting on it. The air–liquid interface in RBC trough and renewed surface of the liquid film (during air-exposure cycle) are responsible for the oxygen transfer into the liquid. It is assumed that in this model surface renewal of the film is proportional to the flow of the liquid film on the disc. Therefore flow of the liquid film is accounted. Oxygen transfer is occurring through the water surface but the bulk-dissolved oxygen concentration is the same in the reactor/tank, i.e.,

\[(N_v) = Q_f / V, \tag{12}\]

where \(N_v\) is the volume renewal number in \((T^{-1})\), \(Q_f\) is the liquid film flow rate, which is the total film flow volume per unit time \((L^3T^{-1})\), and \(V\) is the volume of liquid in RBC \((L^3)\).

\[N_v = (\text{area of disc exposed to air}) \omega \text{ liquid film thickness (δ) / V}.\]

Substituting the liquid film thickness \((δ)\) with the expression given by Zeevalkink et al. (1978), as mentioned in Eq. (5) we get

\[N_v = An2ω1.2(φ/2ω)^{0.5} / V,\]

\[N_v = 1.697Anω1.5φ^{0.5} / V, \tag{13}\]

where, \(A\) is the exposed surface area to air of one disc, \(n\) is the number of discs, \(ω\) is the RPM of disc, \(φ\) is the diameter of the disc and \(V\) is the volume of liquid in the trough. The volume renewal number \((N_v)\) of RBC can be computed for a set of operating conditions.

It may be observed that the volume renewal number is analogous to surface renewal term ‘a’ in Eq. (7) and if the major mechanism of oxygen transfer is only through the RBC discs, then a linear relationship between \(K_{La}\) and \(N_v\) is expected. Fig. 1 shows the plot of \(K_{La}\) versus \(N_v\) and it is observed that the nature of the plot is not
linear and it appears that it would follow a relationship similar to that mentioned in Eq. (10), as proposed by Kim and Molof (1982).

3. Materials and methods

3.1. Experimental work

Specification of the reactor: The laboratory scale RBC consisted of a semicircular trough (volume of 24 L). The tank was divided into three stages of equal volume. High-density polyethylene (HDPE) discs mounted on a one-meter horizontal shaft, which traversed the length of the trough. Each stage carried fourteen discs having a diameter of 0.23 m. The discs were 35% submerged in the trough and rotated at 5.3 RPM. An electric motor provided the driving force for the rotation. The total effective surface area provided by the unit was 3.6 m². The dimensions of the reactor are shown in Table 1.

To evaluate the oxygen transfer coefficient $K_{L}a$, the physical oxygen transfer tests were performed using the RBC unit with clean discs. In the absence of biomass, 24 L of liquid was required to achieve the disc immersion depth of 35%. The volume of liquid was placed in the reactor and nitrogen gas was bubbled until DO concentration decreased to about 0.15 mg/L. Next, nitrogen flow was stopped and the discs were rotated. Instantaneous DO concentrations were measured. The DO probe was calibrated with saturated tap water. The tests were conducted at room temperature (24 ± 1°C).

4. Model comparison

A comparison of various reported models for computing $K_{L}a$, has been done. The models evaluated are, namely, Friedman et al. (1979), Zeevalkink et al. (1979), Kim and Molof (1982) and modified Kim and Molof model (suggested in the present study). The models were evaluated for the set of literature experimental data of researchers, Poalini (1986), Radwan and Ramanujam (1995), Rittmann et al. (1983), Zeevalkink et al. (1979) and Kim and Molof (1982). The models are compared for their goodness of fit to the data and also how close the mathematical formulation considers the probable physical oxygen transfer mechanisms in the RBC.

Friedman et al. (1979) model relates oxygen transfer coefficient only with the RPM of the disc and does not take into account the transfer of oxygen through the thin water film attached to the rotating media. Fig. 2 (legend for data same as Fig. 1) presents the model given by Friedman et al. (1979) applied to the set of literature experimental data. The coefficient of determination ($R^2$) obtained is 0.47.

The experimental approach and condition used by Zeevalkink et al. (1979) were the same as those of Bintanja et al. (1975), with the exception of immersion depth which had been varied in the study. Zeevalkink et al. (1979) reported that the discrepancy between theory and experimental value of oxygen transfer coefficient became larger with decreasing depth of disc immersion. For the range of values, dimensionless film thickness ($t_d$) between 0.8 and 1.7, the experimental values can be in approximation with the values obtained. The values of $K_{L}$ were 30–50% lower than the predicted values given by Yamane and Yoshida (1972) and Bintanja et al. (1975). The difference between experimental $K_{L}$ values and those calculated from the model were explained by incomplete mixing of water film with water in the trough after a revolution. The Zeevalkink et al. (1979) model is applied to the set of the literature experimental data as shown in Fig. 3 (legend for data same as Fig. 1) and it gives the coefficient of determination ($R^2$) of 0.75, which

![Fig. 2. Friedman et al. model for oxygen transfer coefficient in RBC.](image-url)
is higher than that obtained for the model of Friedman et al. (1979).

Kim and Molof (1982) used three differently sized laboratory scale RBC units to find oxygen transfer into mixed liquor. The physical parameters studied included spacing between the discs, size of the discs, rotational velocity, peripheral velocity and number of discs per stage. The oxygen transfer coefficient ($K_{La}$) was correlated with significant physical parameters and suggested the volume renewal number, $N_v = \omega^{3-2}D^{0.5}S^{-1}$. The model proposed by Kim and Molof (1982) neglected the liquid volume between the disc and bottom of trough. Fig. 4 shows the model given by Kim and Molof (1982) applied to set of literature experimental data. The coefficient of determination obtained is 0.85.

In the case of the modified Kim and Molof model the assumption is made that mixing due rotation of disc would occur and thereby the total volume of tank/reactor liquid is considered. As the molecular diffusion of dissolved gases is quite rapid, the dissolved gases would have the same concentration in the turbulent zone of RBC, even though there may be insufficient mixing occurring with respect to other pollutants. Further, studies on hydraulic regime indicate that complete mixing in RBC occurs which has been supported by tracer studies (Boumansour and Vasel, 1998; Shamaraja, 1997). Thereby the modified Kim and Molof model takes into consideration, the tank liquid volume and the actual disc surface area exposed to air. Fig. 5 presents the modified Kim and Molof model applied to the set of data and the coefficient of determination ($R^2$) obtained is 0.95. It can be observed from Fig. 5 that data of Radwan and Ramanujam (1995) are outliers and if they are not considered, then the coefficient of determination ($R^2$) for the modified Kim and Molof model increases to 0.97 (figure not shown). The expression obtained is $K_{La} = 0.01315(N_v)^{0.73778}$. This expression was used to compute the oxygen transfer coefficient value and the experimental and computed values are $K_{La} = 0.031$ and $K_{La} = 0.032$, respectively.

In the modified Kim and Molof model, it is evident that the volume renewal number term does not take into account oxygenation occurring from the air-tank liquid interface, which has been reported to be significant at low RPM (i.e., low $N_v$ values, Rittmann et al., 1983). Further, volume renewal in the modified Kim Molof model is analogous to surface renewal number term ‘$a’ (A/V) in Eq. (7). Therefore when oxygenation from air-reactor liquid interface is insignificant, i.e. when RPM is
high (at high \( N_v \) values), a linear relationship between \( K_{L,a} \) and \( N_v \) is expected. Linearization of a model also simplifies the computations. From Fig. 6 it can be observed that the curve can be approximated by two linear segments, i.e., (a) \( 0 < N_v < 800 \) and (b) \( N_v > 800 \). The data shown in Fig. 1 was linearized for the aforementioned conditions and the \( R^2 \) values obtained were (a) 0.88 for \( 0 < N_v < 800 \) and (b) 0.86 for \( N_v > 800 \) (figures not shown). However, when the data of Radwan and Ramanujam (1995) are excluded, the \( R^2 \) values increased and were 0.93 (for \( 0 < N_v < 800 \)) and 0.92 (for \( N_v > 800 \)). As it can be observed from Fig. 1 the data of Radwan and Ramanujam (1995) are outliers, therefore these were excluded to test the goodness of fit. To test the goodness of fit of the two models, modified Kim and Molof model and segment linearized model, the experimental and computed values of \( K_{L,a} \) were plotted. Fig. 7 shows the computed and experimental values for the modified Kim and Molof’s model with \( R^2 = 0.9623 \) and the slope of the line = 0.9937 which is almost equal to 1 indicating a good fit. Accordingly, a plot was prepared for the segment linearized model and the slope of the line obtained was 0.9847 and \( R^2 = 0.9636 \), which also indicates that linearization in segments also yields satisfactory results.

5. Conclusions

The modified Kim and Molof model is based on the concept of volume renewable number and is applicable to almost all operating conditions of RBC. It is a practical, efficient tool to scale up physical oxygen transfer in RBC. The volume renewal number is based on the liquid film flow rate per unit reactor volume. It is revealed that the modified Kim and Molof model gives good correlation with the significant physical parameters. The value of the coefficient of determination (\( R^2 \)) is also higher than in other reported models. The segmented linearized model can also serve as a simple model with almost the same accuracy as that of the modified Kim and Molof model. Both can be used as practical tools in computing oxygen transfer coefficient for RBC.

References


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