A New Approach to Model Nonquasi-Static (NQS) Effects for MOSFETs—Part I: Large-Signal Analysis
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Abstract—This paper presents a new nonquasi-static (NQS) model for the MOSFET. The model is derived from physics and only relies on the very basic approximation needed for a charge-based model. To derive the model, a popular variational technique named Galerkin’s Method has been used. The model proved to be very accurate even for extremely fast changes in the bias voltages. Simulation results show a very good match even when the rise time of the applied signal is smaller than the transient time of the device.

Index Terms—Large-signal MOS transistor model, MOSFET compact model, nonquasi-static (NQS) effect.

I. INTRODUCTION

During high-frequency operation, the transit time of carriers from source to drain in the MOSFET is comparable to the rise or fall time of the signal. Also, during high-frequency switching, the charge in the MOSFET is in a transient condition and quasi-static (QS) assumptions of steady-state conditions are no longer valid [1]–[3]. A nonquasi-static (NQS) model predicts the terminal currents taking into account the inertia in the charge transport of carriers from source to drain and the charge distribution transient when rapidly varying voltages are applied to the device terminals. The essential difference between QS and NQS formulations can be summarized as follows.

In QS modeling, it is assumed that the channel charge responds instantaneously to any change in the terminal voltages. In other words, the channel charge is only a function of terminal voltages and does not contain time explicitly

\[ Q(t) = Q(V_S(t), V_D(t), V_G(t), V_B(t)). \]  

(1)

In transient response, the following relation gives the total current [3]

\[ I_i(t) = I_{dec}(t) + \frac{d}{dt}(Q_i) \text{ where } i = S, D, G, B \]  

(2)

where \( I_{dec}(t) \) is the dc current, which is determined only by the instantaneous terminal voltages, and \( Q_i \) is the charge partition under \( i \)th terminal. Under QS assumption, the contribution due to the second term becomes

\[ \frac{dQ_i}{dt} = \sum_{j=S,D,G,B} \frac{\partial Q_j}{\partial V_j} \frac{dV_j}{dt} \]  

(3)

which can be modeled by using voltage-dependent capacitors only.

However, the channel charge does have some inertia, and it takes a finite amount of time to follow any changes in the terminal voltages. The channel charge then is not only a function of terminal voltages but also a function of time \( t \). It must contain time explicitly

\[ Q(t) = Q(t, V_S(t), V_D(t), V_G(t), V_B(t)). \]  

(4)

The contribution due to the second term in (2) will then become

\[ \frac{dQ_i}{dt} = \frac{\partial Q_i}{\partial t} + \sum_{j=S,D,G,B} \frac{\partial Q_j}{\partial V_j} \frac{dV_j}{dt} \]  

(5)

If we relax the QS assumption, then an extra \((\partial Q_i)/(\partial t)\) term comes in the transient response. For very fast transients, this term makes a major contribution to the terminal currents. In order to model fast transients and high-frequency behavior of the device, we need to go beyond QS modeling.

Several NQS models have been reported in the literature. The model described in [4] is valid for arbitrary time varying signals at the device terminals. The model involves the solution of a nonlinear partial differential equation derived using time- and position-dependent charge, current and continuity equations. The partial differential equation is solved using suitable initial and boundary conditions. But this solution is done numerically, and therefore this model cannot be implemented in a circuit simulator. Turchetti et al. [5] developed a CAD-oriented model for the MOSFET. In this model, the nonlinear current continuity equation is reduced to an ordinary differential equation by assuming a channel charge profile and using the weighted residue method. But this model assumes a very simple quadratic charge profile. Hwang et al. [6] improved this work by using collocation method instead of weighted residue method, but their work also assumes the same charge profile. Park, Ko, and Hu [7] developed a NQS model with a general charge profile. In their work, they have introduced a position- and time-dependent diffusion coefficient which is the square root of the charge profile. Then they approximated that term as a Fourier cosine expansion with two terms. However, this approximation is only valid when the NQS charge is very small compared to the QS charge. This model also requires a separate treatment when the MOSFET is turned on. Ko et al. [8], [9] proposed a relaxation time based model for both transient and small-signal analysis. This model is not sufficiently physics based and, as we shall see, is quite ineffective when the signal varies sufficiently fast.

In this work, we have developed a completely physics-based NQS model using a variational technique. This model makes only those assumptions which are required for formulation of a charge sheet model. The model equations are reduced to a system of state variable equations; thus, they can be
implemented in a circuit simulator. To validate our model, we have applied a fast ramp with rise time smaller than the transit time of the device. The NQS component of the terminal currents are extracted and compared with device simulation. Even under extreme conditions (i.e., very small rise times), the model shows a good match with device simulation results. A comparison with the BSIM3v3 NQS model is also made, and it is shown that the new model gives more accurate results compared to the BSIM3v3 model.

The paper is organized as follows. The derivation of the model is given in Section II. In Section III, a variational technique called Galerkin’s method has been employed to solve the PDE obtained in Section II, leading to the NQS currents. In Sections IV and V, equations for terminal currents are derived, and results obtained with the new model are compared with device simulation. Finally, in Section VI, some implementation aspects of the model are discussed, and conclusions are drawn.

II. MODEL FORMULATION

The MOSFET inversion layer charge can be expressed in simple form in terms of the surface potential using the charge sheet approximation. Assuming an n-channel device, the inversion charge \( Q(x,t) \) per unit area is given as a function of position \( x \) along the channel (positive \( x \) direction is from source \( x = 0 \) to drain \( x = L \)) and time \( t \) by

\[
Q^i(x,t) = -C_{ox} \Psi_s(x,t) - V_{FB}
\]

where \( \Psi_s(x,t) \) is the surface potential and \( \gamma = ((2\varepsilon_s N_d)^1/2)/(C_{ox}) \) is the body factor. The current \( I(x,t) \) is given by

\[
I(x,t) = W \left[ -D_n \frac{\partial Q^i}{\partial x} + \mu_n Q^i \frac{\partial \Psi_s}{\partial x} \right]
\]

where the first term represents the diffusion component and the second term represents the drift component. In order to get a compact model, we linearize (6) with respect to \( \Psi_s \). The linearization point is chosen at \( \Psi_s = 2\Phi_p + V_F \) where \( V_F \) is the pinchoff voltage [1], [10] and \( \Phi_p \) is the difference between the Fermi level and the intrinsic level. Now

\[
\frac{dQ^i}{d\Psi_s} = \eta C_{ox}
\]

where \( \eta = (1 + \gamma)/(2\sqrt{2\Phi_p + V_F}) \) [10], [11]. By taking into account the current continuity equation (in which the generation-recombination mechanism is neglected),

\[
\frac{\partial I}{\partial x} = -W \frac{dQ^i}{dt}
\]

and we can obtain (by eliminating \( \Psi_s \) from (7) and combining it with (9)) the following nonlinear PDE for inversion layer charge density

\[
\frac{dQ^i}{dt} = k \frac{\partial}{\partial x} \left( Q^i \frac{\partial Q^i}{\partial x} \right)
\]

where \( k \) is given by

\[
k = \frac{\mu_n}{\eta C_{ox}}
\]

where \( Q^i \) and \( Q^f \) are related by \( Q^f = -Q^i + \eta C_{ox} U_t \); \( U_t \) being the thermal voltage.

Next, we separate out \( Q^i \) into two parts

\[
Q^i = Q_{qs} + Q_{nqs}
\]

where \( Q_{qs} \) represents the steady-state charge; it depends only on the terminal voltages and does not have any explicit dependence on time. \( Q_{nqs} \) is the difference between the actual charge and the steady-state charge at the same time instant. Combining (10) and (11)

\[
\frac{dQ_{qs}}{dt} + \frac{dQ_{nqs}}{dt} = k \left( \frac{\partial Q_{qs}}{\partial x} \right)^2 + Q_{qs} \frac{\partial^2 Q_{qs}}{\partial x^2} + k \left( \frac{\partial Q_{nqs}}{\partial x} \right)^2 + 2 \frac{\partial Q_{nqs}}{\partial x} \frac{\partial Q_{qs}}{\partial x} + Q_{qs} \frac{\partial^2 Q_{nqs}}{\partial x^2} + k \left( \frac{\partial Q_{nqs}}{\partial x} \right)^2 + Q_{nqs} \frac{\partial^2 Q_{nqs}}{\partial x^2}.
\]

The first term on RHS represents gradient of steady-state current. Therefore, this term vanishes and the equation reduces to

\[
\frac{dQ_{qs}}{dt} + \frac{dQ_{nqs}}{dt} = k \left( \frac{\partial Q_{qs}}{\partial x} \right)^2 + Q_{nqs} \frac{\partial^2 Q_{nqs}}{\partial x^2} + k \left( \frac{\partial Q_{nqs}}{\partial x} \right)^2 + 2 \frac{\partial Q_{nqs}}{\partial x} \frac{\partial Q_{qs}}{\partial x} + Q_{qs} \frac{\partial^2 Q_{nqs}}{\partial x^2}.
\]

In order to handle the equation, we need to find \( Q_{qs} \) as a function of \( x \), as derived in Appendix I. In solving the PDE (13), we have used a variational method called Galerkin’s method which is described in Appendix II.

III. SOLUTION USING GALERKIN’S METHOD

We now seek an approximate solution of \( Q_{nqs} \) of the form

\[
Q_{nqs}(x,t) = \sum_{n=1}^{N} X_n(t) \varphi_n(x)
\]

where \( \varphi_n \) will be chosen so that it satisfies the essential boundary condition, i.e., it vanishes at \( x = 0 \) and \( x = L \). In addition to this, it is desirable that the functions \( \varphi_n \) form a complete set. Substituting (14) into (13) and noticing the fact that \( Q_{qs} \) does not contain time explicitly, one obtains the following:

\[
\sum_{i=S,D,G,B} \frac{\partial Q_{qs}}{\partial V_i} \frac{dV_i}{dt} + \sum_{n=1}^{N} X_n \varphi_n
\]

\[
= k \sum_{i=1}^{N} \sum_{m=1}^{N} X_n X_m \left( \varphi_n \frac{\partial \varphi_m}{\partial x} + \varphi_m \frac{\partial \varphi_n}{\partial x} \right) + k \sum_{n=1}^{N} X_n \left( \varphi_n \frac{\partial^2 \varphi_n}{\partial x^2} + 2 \frac{\partial \varphi_n}{\partial x} \frac{\partial \varphi_n}{\partial x} \right).
\]
where

\[ \lambda_{ni} = \int_0^L \frac{\partial Q_{qs}}{\partial V_i} \varphi_n \, dx = \frac{\partial}{\partial V_i} \left( \int_0^L Q_{qs} \varphi_n \, dx \right) \quad (17) \]

\[ p_{nm} = \int_0^L \varphi_n \varphi_m \, dx \quad (18) \]

\[ \alpha_{nm} = k \int_0^L \left( \varphi_m \frac{\partial Q_{qs}}{\partial x^2} + 2 \varphi_m \frac{\partial \varphi_m}{\partial x} \frac{\partial Q_{qs}}{\partial x} + \frac{\partial^2 \varphi_m}{\partial x^2} Q_{qs} \right) \varphi_n \, dx \quad (19) \]

\[ \beta_{nm} = k \int_0^L \left( \frac{\partial \varphi_n}{\partial x} \frac{\partial \varphi_n}{\partial x} + \varphi_m \frac{\partial^2 \varphi_n}{\partial x^2} \right) \varphi_n \, dx. \quad (20) \]

Now, inverting the matrix \([p_{nm}]\), the equation can be reduced to a state variable equation. However, if \(\phi_n\) are so chosen that they are mutually orthogonal, then there is no need for the last step. In that case, \(p_{nm} = c_n\delta_{nm}\), where \(c_n = \int_0^L \varphi_n^2 \, dx\). Therefore, \(\sum p_{nm} \varphi_m \varphi_n = c_n \varphi_n\) which implies that (16) itself is a state equation. In our work, we have taken advantage of this fact and chosen \(\varphi_n(x) = \sin(n \pi x/L)\) which are mutually orthogonal functions in the interval \([0, L]\).

The number of terms \(N\) in the approximation (14) should be chosen such that the results are sufficiently accurate. It is difficult to estimate \(N\) analytically for transient simulation. However, we have outlined a procedure to estimate \(N\) in an accompanying paper [12] for small-signal analysis.

In the rest of the paper, results are presented using the model equations described above, and comparisons with device simulation and BSIM3 are carried out. We have used a 4th-order Runge-Kutta method to solve the differential equations involved in our model. The model has not been implemented in a circuit simulator so far. We will report on the computational overhead of our model in a future publication after implementation of the model in a circuit simulator. It may be pointed out that the “stamp” or the “template” for the device would be rather complex when the above model equations are used. However, a computer implementation is relatively straightforward. The additional number of variables required with the new model is \(2N\), and \(N\) time derivatives of the \(X\)’s.

An example with \(N = 2\) is included here to illustrate the model equations further

\[ \dot{X}_1 = \alpha_{111} X_1 + \alpha_{112} X_2 + \beta_{111} X_1 X_1 + \beta_{112} X_2 X_2 \]

\[ + \sum_{i=2, \text{DG}, \text{B}} \lambda_{i} \frac{dV_i}{dt} \]

\[ \dot{X}_2 = \alpha_{211} X_1 + \alpha_{212} X_2 + \beta_{211} X_1 X_1 + \beta_{212} X_1 X_2 \]

\[ + \sum_{i=2, \text{DG}, \text{B}} \lambda_{i} \frac{dV_i}{dt}. \quad (21) \]

Note that \(\beta_{112}, \beta_{121}, \beta_{211}, \text{and} \beta_{222}\) are zero. In general \(\beta_{i, nm}\) vanishes when \(n\) is even and both \(I\) and \(m\) are even or odd. When \(n\) is odd, the term vanishes when only one (not both) \(I\) and \(m\) are even. For a general case, the number of \(\alpha\), \(\beta\), and \(\lambda\) terms would be \(N^2, N^3/2, \text{and} 4N\), respectively.

IV. DETERMINATION OF TERMINAL NQS CURRENTS

The drain and source currents are given by [1], [3]

\[ I_D(t) = I_{DK,D}(t) + W \frac{d}{dt} \left[ \int_0^L \frac{x}{L} Q_{qs} \, dx \right] \]

\[ I_S(t) = I_{DK,S}(t) + W \frac{d}{dt} \left[ \int_0^L (1 - \frac{x}{L}) Q_{qs} \, dx \right] \quad (22) \]

where \(I_{DK,D/S}\) is the dc transport current at time \(t\). This term only depends on the terminal voltages at time \(t\). Substituting (11) into (22), we get

\[ I_D(t) = I_{DK,D}(t) + W \frac{d}{dt} \left[ \int_0^L \frac{x}{L} Q_{qs} \, dx \right] \]

\[ - W \frac{d}{dt} \left[ \int_0^L \frac{x}{L} Q_{NQS} \, dx \right]. \quad (23) \]

But the first two terms on RHS represent the current \(I_{qs,D}\) predicted by the QS model. Hence, due to the NQS effect, an extra current component appears

\[ I_{NQS,D} = - W \frac{d}{dt} \left[ \int_0^L \frac{x}{L} Q_{NQS} \, dx \right]. \quad (24) \]

By similar analysis

\[ I_{NQS,S} = - W \frac{d}{dt} \left( \int_0^L \frac{1-x}{L} Q_{NQS} \, dx \right). \quad (25) \]

For the gate current, one has

\[ I_g(t) = W \frac{d}{dt} \left( C_{ox} \int_0^L (V_G - \Psi_s) \, dx \right) \quad (26) \]

\[ I_{g}(t) = C_{ox} W \int_0^L \left( \frac{dV_G}{dt} - \frac{1}{\eta C_{ox}} \frac{dQ_{qs}}{dt} \right) \, dx \]

\[ + W \frac{1}{\eta} \frac{d}{dt} \left( \int_0^L Q_{NQS} \, dx \right). \quad (27) \]

But the first term is again the QS component of the current, and we have

\[ I_{NQS,G}(t) = - (I_{NQS,S}(t) + I_{NQS,D}(t) + I_{NQS,G}(t)). \quad (28) \]

One can obtain the bulk component of the NQS current by charge conservation

\[ I_{NQS,B}(t) = -(I_{NQS,S}(t) + I_{NQS,D}(t) + I_{NQS,G}(t)). \quad (29) \]

V. SIMULATION RESULTS

To validate our model, we have compared it with device simulation. A MOSFET is designed using process simulator ISE-DIOS [13] with \(N_{sub} = 1.0 \times 10^{16} \text{ cm}^{-3}\), \(t_{ox} = 18 \text{ nm}\), \(V_{FB} = -0.8 \text{ V}\), \(\mu_0 = 540 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}\), \(W = 1 \mu\text{m}\). A
relatively slow device, with $L = 6 \mu m$, was simulated for this study to ensure that substantial NQS effects would be observed for moderate rise and fall times. Device simulation is done using ISE-DESSIS [13]. BSIM3v3 parameters of the device were extracted using ISE-EXTRACT [13]. The NQS component of the current was obtained by subtracting the QS current predicted by the BSIM3v3 QS model from the current obtained by device simulation. The transit time of the device was estimated to be 0.4 ns [1]. In our validation process, we have applied a ramp to the gate of the transistor, with a rise time less than the transit time of the electrons. As we shall see, even under this extreme condition, the NQS component obtained with our model (with 6 harmonic terms in (14)) matches very well with the device simulation results.

A. Turnon Transient

1) Transient in Saturation Region: Fig. 1(a) shows the drain NQS current when the device is in saturation with $V_D = 2 V$, $V_S = 0 V$, $V_B = 0 V$, and a ramp (0 to 2 V) with rise time of 0.1 ns applied to the gate. The drain partition of the QS charge is expected to increase linearly with gate voltage; but due to channel inertia, the growth of the actual charge is less. The difference between the QS charge and the actual charge only increases during the rise time of the ramp. Thus, the NQS component of the drain current is positive during this time. During this period, the electrons injected by source have not yet reached the drain terminal, and due to very high drain barrier, drain cannot inject electrons into the channel. Therefore, the total intrinsic drain current is still zero. After the rise of the pulse, the channel charge gradually reaches its equilibrium value. The difference between the QS charge and actual charge decreases, so the NQS current becomes negative. Here, an important point must be noted. No drain current can flow before the electrons reach the drain terminal. So, during this period of time, the NQS current must have a constant value whose magnitude is equal to the QS current (with opposite sign). Our model correctly predicts this behavior and matches simulation results. After this, the NQS current gradually decays and the actual current tends to the QS current. It can be seen from the figure that the BSIM3v3 NQS model (with $elm = 5$) differs from the device simulation result. It fails to predict the flat portion of the waveform. This part of the waveform arises because of the mutual cancellation of the current due to different harmonics in our model. Since BSIM3v3 uses a single time constant, it is impossible for the BSIM3v3 NQS model to predict the flat portion.

Fig. 1(b) shows the source NQS current waveform under the same condition. Here also, due to channel inertia, source partition of the NQS charge increases during the rise time of the pulse and produces a positive NQS source current. After the rise time, the channel charge gradually reaches its steady-state value. The NQS charge decreases and produces a negative NQS source current. The source end electron barrier is very low, so the source continues to inject electrons, and, therefore, we see a gradual decrease in the source NQS current after the rise time of the pulse.

![Fig. 1.](image-url)
Fig. 2. Turnon transient of a MOSFET kept in linear region with $V_D = 0.05$ V, $V_B = 0$ V, $V_G = 0$ V, a ramp (0 to 2 V) with rise time 0.1 ns is applied at gate.

Fig. 1(c) shows the gate NQS current waveform under the same condition. It shows a trend similar to the source current but there is a difference. The gate NQS current is negative during the rise time of the pulse and positive during the steady state of the pulse. This is simply because of the fact that gate charge has opposite sign. Fig. 1(d) shows the substrate NQS current waveform under the same condition.

2) Transient in Linear Region:

Fig. 2(a) shows the drain NQS current waveform when the device is in deep triode region with $V_S = 0$ V, $V_B = 0$ V, $V_D = 0.05$ V, and a ramp (0 to 2 V) with rise time of 0.1 ns applied to gate. The important observation here is that, the magnitude of the drain NQS current is greater than that in saturation. This is because of the fact that, in the linear region, the QS charge increases at a faster rate, and that makes both the magnitude and the rate of change of the NQS charge larger. This is reflected in our model with higher values of $\chi$'s (17).

Fig. 2(b) shows the source NQS current waveform when the device is in deep triode region with $V_S = 0$ V, $V_B = 0$ V, $V_D = 0.05$ V, and a ramp (0 to 2 V) with rise time of 0.1 ns applied to gate. This plot shows a characteristic similar to that in the saturation case. However, the magnitude of the source NQS current is greater. This is also because of the fact that, in linear region, the QS charge decreases at a faster rate, and that makes both the magnitude and the rate

B. Turnoff Transient

1) Transient in Saturation Region:

Fig. 3(a) shows the drain NQS current when the device is in saturation with $V_S = 0$ V, $V_B = 0$ V, $V_D = 2$ V, and a ramp (2 to 0 V) with a fall time of 0.1 ns applied to gate. The drain partition of the QS charge decreases linearly with gate voltage, but due to channel inertia, the decay of actual charge is less. The difference between the QS charge and actual charge only increases during the fall time of the ramp, but in this case, the actual charge is greater than the QS charge. That makes the NQS component of the drain current negative during that time. After the fall of the pulse the channel charge gradually reaches its equilibrium value. The difference between the QS charge and actual charge decreases, so the NQS current becomes positive. One observes a longer tail of the NQS current, because, during this period of time, the QS channel charge is very small, so the NQS charge decays very slowly. This fact is reflected in our model with a very low value of $\chi$ (19).

Figs. 3(b), (c), and (d) show the source, gate and substrate NQS current waveforms under the same condition.

2) Transient in Linear Region:

Fig. 4(a) shows the drain NQS current waveform when the device is in deep triode region with $V_S = 0$ V, $V_B = 0$ V, $V_D = 0.05$ V, and a ramp (0 to 2 V) with a fall time of 0.1 ns applied to gate. This plot shows a characteristic similar to that in the saturation case. However, the magnitude of the drain NQS current is greater. This is also because of the fact that, in linear region, the QS charge decreases at a faster rate, and that makes both the magnitude and the rate
Fig. 3. Turnoff transient of a MOSFET kept in saturation with $V_D = 2 \text{ V}, V_S = 0 \text{ V}, V_B = 0 \text{ V}$, a ramp (0 to 2 V) with fall time 0.1 ns is applied at gate.

Fig. 4. Turnoff transient of a MOSFET kept in linear region with $V_D = 0.5 \text{ V}, V_S = 0 \text{ V}, V_B = 0 \text{ V}$, a ramp (0 to 2 V) with fall time 0.1 ns is applied at gate.


of change of drain partition of NQS charge larger. Figs. 4(b), (c), and (d) show the source, gate, and substrate NQS current waveforms under the same condition.

We notice that the BSIM3v3 NQS currents have a much longer tail. In steady state, the NQS component of all currents must go to zero. In Fig. 4, while both NQS current predicted by our model and device simulation reaches zero, the NQS current predicted by BSIM3v3 NQS model has a considerable value and it is reaching zero very slowly (see particularly the source current). This is because of the fact that the BSIM3v3 model does not consider the drift current due to the excess non-equilibrium charge. In reality, the spatial derivative of this term has a major contribution to the time derivative of the charge. In our model, this fact is modeled by the $\beta$ (20) terms which arise due to the drift of excess charge.

VI. CONCLUSION

In this work, a new physics-based NQS model for the MOSFET has been presented. The model is derived starting from basic semiconductor equations and without using any fitting parameters. The NQS charge profile has been expressed as a linear combination of basis functions. The model results in a nonlinear state equation, with nonlinearity only of second order. It is therefore possible to implement the new model into a circuit simulator. This calls for defining new “system variables” for the MOS transistor, in addition to the bias voltages. Since the introduction of more variables will generally slow down circuit simulation, a practical compromise may be employed: The “slow” devices (with larger gate lengths) could be treated with the NQS model whereas the normal devices could be simulated with the standard QS models.

A remark about the boundary conditions used in the model derivation is in order. In this work, it has been assumed that the source and drain charge can always be expressed in terms of terminal voltages which results in a homogeneous boundary condition for $Q_{NQS}$. However, it should be pointed out that this assumption is valid only when the Gradual Channel Approximation (GCA) is valid. For extremely scaled devices, the CGA is not valid and the drain end of the charge is not uniquely determined by terminal voltages. In that case, the boundary condition at the drain end is much more complex and nonlinear. This is a difficult problem which needs further work. Fortunately, the simple boundary condition used in this work is justified except for devices with very short channel lengths, and for these devices, the NQS effects are not likely to be important anyway.

Using (30), we get

$$\int_0^L \frac{I_{dc}(x)}{dx} \frac{dx}{I_{dc}(x)dx} = \frac{\int_0^L kQ_{NQS} \frac{\partial Q_{NQS}}{\partial x} dx}{\int_0^L kQ_{NQS} \frac{\partial Q_{NQS}}{\partial x} dx}. \quad (31)$$

But the dc current $I$ is constant along the channel; therefore, we have

$$\frac{1}{L} \int_0^L \frac{Q_{NQS}^2(x)}{Q_{NQS}(L)^2} - \frac{Q_{NQS}(0)^2}{Q_{NQS}(L)^2} \quad \text{and} \quad Q_{NQS}(x) = Q_{NQS}(0) \sqrt{1 - \frac{x}{L}}$$

where $\lambda = 1 - \left(\frac{Q_{NQS}(L)}{Q_{NQS}(0)}\right)^2 \quad (32)$

In order to have a set of compact equations, $Q_{NQS}$ is expanded in Taylor series

$$Q_{approx}(x) = Q(0) \left(1 - \frac{1}{2} \lambda \left(\frac{x}{L}\right)^2 - \frac{1}{8} \lambda^2 \left(\frac{x}{L}\right)^2 + \theta \lambda^3 \left(\frac{x}{L}\right)^3\right) \quad (34)$$

where $\theta$ in the Taylor remainder has to be chosen or to be used as a parameter. We have chosen $\theta$ so that the following condition is satisfied

$$\int_0^L Q_{approx}(x) \frac{dx}{dx} = \int_0^L Q_{NQS}(x). \quad (35)$$

The reason behind this choice is that it produces a reasonable match between $Q$ and $Q_{approx}$ for the entire range of $x$. For weak inversion region, we used a different charge profile. Recognizing the fact that, in weak inversion region, the current is mainly because of diffusion, we can say that the double derivative of $Q_{NQS}$ will vanish. Therefore, $Q_{NQS}$ in weak inversion will be linear as given by

$$Q_{NQS}(x) = Q_{NQS}(0) + (Q_{NQS}(L) - Q_{NQS}(0)) \frac{x}{L}. \quad (36)$$

APPENDIX II

GALERKIN’S METHOD

Galerkin’s method can be described as follows [14]: Let a differential equation be described as $O(u(x,t)) = f(x,t)$, where $O$ is an operator consisting of spatial and temporal derivatives and constant terms, and $f(x,t)$ is a function. A residue is defined as $R(x,t) = O(u(x,t)) - f(x,t)$. Next, we seek an approximate solution of the equation in the form of

$$u(x,t) = \sum \Phi_n(x) \Phi_n(x)$$

where $\Phi_n(x)$ is a set of known functions that satisfy the essential boundary condition.

Now Galerkin’s method states that the optimum values of $X_n(t)$ (values of $X_n(t)$ which will minimize the residue) will occur if the following condition is satisfied

$$\int R(x,t) \Phi_n(x) dx = 0 \quad \forall n. \quad (38)$$

By using Galerkin’s method, any partial differential equation can be reduced to a system of differential equations. The order of the system is equal to the number of terms used to approximate the solution. Thus, by increasing the number of terms,
the accuracy of the solution can be increased but the choice of basis function is also an important factor. In our work, we have chosen $\sin(n \pi x)$ as our basis functions. We have also tried several other basis functions satisfying the essential boundary conditions (i.e., the function is zero at 0 and 1). For example, as the simplest case, we tried functions of the form $x^m (1-x)^m$ with several choices of $n$ and $m$. However, with this class of functions, we were unable to produce a good match between our model and simulation results. We also tried functions which are complete in the interval $[0, 1]$. We tried to express the NQS charge profile in a Fourier-Bessel expansion using Bessel functions of different orders. But we found that this approach takes a larger number of terms. Thus, in our experience, the functions used in this work, viz., $\sin(n \pi x)$, are more appropriate than other possibilities. Another advantage of using $\sin(n \pi x)$ as the basis functions is the following. In many cases, $\beta_{1m}(20)$ vanishes. It vanishes when $n$ is even and both $l$ and $m$ are even or odd. When $n$ is odd, the term vanishes when only one of (not both) $l$ and $m$ is even. However, it cannot be said that $\sin(n \pi x)$ is the optimum choice of basis functions; it may be possible to find some other basis functions which give the same accuracy but with fewer terms. This needs to be explored further.

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REFERENCES


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