Seismic analysis of structures connected with friction dampers

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Abstract

Analytical seismic responses of two adjacent structures, modeled as single-degree-of-freedom (SDOF) structures, connected with a friction damper are derived in closed-form expressions during non-slip and slip modes and are presented in the form of recurrence formulae. However, the derivation of analytical equations for seismic responses is quite cumbersome for damper connected multi-degree-of-freedom (MDOF) structures as it involves some dampers vibrating in sliding phase and the rest in non-sliding phase at any instant of time. To overcome this difficulty, two numerical models of friction dampers are proposed for MDOF structures and are validated with the results obtained from the analytical model considering an example of SDOF structures. It is found that the proposed two numerical models are predicting the dynamic behavior of the two connected SDOF structures accurately. Further, the effectiveness of dampers in terms of the reduction of structural responses, namely, displacement, acceleration and shear forces of connected adjacent structures is investigated. A parametric study is also conducted to investigate the optimum slip force of the damper. In addition, the optimal placement of dampers, rather than providing dampers at all floor levels is also studied to minimize the cost of dampers. Results show that using friction dampers to connect adjacent structures of different fundamental frequencies can effectively reduce earthquake-induced responses of either structure if the slip force of the dampers is appropriately selected. Further, it is also not necessary to connect two adjacent structures at all floors but lesser dampers at appropriate locations can significantly reduce the earthquake response of the combined system.

Keywords: Adjacent structures; Seismic response; Friction damper; Analytical modeling; Numerical modeling; Non-slip mode; Slip mode; Optimum slip force; Optimal placement

1. Introduction

Structural vibration control, as an advanced technology in engineering, is to implement energy dissipation devices or control systems into structures to reduce excessive structural vibration, enhance human comfort and prevent catastrophic structural failure due to strong winds and earthquakes. Structural control technology can also be used for retrofitting of historical structures especially against earthquakes. The common sense approach to vibration control of structures is with vibration damping that is added to a structure either passively or actively. The damping dissipates some of the vibration energy of a structure by either transforming it to heat or transferring it directly to a connected structure. By utilizing viscoelastic material as well as dashpots, and appending the structures with control devices are the most common ways of adding damping treatment to structures. Effective damping can result by properly treating the structure, which is not damped adequately with viscoelastic materials. In addition, viscous dampers, tuned-mass dampers, friction dampers, dynamic absorbers, shunted piezoceramics dampers, and magnetic dampers are other mechanisms that are used for passive vibration control [1].

Connecting the adjacent structures with passive energy dissipation devices has attracted the attention of many researchers due to its ability in mitigating the dynamic responses as well as to reduce the chances of pounding. Installation of such devices does not require additional space and the free space available between two adjacent structures can be effectively utilized for placing the control devices. Such types of arrangement are also helpful in reducing the mutual pounding of structures which occurred in past major seismic events such as the 1985 Mexico City and 1989 Loma Prieta
earthquakes. Westermo [2] investigated the effectiveness of hinged links for connecting two neighboring floors of buildings to prevent mutual pounding. The hinged link alters the dynamic characteristics of the connected structures and reduces the chances of the pounding phenomenon. Luco and Barros [3] investigated the optimal values for the distribution of viscous dampers interconnecting two adjacent structures of different heights. It was observed that under certain conditions apparently high damping ratios could be achieved by the dampers in various modes of lightly damped structures. Xu et al. [4] studied the effectiveness of fluid dampers connecting multi-story buildings under earthquake excitation. Zhu and Iemura [5] examined the dynamic characteristics of two single-degree-of-freedom systems coupled with a viscoelastic damper under stationary white-noise base excitation. Ni et al. [6] developed a method for analyzing the random seismic response of a structural system consisting of two adjacent buildings interconnected by non-linear hysteretic damping devices.

Although the above studies confirm the effectiveness of different passive dampers in reducing the seismic response of connected structures, however, the dynamic behavior of two adjacent structures connected with friction dampers is not yet investigated. The friction dampers have advantages such as simple mechanism, low cost, less maintenance and powerful energy dissipation capability as compared to other passive dampers. They were found to be very effective for the seismic design of structures as well as the rehabilitation and strengthening of existing structures [7–10]. They provide a practical, economical and effective approach for the design of structures to resist excessive vibrations. However, modeling of frictional force in the damper is quite a cumbersome process, as the number of equations of motion varies depending upon the non-slip and slip modes of vibration.

In this paper, an attempt is made to investigate the effectiveness of friction dampers in mitigating the seismic responses of connected structures under various earthquakes. The specific objectives of the study are: (i) to formulate the equations of motion and to derive the closed-form expressions for the seismic responses of the connected SDOF system with a friction damper; (ii) to propose numerical models for the evaluation of frictional force in the connected dampers for the
MDOF system; (iii) to ascertain the optimum slip force in friction dampers; and (iv) to investigate the optimal placement of dampers instead of providing them at all floors to minimize the cost of dampers.

2. Two SDOF structures connected with friction damper

Consider two adjacent structures connected with friction damper as shown in Fig. 1(a). The adjacent structures are idealized as single-degree-of-freedom systems and referred to as Structure 1 and 2. The frictional force mobilized in the damper has typical Coulomb-friction characteristics. The corresponding mechanical model of the structures connected with friction damper is shown in Fig. 1(b). Let \( m_1, c_1, \) and \( k_1 \) be the mass, damping coefficient and stiffness, respectively of Structure 1. Similarly, \( m_2, c_2, \) and \( k_2 \) denote the corresponding parameters of Structure 2. The system is subjected to earthquake motion. Depending upon the system parameters and excitation level, the connected structures may vibrate together without any slip in the friction damper (referred to as non-slip mode) or vibrate independently if frictional force in the damper exceeds the limiting value. The formulation of the equations for this system and the derivation of equations for the analytical responses of the two connected structures are given here.

2.1. Non-slip mode

During the non-slip mode, both structures vibrate together as a single-degree-of-freedom system under ground excitation. Thus, the governing equation of motion of the combined system is expressed by

\[
m_0 \ddot{x}_0 + c_0 \dot{x}_0 + k_0 x_0 = -m_0 \ddot{x}_g
\]

where \( m_0 = m_1 + m_2, c_0 = c_1 + c_2 \) and \( k_0 = k_1 + k_2 \) are mass, damping coefficient and stiffness of the combined system, respectively; \( x_0, \dot{x}_0 \) and \( \ddot{x}_0 \) are displacement, velocity and acceleration of the combined system, respectively; and \( \ddot{x}_g \) is the ground acceleration.

The coupled system remains in the non-slip mode until the frictional force in the damper is less than the limiting frictional force. The frictional force in the damper can be obtained by considering the dynamic equilibrium of either Structure 1 or 2. Thus, the non-slip mode of the damper is valid until the following inequalities hold good

\[
|m_1 (\ddot{x}_0 + \ddot{x}_g) + c_1 \dot{x}_0 + k_1 x_0| \leq f_s
\]
Fig. 5. Structural model of two MDOF structures connected with friction dampers.

Table 1
Peak displacement responses of two SDOF structures ($f_s = 0.173$)

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Structure</th>
<th>Displacement (cm)</th>
<th>Unconnected</th>
<th>Connected</th>
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<tr>
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<tr>
<td>Kobe, 1995</td>
<td>1</td>
<td>37.72</td>
<td>28.20 (25.25)</td>
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<td></td>
<td>2</td>
<td>9.69</td>
<td>9.43 (2.74)</td>
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<td>Northridge, 1994</td>
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<td>23.83</td>
<td>13.48 (43.43)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>14.53</td>
<td>10.85 (25.34)</td>
<td></td>
</tr>
<tr>
<td>Loma Prieta, 1989</td>
<td>1</td>
<td>28.06</td>
<td>13.31 (52.56)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>13.79</td>
<td>10.84 (21.41)</td>
<td></td>
</tr>
</tbody>
</table>

Quantity within the parenthesis denotes the percentage reduction.

or

$|m_2(\ddot{x}_0 + \ddot{x}_g) + c_2\dot{x}_0 + k_2x_0| \leq f_s$  \hspace{1cm} (2b)

Fig. 6. Modeling of force in the friction damper using fictitious spring concept.

where $f_s$ is the limiting force in the friction damper and it is referred to as slip force.

2.2. Slip mode

Whenever the force in the friction damper attains its slip force, the system moves into the slip mode. The condition for initiation of slippage is written as

$|m_1(\ddot{x}_1 + \ddot{x}_g) + c_1\dot{x}_1 + k_1x_1| > f_s$ \hspace{1cm} (3a)

or

$|m_2(\ddot{x}_2 + \ddot{x}_g) + c_2\dot{x}_2 + k_2x_2| > f_s$ \hspace{1cm} (3b)

where $x_1$ and $x_2$ are the displacements of Structure 1 and Structure 2, respectively. The governing equations of motion of the two connected structures are given by

$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 = -m_1\ddot{x}_g - f_s\text{sgn}(\dot{x}_2 - \dot{x}_1)$ \hspace{1cm} (4)

$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = -m_2\ddot{x}_g - f_s\text{sgn}(\dot{x}_2 - \dot{x}_1)$ \hspace{1cm} (5)

where sgn denotes the signum function.

The coupled system remains in the slip mode till the relative velocity in the friction damper becomes zero i.e. $\dot{x}_1 = \dot{x}_2$. At this point in time, there are two possibilities depending upon the system parameters and excitation level namely, (i) the reattachment of the two structures which is referred to as stick–slip mode and (ii) occurrence of another slip mode in which the damper starts slipping in the opposite direction immediately and this is referred to as slip–slip mode.

The condition for the reattachment of the two structures is expressed as,

$|m_1(\ddot{x}_1 + \ddot{x}_g) + c_1\dot{x}_1 + k_1x_1| \leq f_s$ \hspace{1cm} (6a)

or

$|m_2(\ddot{x}_2 + \ddot{x}_g) + c_2\dot{x}_2 + k_2x_2| \leq f_s$. \hspace{1cm} (6b)

2.3. Solution of equations of motion

To enable the derivation of equations for the analytical seismic responses, the earthquake motion needs to be
represented as a function of time. Thus, it is assumed that the earthquake motion varies linearly in between any two time intervals $t_i$ and $t_{i+1}$ as shown in Fig. 2. Therefore, the earthquake motion at any time $\tau$, $\ddot{x}_g(\tau)$ is given by

$$\ddot{x}_g(\tau) = \ddot{x}_g^i + \frac{\ddot{x}_g^{i+1} - \ddot{x}_g^i}{\Delta t} \tau$$

(7)

where $\ddot{x}_g^i$ and $\ddot{x}_g^{i+1}$ are the earthquake accelerations at times $t_i$ and $t_{i+1}$, respectively; and $\Delta t$ is the sampling time of the earthquake time history.

The general exact solution for the responses of the structures, applicable both for non-slip mode as well as slip mode, can be written as recurrence formula [11] as below.

$$\begin{align*}
\left\{ \begin{array}{c} x_j^{i+1} \\
\dot{x}_j^{i+1} \\
\ddot{x}_j^{i+1} \\
\end{array} \right\} & = A_j \left( \begin{array}{c} A_{j} \\
B_{j} \\
D_{j} \\
\end{array} \right) \left\{ \begin{array}{c} x_j^i \\
\dot{x}_j^i \\
\ddot{x}_j^i \\
\end{array} \right\} + \left\{ \begin{array}{c} E_{j} \\
D_{j} \\
C_{j} \\
\end{array} \right\} f_{sj} \text{sgn}(\ddot{x}_g^2 - \ddot{x}_j^2) \quad \text{for } j = 0, 1 \text{ and } 2.
\end{align*}$$

(8)

The superscripts ‘$i$’ and ‘$i+1$’ denote the quantities at times ‘$i$’ and ‘$i+1$’, respectively. The expressions for the coefficients $A_j, B_j, C_j, D_j, E_j$ are given by

$$A_j = e^{-\xi_j/\omega_j \Delta t} \left( \frac{\xi_j}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t + \cos \omega_j \Delta t \right)$$

(9)

$$B_j = e^{-\xi_j/\omega_j \Delta t} \left( \frac{1}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t \right)$$

(10)

$$C_j = -\frac{1}{\omega_j} \left\{ \frac{2\xi_j}{\omega_j \Delta t} + e^{-\xi_j/\omega_j \Delta t} \left[ \left( \frac{1 - 2\xi_j^2}{\omega_j^2 \Delta t} - \frac{\xi_j}{\sqrt{1 - \xi_j^2}} \right) \times \sin \omega_j \Delta t - \left( 1 + 2\xi_j^2 \right) \cos \omega_j \Delta t \right] \right\}$$

(11)

$$D_j = -\frac{1}{\omega_j} \left\{ 1 - \frac{2\xi_j}{\omega_j \Delta t} + e^{-\xi_j/\omega_j \Delta t} \left( \frac{2\xi_j^2 - 1}{\omega_j^2 \Delta t} \sin \omega_j \Delta t \right) \right.$$  

$$+ \left. \frac{2\xi_j}{\omega_j \Delta t} \cos \omega_j \Delta t \right\}$$

(12)

$$E_j' = \frac{1}{k_j} \left\{ 1 - e^{-\xi_j/\omega_j \Delta t} \left( \frac{\xi_j}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t \right) 
\right.$$  

$$+ \left. \cos \omega_j \Delta t \right\}$$

(13)

$$A_j' = -e^{-\xi_j/\omega_j \Delta t} \left( \frac{\omega_j}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t \right)$$

(14)

$$B_j' = e^{-\xi_j/\omega_j \Delta t} \left( \cos \omega_j \Delta t - \frac{\xi_j}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t \right)$$

(15)

$$C_j' = -\frac{1}{\omega_j} \left\{ -\frac{1}{\Delta t} + e^{-\xi_j/\omega_j \Delta t} \left[ \left( \frac{\omega_j}{\sqrt{1 - \xi_j^2}} \right) 
\right.$$  

$$+ \left. \frac{\xi_j}{\Delta t \sqrt{1 - \xi_j^2}} \cos \omega_j \Delta t \right] \right\}$$

(16)

$$D_j' = -\frac{1}{\omega_j^2 \Delta t} \left\{ 1 - e^{-\xi_j/\omega_j \Delta t} \left( \frac{\xi_j}{\sqrt{1 - \xi_j^2}} \cos \omega_j \Delta t \right) 
\right.$$  

$$+ \left. \cos \omega_j \Delta t \right\}$$

(17)

$$E_j' = \frac{1}{k_j} \left\{ e^{-\xi_j/\omega_j \Delta t} \left( \frac{\omega_j}{\sqrt{1 - \xi_j^2}} \sin \omega_j \Delta t \right) \right\}$$

(18)

where $\xi_j = c_j/2\sqrt{k_j/m_j}, \omega_j = \sqrt{k_j/m_j}$ and $\omega_j' = \omega_j \sqrt{1 - \xi_j^2}$ denote the damping ratio, natural frequency and damped natural frequency of the structure, respectively; and $f_{sj} = 0, f_s$ and $-f_s$, for the combined structure, Structure 1 and Structure 2, respectively.

2.4. Numerical example

A numerical example is considered to study the influence of slip force on the seismic responses of two adjacent connected SDOF structures under various earthquake excitations. The earthquake time histories selected to examine seismic behavior of the two structures are: E00W component of Kobe, 1995, N90S component of Northridge, 1994 and N00E component of Loma Prieta, 1989. The peak ground acceleration of Kobe, Northridge and Loma Prieta earthquake motions are 0.63g, 0.84g and 0.57g, respectively (g is the acceleration due to gravity). The slip force is normalized with the weight of the floor to get the normalized slip force, $\bar{f}_s$ (i.e. $\bar{f}_s = f_s/m_1 g$). The masses of the two structures are assumed to be the same and the damping ratio in each structure is taken as 2%. The values of the stiffness in the two structures are chosen such as to provide fundamental time periods of 1 and 0.5 s for Structure 1 (also termed as the softer structure) and Structure 2 (also termed as the stiffer structure), respectively.

The variation of the peak displacements of the two structures against normalized slip force is shown in Fig. 3 for various earthquake motions. It is observed that the peak displacements of the structures reduce up to a certain increase in value of the slip force. However, with further increase in the slip force, the peak displacements increase. This shows that there exists an optimum value of slip force of friction damper for which the peak displacements of the structures attain a minimum value. The time histories of the displacements of the two structures with and without damper at optimum slip force are shown in Fig. 4. It is seen that the friction dampers are very effective in mitigating the seismic response of the two adjacent connected structures. The reductions in the peak displacements corresponding to optimum slip force are presented in Table 1.
The table indicates the reduction in the displacement of the softer structure is significantly more in comparison to the stiffer structure.

3. Two MDOF structures connected with friction dampers

The system of two adjacent MDOF structures connected with friction dampers is relatively complicated, as some dampers may be in non-sliding phase and some may be in sliding phase at a particular instant of time. After each time step, the state of each damper has to be checked and the forces in the dampers have to be calculated depending upon the sliding and non-sliding phases of the friction dampers at various locations. Thus, to find out the seismic responses of connected adjacent structures, two numerical models are employed in which the number of degrees-of-freedom remains the same irrespective of the mode of vibration of the friction dampers. The formulation of equations of motion and evaluation of the force in friction dampers using the proposed two numerical models are described below.

3.1. Governing equations of motion

The two MDOF structures are assumed to be symmetric with their symmetric planes in alignment. The floors of each structure are at the same level, but the number of storeys in each structure can be different. Each structure is modeled as a linear MDOF flexible shear type structure with lateral degree-of-freedom at their floor levels. Let Structure 1 and Structure 2 have \( n + m \) and \( n \) storeys, respectively as shown in Fig. 5. Note that the connected friction dampers are not shown at all floor levels to indicate that they need not be provided at all floor levels and can be provided even at fewer floor levels. Though the slip force in all dampers is taken to be the same in the present study, note that it need not be the same and can be different. The mass, damping coefficient and shear stiffness values for the \( i \)th storey are \( m_{1i} \), \( c_{1i} \) and \( k_{1i} \) for Structure 1 and \( m_{2i} \), \( c_{2i} \) and \( k_{2i} \) for Structure 2, respectively. The combined system will then be having a total number of degrees-of-freedom equal to \( (2n + m) \). The governing equations of motion for the connected system are expressed as

\[
M \ddot{X} + C \dot{X} + KX = -ML \ddot{x} + F_D
\]  

where \( M \), \( C \) and \( K \) are the mass, damping and stiffness matrices of the combined system, respectively; \( F_D \) is a vector consisting of the forces in the friction dampers; \( X \) is the relative displacement vector with respect to the ground and consists of Structure 1’s displacements in the first \( n + m \) positions and
Structure 2’s displacements in the last $n$ positions; and $I$ is a vector with all its elements equal to unity. The details of each matrix in Eq. (19) are given as

$$M = \begin{bmatrix} m_{n+m,n+m} & o_{n+m,n} \\ o_{n+m,n} & m_{n,n} \end{bmatrix};$$

$$K = \begin{bmatrix} k_{n+m,n+m} & o_{n+m,n} \\ o_{n+m,n} & k_{n,n} \end{bmatrix};$$

$$C = \begin{bmatrix} c_{n+m,n+m} & o_{n+m,n} \\ o_{n+m,n} & c_{n,n} \end{bmatrix}$$

$$m_{n+m,n+m} = \begin{bmatrix} m_{11} \\ m_{21} \\ \cdots \\ m_{n+m-1,1} \\ m_{n+m,1} \end{bmatrix};$$

$$m_{n,n} = \begin{bmatrix} m_{12} \\ m_{22} \\ \cdots \\ m_{n-1,2} \\ m_{n2} \end{bmatrix};$$

$$K_{n+m,n+m} = \begin{bmatrix} k_{11} + k_{21} & -k_{21} & \cdots & -k_{31} \\ -k_{21} & k_{21} + k_{31} & \cdots & -k_{41} \\ \vdots & \vdots & \ddots & \vdots \\ -k_{n+m-2,1} & -k_{n+m-1,1} & \cdots & k_{n+m-1,n} \end{bmatrix}$$

(22)

$$k_{n,n} = \begin{bmatrix} k_{12} + k_{22} & -k_{22} & \cdots & -k_{32} \\ -k_{22} & k_{22} + k_{32} & \cdots & -k_{42} \\ \vdots & \vdots & \ddots & \vdots \\ -k_{n-1,2} & -k_{n-1,2} & \cdots & k_{n-1,n} \end{bmatrix}$$

(23)

$$c_{n+m,n+m} = \begin{bmatrix} c_{11} + c_{21} & -c_{21} & \cdots & -c_{31} \\ -c_{21} & c_{21} + c_{31} & \cdots & -c_{41} \\ \vdots & \vdots & \ddots & \vdots \\ -c_{n+m-2,1} & -c_{n+m-1,1} & \cdots & c_{n+m-1,n} \end{bmatrix}$$

(24)

$$c_{n,n} = \begin{bmatrix} c_{12} + c_{22} & -c_{22} & \cdots & -c_{32} \\ -c_{22} & c_{22} + c_{32} & \cdots & -c_{42} \\ \vdots & \vdots & \ddots & \vdots \\ -c_{n-1,2} & -c_{n-1,2} & \cdots & c_{n-1,n} \end{bmatrix}$$

(25)

$$F_D^T = \{f_{d(n,1)} - f_{d(n,1)}\}$$

(26)

$$f_D^T = \{f_{d1}, f_{d2}, \ldots, f_{dn-1}, f_{dn}\}$$

(27)

$$X^T = \{x_{11}, x_{21}, x_{31}, \ldots, x_{n+m-1,1}, x_{n+m,1}, x_{12}, x_{22}\}$$

Fig. 8. Comparison of displacement time histories of Structure 2 for three models of friction damper.
where \( f_{di} \) is the force in any \( i \)th damper connecting the floors; \( x_{i1} \) and \( x_{i2} \) of Structure 1 and 2, respectively; and \( 0 \) is the null matrix.

3.2. Fictitious spring model (Model 1)

Utilizing a fictitious spring, Yang et al. [12] studied the response of MDOF structures on sliding supports. In their study, they used the force in the fictitious spring to model the friction force under the foundation raft. The spring was assumed to have a very large stiffness in the non-sliding mode and zero stiffness in the sliding mode. Vafai et al. [13] modeled the friction in the sliding support of the MDOF structure as a rigid-plastic link. The link is assumed to be having infinite stiffness during the non-slip mode and zero stiffness during the slip mode. The model described by Yang et al. [12] is used here to model the friction dampers connecting the adjacent buildings. Thus, the friction damper is modeled as a fictitious spring with very high stiffness \( k_{di} \) during non-slip mode and zero stiffness during slip mode as shown in Fig. 6.

The force in the friction damper, \( f_{di} \), equal to the force in the fictitious spring, is then equal to the product of its stiffness and the relative displacement between the two connected floors. The slip takes place whenever the force in the damper exceeds its slip force, \( f_{si} \). When the relative velocities of two connected floors are zero and the force in that connecting damper becomes less than its slip force or when the work done on the incremental relative displacement of the floors is negative, the two floors again move into the non-slip mode. Thus, the force in the friction damper is arrived at as explained below.

Let \( \Delta x_{i1} \) and \( \Delta x_{i2} \) be the incremental displacements in \( x_{i1} \) and \( x_{i2} \), respectively. Initially, the force in the damper is assumed to be zero, i.e. \( f_{di} = 0 \). The incremental force in the damper is calculated from

\[
\Delta f_{di} = k_{di}(\Delta x_{i2} - \Delta x_{i1}).
\]  

Now,

\[
f_{di} = f_{di} + \Delta f_{di}.
\]  

If the absolute value of \( f_{di} \) is less than or equal to the slip force, \( f_{di} \), then the damper is in non-slip mode and the stiffness in the damper is kept equal to \( k_{di} \); otherwise it is in slip mode and the stiffness in the damper is made equal to zero, and the force in the damper is limited to the slip force with proper sign.
### Table 2
Peak responses of two MDOF structures connected with friction dampers

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<th>Structure</th>
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<th>Top floor acceleration (g)</th>
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<tr>
<td></td>
<td></td>
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</tbody>
</table>

![Fig. 10. Comparison of two numerical models of friction dampers for Northridge earthquake for MDOF system.](image)

Thus, in slip mode the force in any \( i \)th damper is given by

\[
f_{di} = f_{s_i} \text{sgn}(\dot{x}_i - \dot{x}_{i1}).
\]  

(31)

Whenever the work done on the incremental relative displacement of the floors, given by

\[
f_{di}(\Delta x_{i2} - \Delta x_{i1})
\]  

(32)

is negative, the two floors again move into the non-slip mode and the stiffness \( k_d \) is considered in the damper. After each time step, the mode of the damper is checked and accordingly the stiffness in the damper is considered.

### 3.3. Hysteretic model (Model 2)

The hysteretic model is a continuous model of the frictional force proposed by Constantinou et al. [14] using Wen's equation [15]. The frictional forces mobilized in the dampers are expressed by

\[
f_{di} = f_{s_i} Z_i \quad (i = 1 \text{ to } n)
\]  

(33)
where $Z_i$ is a non-dimensional hysteretic component satisfying the following non-linear first order differential equation, which is expressed as

$$q \frac{dZ_i}{dt} = A(\dot{x}_i - \dot{x}_{i1}) - \beta |(\dot{x}_i - \dot{x}_{i1})|Z_i|Z_i|^{n-1} - \tau (\dot{x}_i - \dot{x}_{i1})|Z_i|^n$$

where $q$ is the yield displacement; $\beta$, $\tau$, $n$ and $A$ are non-dimensional parameters of the hysteresis loop. The parameters $\beta$, $\tau$, $n$ and $A$ control the shape of the loop and are selected such as to provide a rigid-plastic behavior (typical Coulomb-friction behavior). The recommended values for the above parameters are: $q = 0.1$ mm, $A = 1$, $\beta = 0.5$, $\tau = 0.5$ and $n = 2$. The hysteretic displacement component, $Z_i$ is bounded by its
peak values of $\pm 1$ to account for the conditions of sliding and non-sliding phases.

3.4. Solution of equations of motion

The frictional force mobilized in the dampers is a non-linear function of the displacement and velocity of the system. As a result, the governing equations of motion are solved in the incremental form using Newmark’s step-by-step method assuming linear variation of acceleration over a small time interval, $\Delta t$. For the hysteretic model, an iterative procedure is used for evaluating the incremental hysteretic displacement component. The iterations are required due to the dependence of the $Z_i$ on the response of the system at the end of each time step. In addition, a fourth-order Runge–Kutta method is employed for the solution of the first-order differential equation given in Eq. (34).

Initially all dampers are considered in non-slip mode and hence, $f_{di}$ is equal to $f_i$ for all the dampers. If a damper is not provided at any floor level, then the corresponding force in the damper becomes zero. Eq. (19) is so adaptable and powerful that it can be used whether all dampers are in non-slip mode or in slip mode or some dampers are in non-slip mode and the rest in slip mode. After each time step, the modes of all dampers are checked and accordingly the forces in the dampers are considered. Here, the value of $k_{di}$ is taken as 5000 times the inter-storey stiffness of Structure 1.

4. Results and discussion

The same numerical example, described in Section 2.4, is considered to compare the three models of the friction damper connecting two adjacent SDOF structures under various earthquake excitations. The comparison of the time histories of the displacements obtained from the three models of the friction damper for all earthquakes considered is made in Figs. 7 and 8 for Structure 1 and 2, respectively. From these plots, it is seen that the results obtained from the two numerical models are in very good agreement with that obtained from the analytical model. However, the computational times required by the analytical, fictitious spring and hysteretic models are approximately in the ratio of 1:20:100 for the connected SDOF structures. Thus, the computational time required for the analytical model is less than that required for the numerical models and among the two numerical models, the computational time required for the fictitious spring model is less than that required
Fig. 13. Variation of floor displacements for different damper arrangements in MDOF structures.

for the hysteretic model. The more computational time is required in the hysteretic model due to the additional iterations required in each time step for the convergence of the hysteretic displacement component (refer to Eq. (34)).

Having validated the proposed two numerical models with the analytical model considering two adjacent connected SDOF structures, the seismic responses of two adjacent connected MDOF structures are then obtained using the numerical models. For the purpose of numerical analysis, two adjacent structures with 20 and 10 storeys are considered. The floor mass and inter-storey stiffness are considered to be uniform for both structures. The mass and stiffness of each floor are chosen such as to yield fundamental time periods of 1.9 and 0.9 s for Structure 1 and 2, respectively. The damping ratio of 2% is considered for both structures in all modes of vibration. Thus, Structure 1 may be considered as the softer structure and Structure 2 as the stiffer structure. For the uncontrolled system the first three natural frequencies corresponding to the first three modes of Structure 1 are 3.3069, 9.9014, 16.4378 rad/s and that of Structure 2 are 6.9813, 20.7880, 34.1303 rad/s, respectively. These frequencies clearly show that the modes of the structures are well separated.

The seismic responses of these two MDOF structures connected with friction dampers, using the proposed two numerical models, are compared first in Figs. 9–11 for different earthquakes. The time histories of the top floor relative displacement, top floor absolute acceleration and the normalized base shears of the two structures using both models under Kobe earthquake motion are compared in Fig. 9 for a normalized slip force of 0.204. It is observed that the results are agreeing with each other quite satisfactorily. Similarly, the results obtained for Northridge and Loma Prieta earthquake motions are compared in Figs. 10 and 11 and found that they are matching with each other very well. The comparison of the peak top floor displacements, top floor accelerations and the normalized base shears is made in Table 2. It is found that the results from both models are in excellent agreement with each other. Thus, it is concluded from the results that both proposed numerical models are working very well and can be used to model the frictional force in the dampers, connecting the adjacent MDOF structures.

A thorough study is conducted to arrive at the optimum slip force in the friction dampers for MDOF adjacent structures under various earthquake excitations described earlier. The response quantities of interest are peak relative displacements, peak top floor absolute accelerations and peak base shears. The peak base shears is normalized with the weight of the structure and the slip force is normalized with the weight of the floor
Fig. 14. Variation of floor shears for different damper arrangements in MDOF structures.

to get the normalized base shear and normalized slip force, respectively.

To arrive at the optimum slip force in the friction dampers, the variation of the top floor relative displacements, top floor absolute accelerations and base shears of the two structures are plotted against the normalized slip force and are shown in Fig. 12. It is observed that the responses of both structures for all three earthquakes are reduced up to a certain increase in the value of the slip force and with a further increase in the value of the slip force they are again increased. Therefore, it is clear from the figures that the optimum slip force exists to attain the minimum responses in both structures. As the optimum slip force is not exactly the same for both structures, the optimum value is taken as the one, which gives the minimum sum of the responses of the two structures. In arriving at the optimum value, the emphasis is given to displacements and base shears of the two structures and at the same time care is taken that the accelerations of the structures, as far as possible, are not increased. From Fig. 12, it is observed that the responses are reduced significantly when the normalized slip force is 0.204. For a normalized slip force higher than this, the performance of the dampers is reduced. At very high slip force, the two structures behave as though they are rigidly connected. As a result, the relative displacements and the relative velocities of the connected floors become almost zero and the damper totally loses its effectiveness. On the other hand, if the normalized slip force is reduced to zero, the two structures act as if unconnected.

In order to minimize the cost of dampers, the responses of two adjacent structures are investigated by considering only five dampers (i.e. 50% of the total) with optimum slip force obtained as above at selected floor locations. The floors which have more relative displacements are selected to place the dampers. Many trials are carried out to arrive at the optimal placement of the dampers, among which Figs. 13 and 14 show the variation of the displacements and shear forces in all the floors for four different cases, namely, when case (i) unconnected, case (ii) connected at all the floors, case (iii) connected at 6, 7, 8, 9 and 10 floors and case (iv) connected at 2, 4, 6, 8 and 10 floors. It can be observed from the figures that the dampers are more effective when they are placed at 6, 7, 8, 9 and 10 floors. When the dampers are attached to these floors, the displacements and shear forces in all the storeys are reduced almost as much as when they are connected at all the floors. Hence, 6, 7, 8, 9 and 10 floors are considered for optimal placement of the dampers. The reductions in the peak top floor
displacements, peak top floor accelerations and normalized base shears of the two structures without dampers, connected with friction dampers at all floors and connected with only five friction dampers at optimal locations are shown in Table 3. It is observed from the table that there is a similar reduction in the responses for two damper arrangements and the decrease in the reduction of the responses of the two structures with only 50% dampers is not more than 10% of that obtained for the structures with dampers connected at all the floors. Thus, it is concluded that it is not necessary to connect two adjacent structures by dampers at all floors but lesser dampers at appropriate locations can significantly reduce the earthquake responses of the combined system.

5. Conclusions

Closed form expressions for the analytical responses of two adjacent SDOF structures connected with friction dampers are derived under earthquake excitation. Two numerical models for the evaluation of frictional force in the damper connecting MDOF structures are also proposed and are validated with the results obtained from the analytical model. From the trends in the results of the present study, the following conclusions are drawn:

(1) The seismic responses predicted by the analytical and the numerical models of frictional force in the connected damper closely match.
(2) The friction dampers are found to be very effective in reducing the earthquake responses of the adjacent connected structures.
(3) There exists an optimum slip force of friction dampers for minimum earthquake response of two adjacent connected structures.
(4) It is not necessary to connect two adjacent structures by dampers at all floors but lesser dampers at appropriate locations can significantly reduce the earthquake responses of the combined system.
(5) The neighboring floors having more relative displacement should be chosen for optimal damper locations.
(6) The computational time required for the analytical model is significantly less in comparison to that required for the numerical models and among the two numerical models, the computational time required by the fictitious spring model is less than that required by the hysteretic model.

References