A Heuristic for Decomposition in Multilevel Logic Optimization

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Abstract—In this paper, we propose a new heuristic to find common subexpressions of given Boolean functions based on Shannon-type factoring. This heuristic limits the search space of finding common subexpressions considerably by applying a top-down approach. In this top-down approach, synthesis of a Boolean network flows from the primary outputs to the primary inputs. The common subexpressions and their complements in N variables are extracted before common subexpressions and their complements in $(N - 1)$ variables. This decomposition of the network depends upon a permutation of Boolean variables and has a polynomial complexity for restricted extraction of complements.

A multilevel logic optimization system, MULTI, has been implemented using this heuristic. Good results on several benchmark circuits show its effectiveness.

I. INTRODUCTION

COMMON subexpression extraction and factoring are two basic optimization methods for technology-independent multilevel logic optimization [1]. In many multilevel logic synthesis systems [2], [6], algorithms of [1] have been used for multilevel logic optimization. These algorithms are based on the notion of a kernel which is a subexpression that cannot be factored fully. Common subexpressions are computed by finding common kernels and kernel intersections. Since there can be an exponential number of kernels in the number of Boolean variables, these operations take an impractical amount of time for many circuits. Because of this, in optimization of large circuits, these operations are restricted to level-0 kernels only. This problem is particularly evident in the case of symmetric Boolean functions [5].

The approach of [1] with level-0 kernels can be viewed as a bottom-up approach to synthesizing given Boolean functions. In the first step, common kernels of primary inputs are found. There kernels are used recursively to synthesize higher level kernels and finally the primary outputs. As there will be too many kernels at low levels for large circuits, getting a good kernel is more costly and difficult. This limitation of a kernel-based algorithm has been addressed in [8], in which only two literal single cubes and double cube divisors and their complements are

considered. This speeds up the common subexpression extraction process considerably. In this paper, we have proposed a new heuristic which handles this problem efficiently.

The proposed heuristic is a top-down approach to synthesizing a multilevel logic and starts by finding common subexpressions equivalent to higher-level kernels. In this heuristic, the problem of finding common subexpressions of a given circuit has been transformed into the problem of finding a good ordering on the Boolean variables of the circuit. It has a polynomial time complexity in the size of inputs if finding complements is restricted to functions of a constant number of variables.

The system MULTI uses this heuristic to find global common subexpressions. A given Boolean network is globally optimized when this heuristic is used. Local transformation of *misII* [2] are used on the resultant network. This system has been found to give excellent results for symmetric function and large circuits.

II. DEFINITIONS

This section contains definitions which have been used to describe the algorithms. The representation of a circuit in the form of Boolean expressions has been taken from [2].

A *variable* or a *Boolean variable* is a symbol representing a signal which can take the value 0 or 1. A variable v in its complemented (i.e., v') or uncomplemented (i.e., v) form is a *literal*.

A *cube* is a set of literals such that if literal l is present, then literal l' is not present. A cube represents the conjunction of its literals. For example, set $\{a, b\}$ is a cube representing conjunction $a * b$. The Boolean functions 0 and 1 are represented by cubes 0 and 1, respectively.

An *expression* or *subexpression* is a set of cubes. An expression represents a disjunction of cubes. For example, set $\{\{a, b\}, \{b, c\}\}$ represents disjunction of cube $a * b$ and $b * c$ (i.e., $a * b + b * c$). The symbolic representation (e.g., $a * b + b * c$) will be used interchangeably for the set representation (e.g., $\{\{a, b\}, \{b, c\}\}$). The size of an expression e is defined as the number of cubes in e . For example, the size of expression $a * b + b * c$ is 2.

The *support* of an expression is a set of variables used in the expression. The support of expression f is set V , $V = \text{support}(f) = \{v | v \in C \text{ or } v' \in C \text{ and cube } C \in f\}$.

A *factored* expression is defined as follows.

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- 1) A literal is a factored expression.
- 2) A conjunction of factored expressions is a factored expression.
- 3) A disjunction of factored expressions is a factored expression.

For example, $a * (b' * c' + b * c) + a' * (b' * c + b * c')$ is a factored expression.

The *literal factor* of an expression f with respect to a literal l is a set of cubes of f containing l with l deleted: i.e., $LF(f, l) = \{C - \{l\} \mid \text{cube } C \in f \text{ and } C \cap \{l\} \neq \emptyset\}$.

The *variable factor* of expression f with respect to a variable v is a set of expressions, defined as

$$VF(f, v) = \{h_1, h_2, r\}$$

where

$$\begin{aligned} h_1 &= LF(f, v) \\ h_2 &= LF(f, v') \\ r &= \{C \mid \text{cube } C \in f \text{ and } C \cap \{v\} = \emptyset \text{ and } C \cap \{v'\} = \emptyset\}. \end{aligned}$$

For example, the variable factor of expression $a * b * c + a * b' * c' + a' * b * c' + a' * b' * c$ with respect to variable a is set $\{b * c + b' * c', b * c' + b' * c\}$.

A node is defined by the Boolean equation $f = e$, where f is the output signal of the node and expression e represents a Boolean relation between f and support (e).

A *Boolean network* [2] represents a cycle-free combinational multilevel circuit. It is represented by a set of nodes. We define a Boolean network to be a set of Boolean equations.

The cost of a Boolean equation $f = e$ is defined as the number of literals in expression e . For example, the cost of Boolean equation $f = a * b * c + a * b' * c' + a' * b * c' + a' * b' * c$ is 12.

The cost of a Boolean network is defined as the sum of costs of each node of the network. For example, the cost of Boolean network $N = \{f_1, f_2\}$

where

$$\begin{aligned} f_1 &= a * b * c + a * b' * c' + a' * b * c' \\ &\quad + a' * b' * c \end{aligned}$$

and

$$\begin{aligned} f_2 &= a' * b' * c' + a * b * c' + a * b' * c \\ &\quad + a' * b * c \text{ is} \end{aligned}$$

$$\text{cost}(N) = \text{cost}(f_1) + \text{cost}(f_2) = 12 + 12 = 24.$$

III. PERMUTATION DECOMPOSITION

Permutation decomposition is a fast heuristic to extract common subexpressions of a Boolean network. In permutation decomposition, a Boolean network N is transformed into an equivalent Boolean network N' , such that all common subexpressions in variables $V_i, \dots, V_n$ for $1 < i < n$ and $n = |V|$ for a given permutation V of

```
perm_decomp(N, V)
/* Let N be a boolean network
and V be a given ordering
of variables of N. This
algorithm computes an optimized
boolean network N'. */
{
1. N' = φ;
2. for each variable v of V in order do
2.1 N' = φ;
2.2 for each node f of N do
2.2.1 S = {h1, h2, r} = VF(f, v);
2.2.2 for each expression h of S do
2.2.2.1 if( h ∈ N' ) then
name(h) = name(k), k ∈ N' and k == h;
2.2.2.2 else if( h' ∈ N' ) then
name(h) = comp_name(k), k ∈ N' and k == h';
2.2.2.3 else
name(h) = new_name(h);
N' = N' ∪ {h};
2.2.3 f = a * name(h1) + a' * name(h2) + name(r);
2.2.4 N' = N' ∪ {f};
2.3 N = N';
3. return(N');
}
```

Fig. 1. Algorithm perm_decomp.

variables N are expressed as intermediate nodes, and these intermediate nodes are substituted in the nodes of N .

The algorithm for permutation decomposition is given in Fig. 1. The functions used are as follows:

name(): the *name* of a node is the output signal associated with it, whereas the *name* of an expression is the output signal of the node with this expression.

comp_name(): the *comp_name* of a node is the complement of the output signal of the node.

Example 1: Let

$$\begin{aligned} f_1 &= a * b * c + a * b' * c' + a' * b * c' \\ &\quad + a' * b' * c \\ f_2 &= a' * b' * c' + a * b * c' + a * b' * c \\ &\quad + a' * b * c \end{aligned}$$

be a Boolean network and $V = \{a, b, c\}$ a permutation. The Boolean network obtained after permutation decomposition is

$$\begin{aligned} f_1 &= a * [0] + a' * [0]' \\ f_2 &= a * [0]' + a' * [0] \\ [0] &= b * c + b' * c'. \end{aligned}$$

□

Step (2.2.2.2) in the algorithm perm_decomp shown in Fig. 1 requires determining whether two expressions are a complement of each other. This problem is equivalent to the problem of finding a dissatisfying assignment to a DNF formula, which is NP-complete [4]. In this algorithm, this step is restricted to the expressions of a fixed number of variables.

The optimization done by permutation decomposition depends upon the given permutation of the Boolean vari-

ables. Exploring all possible permutations is a costly operation. We have used a greedy algorithm to decide upon a good permutation.

If an expression is symmetric, then the permutation decomposition on any permutation of variables is the best decomposition. In this case, common subexpressions of all possible weights [5] are identified.

The optimization by permutation decomposition can be viewed as a top-down decomposition, in which common expressions with more variables are extracted first. These common expressions are decomposed recursively until common expressions in two inputs are obtained.

IV. MULTILEVEL LOGIC OPTIMIZATION

The multilevel logic optimization algorithm based on permutation decomposition is given in Fig. 2. The functions used in the algorithm are as follows.

simplify(): *simplify* is *misII* transformation which makes each node of the network irredundant with single cubic containment.

partition(): *partition* splits a network N into a set of networks $\{N_1, N_2, \dots, N_n\}$, such that at most one variable is shared between N_i and N_j , $i \neq j$ and $1 \leq i, j \leq n$. This speeds up optimization specifically for large multilevel combinational circuits.

select_perm(): *select_perm* uses a greedy algorithm to select a good permutation of variables of the Boolean network.

factor(): *factor* is the same as the literal factoring algorithm of [2]. Kernel factoring [6] is not required since all good common subexpressions have already been found.

Example 2: Let

$$f_1 = a * c + a * d + a * e$$

$$f_2 = b * c + b * d + b * e$$

$$f_3 = a * c + b * c \quad f_4 = a * d + b * d$$

be a Boolean network. The permutation $V = \{a, b, c, d, e\}$ is selected by *select_perm*. The optimized Boolean network obtained after permutation decomposition on this permutation is

$$f_1 = a * [0]$$

$$f_2 = b * [0]$$

$$f_3 = a * c + b * c$$

$$f_4 = a * d + b * d$$

$$[0] = c + d + e.$$

The optimization steps are repeated again since the cost of the new network has improved. The final optimized Boolean network is

$$f_1 = a * [0] \quad f_2 = b * [0]$$

$$f_3 = c * [1] \quad f_4 = d * [1]$$

$$[0] = c + d + e \quad [1] = a + b.$$

□

```

/* Let N be a given boolean network,
this algorithm returns an optimized
boolean network. */
multi_opt(N)
{
1. N' = simplify(N);
2. N = partition(N');
3. for each network N' of N
3.1 min_cost = cost(N);
3.2 V = select_perm(N);
3.3 N' = perm_decomp(N, V);
3.4 cost = cost(N');
3.5 if(cost < min_cost)
3.5.1 min_cost = cost;
3.5.2 N = N';
3.5.3 goto step 3.2;
3.6 N'' = N'' ∪ factor(N');
4. return(N'');
}

```

Fig. 2. Algorithm multi_opt.

```

select_perm(N)
{
1. V = φ;
2. if( v = select_var(N) )
V = V ∪ {v};
3.else
return( V );
4. N' = φ;
5. for each node f in N do
5.1 S = {h1, h2, r} = VF(f, v);
5.2 N' = N' ∪ S;
6. N = N';
7. goto step2;
}

```

Fig. 3. Algorithm select_perm.

```

select_var(N)
{
1. V = ∪ support(f), f ∈ N;
2. for each v of V in order do
3. N' = φ;
3.1 for each node f of N do
3.1.1 S = {h1, h2, r} = VF(f, v);
3.1.2 for each expression h in S do
3.1.2.1 if(h/h' ∈ N')
saving[v] = saving[v] + cost(h) + size(h) - 1;
3.1.2.2 else
N' = N' ∪ {h};
saving[v] = saving[v] + size(h) - 1;
4. return (variable of max saving);
}

```

Fig. 4. Algorithm select_var.

A. Selection of Permutation

A greedy algorithm to find a good permutation of a Boolean network is a given in Fig. 3. A variable which gives maximum saving in literal count is selected by *select_var* (Fig. 4) when variable factor *VF* is applied to each node of the given Boolean network. The nodes of the network are decomposed using *VF* for the selected variable and a new network is obtained. This takes into consideration the changed savings of remaining variables. The next best variable is selected by *select_var* and the steps are repeated similarly for the remaining variables.

TABLE I

BENCH-MARK CIRCUIT	INITIAL # LITERALS	# LITERALS IN SOP			CPU TIME*		
		multi	sis-1.1**	sis-1.1***	multi	sis-1.1**	sis-1.1***
tst	2028	163	202	217	29.7	102.0	239.2
sym10	8370	105	195	248	58.4	710.3	147.3
sym9	3780	81	275	229	17.2	272.5	28.5
alu	6150	368	401	461	95.5	482	405.2
8bitadder	21692	104	136	215	1301.5	992.4	750.4

*CPU time is in seconds on a Sun SPARC IPC.

**sis-1.1 with rugged script.

***sis-1.1 with Boolean script.

TABLE II

BENCH-MARK CIRCUIT	INITIAL # LITERALS	# LITERALS IN SOP			CPU TIME*		
		multi	sis-1.1**	sis-1.1***	multi	sis-1.1**	sis-1.1***
5xp1	303	108	147	168	5.8	13.2	6.7
apex1	9133	1722	1676	1822	1041	1033.3	2273.6
apex2	14871	456	-	-	328.4	-	-
apex3	8397	1961	-	2152	870	-	739
alupla	211	160	144	169	11.8	10.4	5.6
f51m	327	127	109	172	7	10.0	7
sao2	493	171	192	223	22.1	24.7	7
rd73	843	135	73	112	11.2	12.3	17.5

*CPU time is in seconds on a Sun SPARC IPC.

**sis-1.1 with rugged script.

***sis-1.1 with Boolean script.

—System ran out of space (20 MB main memory and 200 MB swap space).

TABLE III

BENCH-MARK CIRCUIT	INITIAL # LITERALS	# LITERALS IN SOP		CPU TIME*	
		multi	sis-1.1**	multi	sis-1.1**
c2670	2075	815	995	28	71
c3540	2936	1421	2319	55	309
c5315	4386	1970	2007	182	159
c6288	4800	3328	4619	80	728
c7552	6144	2501	2838	119	638

*CPU time is in seconds on a Sun SPARC IPC.

**sis-1.1 with Boolean script.

V. EXPERIMENTAL RESULTS

The results of our system MULTI on ISCAS [3] and MCNC benchmarks are presented in Tables I, II, and III. The results have been compared with the best scripts of *sis-1.1*, which has a fast common subexpression extraction algorithm [8] in *misII* framework.

Table I shows the results on some large circuits which have a large number of kernels. The number of kernels for these circuits is shown in Table IV.

The benchmarks *sym9*, *sym10*, and *alu* are ISCAS and *8bitadder* is an 8 bit adder in which outputs and carry out are expressed as the sum of products of inputs and carry in.

The circuits *tst*, *sym10*, and *sym9* are symmetric functions and *8bitadder* and *alu* have good higher level kernels. The excellent results for these circuits can be explained by the fact that being a top-down approach, by our system is able to extract higher level common expressions and their complements efficiently. This system synthesizes ripple carry adder for the circuit *8bitadder*.

TABLE IV

BENCH-MARKS	PI	PO	# LITERALS	# KERNELS
tst	3	3	2028	1478
sym10	10	1	8370	53745
sym9	9	1	3780	17761
alu	14	3	6150	51468
8bitadder	17	9	21692	63079

Table II shows results on MCNC benchmarks. For the circuits *5xp1*, *apex2*, and *apex3*, MULTI produces smaller circuits as compared to the best circuits produced by *sis-1.1*.

Table III presents the results on large multilevel benchmarks of [3]. These circuits have a small number of kernels but have many partitions. Reduction in the circuit size by partitioning results in the fast optimization of these circuits.

VI. CONCLUSION

Generation of good common subexpressions is very important for optimizing the global structure of a Boolean network, but it is a computationally hard problem. The permutation decomposition which we have proposed models this problem effectively and restricts the search space considerably. The optimization by this decomposition algorithm depends upon the permutation of Boolean variables on which the decomposition is done. An effective greedy algorithm for finding a permutation of a Boolean variable is discussed.

Future work will involve exploring the following issues for possible improvements.

- 1) A better heuristic for deciding upon the permutation of variables.
- 2) Restricting a permutation decomposition to a fixed depth and searching for another permutation.
- 3) Use of Boolean properties.
- 4) A mixed-top-down and bottom-up approach.

REFERENCES

- [1] R. K. Brayton and C. McMullen, "The decomposition and factorization of Boolean expressions," in *Proc. Int. Symp. Circuits Syst. (ISCAS-82)*, May 1982, pp. 49-54.
- [2] R. K. Brayton et al., "MIS: A multiple-level logic optimization system," *IEEE Trans. Comput.-Aided Design*, vol. CAD-6, pp. 1062-1081, Nov. 1987.
- [3] A. J. DeGeus, "Logic synthesis and optimization benchmark for the 1986 design automation conference," in *Proc. 23rd IEEE Design Automation Conf.*, 1986, p. 78.
- [4] M. R. Garey and D. S. Johnson, *Computers and Intractability—A Guide to Theory of NP-Completeness*. San Francisco: Freeman, 1979.
- [5] B. G. Kim and D. L. Dietmeyer, "Multilevel logic synthesis of symmetric switching functions," *IEEE Trans. Comput.-Aided Design*, vol. 10, pp. 436-446, Apr. 1991.
- [6] G. De Micheli et al., *Design Systems for VLSI Circuits*. Dordrecht, The Netherlands: Martinus Nijhoff, 1987.
- [7] Y. Nakamura et al., "Area optimization for large circuits by partial collapsing," in *Proc. Int. Symp. Logic Synthesis and Microprocessor Architecture*, July 1992, pp. 28-33.
- [8] J. Rajski and J. Vasudevamurthy, "The testability-preserving concurrent decomposition and factorization of Boolean expressions," *IEEE Trans. Comput.-Aided Design*, vol. 11, pp. 778-793, June 1992.

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