Aseismic design of structure–equipment systems using variable frequency pendulum isolator

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Abstract

Sliding isolation systems have emerged as very useful vibration control technique that incorporate isolation, energy dissipation and restoring mechanism in one unit. However, most currently available systems such as friction pendulum system (FPS) and pure friction (PF) system have practical limitations and are of limited effectiveness when input excitation level is significantly different from its design level. To overcome these limitations while preserving the advantages, a new system called the variable frequency pendulum isolator (VFPI) has been developed by the authors. The isolation period of VFPI continuously decreases with increase in horizontal sliding displacement which results in a more robust isolation system. The VFPI also limits the maximum force transmitted to the structure by providing a restoring force-softening mechanism. In this paper, it has been shown that isolating a structure using VFPI is very effective for vibration control of structure–equipment and other primary–secondary systems. An example five-storey structure with equipment mounted at its top has been analysed to demonstrate the effectiveness of VFPI. It is shown that VFPI provides better vibration control properties compared to other friction isolation systems, and excellent response reduction is observed for a wide range of equipment properties and excitation characteristics. The performance of the example structure with VFPI has been compared with the other frictional systems such as FPS and PF system.

1. Introduction

Base isolation systems are now well established as an effective technique for aseismic design of structures. In base isolation, a flexible layer (or isolator) is placed between the structure and its foundation such that relative deformations are permitted at this level. Since the isolator is flexible, the time period of motion of the isolator is relatively long compared to that of the structure and the isolator time period governs the fundamental period of isolated structure. For properly designed isolation systems, the isolator time period is much longer than those containing significant ground motion energy. As a result, use of isolator shifts the fundamental period of the structure away from predominant periods of ground excitation thereby decreasing the energy introduced into the structure. Extensive review of base isolation systems and its applicability is available in literature (Buckle and Mayes, 1990; Naeim and Kelly, 1999).

Isolation systems that use a sliding layer incorporate isolation and energy dissipation in one unit since energy is dissipated during sliding through friction. Such
systems are very effective in controlling the vibrations of structures due to earthquake and their behaviour is relatively independent of frequency and amplitude of ground motion (Mostaghel and Tanbakuchi, 1983). However friction systems that use a horizontal sliding surface (pure friction system) typically experience large sliding and residual displacements due to absence of restoring force mechanism and are difficult to incorporate in structural design. A modified isolation system called the friction-pendulum system (FPS) has been extensively used in which the sliding and re-centring mechanisms are integrated in one unit and the sliding surface takes a concave spherical shape (Zayas et al., 1990). The restoring force mechanism is provided by a component of vertical load that helps to restore the structure to its original position at the end of excitation. The spherical shape of the sliding surface results in a fairly constant isolator time period (for small displacements relative to radius of the spherical sliding surface) and re-centring mechanism through gravity. The FPS isolators have many advantages over pure friction system; however, severe practical difficulties of aseismic design using FPS isolator due to its fixed time-period have also been observed (Pranesh, 2000). Gravity has been effectively used as a restoring force mechanism in other isolation systems such as antifriction and multi-step base isolation unit (AF&MSBI) and ball-n-cone (BNC) isolator (Shustov, 1992; Tekton, 1994). However in all the above systems proper control over the geometrical properties cannot be achieved so as to obtain desired properties of the isolation system for wide range of structure and ground motion characteristics.

The authors have recently developed a new isolator called the variable frequency pendulum isolator (VFPI), in which a progressive period lengthening takes place in sliding displacement, and which also has a restoring force softening mechanism (Pranesh and Sinha, 2000a, 2002). The innovative feature of VFPI lies in the geometry of its sliding surface which enables the isolator to achieve the desirable isolator properties. The performance of VFPI has been found to be very good under a variety of earthquake excitations and useful for many different applications (Pranesh and Sinha, 2000b; Sinha and Pranesh, 1998). In the present paper, the performance of VFPI for aseismic design of structure–equipment system has been discussed. A structure–equipment system with structure isolated by VFPI, with attached equipment on the top of the structure has been considered. The basic mathematical formulations of structure and equipment response have also been derived. The response of a multi-storey shear building-equipment system isolated with VFPI has been evaluated for different intensities of earthquake excitations. The response have been compared with the response obtained for fixed-base structure, and structure isolated by FPS and pure friction (PF) isolators. It is found that a substantial reduction in the equipment response can be achieved by isolating the structure with VFPI, when compared to other isolation systems.

2. VFPI geometry

Consider a rigid block of mass $m$ sliding on a curved surface of any geometry, $y = f(x)$ representing the sliding surface of the isolator. The origin of the coordinate axis is at the centre of the sliding surface where the sliding displacement is zero. At any instant the restoring force is given by

$$f_R = mg \frac{dy}{dx}$$

(1)

Assuming that this restoring force is provided by an equivalent spring, the spring force can be expressed as the product of spring stiffness and spring deformation. The spring stiffness in turn may be expressed as product of mass and square of isolator frequency. So,

$$f_R = m\omega_b(x)^2$$

(2)

where $\omega_b(x)$ can be called the instantaneous isolator frequency, which solely depends on geometry of the sliding surface. In case of the FPS that has a spherical sliding surface, the isolator frequency is approximately a constant and the restoring force is linear for small sliding displacement. For large sliding displacements the time period decreases sharply and the restoring force increases. These properties of FPS have been found to be responsible for the ineffectiveness of the isolator under high intensity excitations. To eliminate these disadvantages the geometry of sliding surface in case of VFPI has been modified. Its geometry has been derived from the basic equation of an ellipse, with its semi-major axis being a linear function of sliding displacement (Pranesh and Sinha, 2000a). The
The ratio \( b/d \) of the geometry of sliding surface of VFPI can be represented as
\[
y = b \left[ 1 - \frac{\sqrt{d^2 + 2dx \text{sgn}(x)}}{d + x \text{sgn}(x)} \right]
\]  
(3)

where \( b \) and \( d \) are the geometrical parameters defining the profile of the sliding surface. The signum function, \( \text{sgn}(x) \), has been incorporated to maintain symmetry of the sliding surface about the central vertical axis. The slope at any point on the sliding surface is given by
\[
dy = \frac{bd}{(d + x \text{sgn}(x))^2 \sqrt{d^2 + 2dx \text{sgn}(x)}}
\]  
(4)

Defining \( r = x \text{sgn}(x)/d \) and the initial frequency when \( x = 0 \) as \( \omega_0^2 = gb/d^2 \), the instantaneous frequency can be expressed as
\[
a_{\omega}^2(x) = \frac{\omega_0^2}{1 + r^2 \sqrt{1 + 2r}}
\]  
(5)

In the above equations, parameters \( b \) and \( d \) completely define the isolator properties. The ratio \( b/d^2 \) decides the initial frequency of the isolator and the value of \( d \) decides the rate of variation of the isolator frequency. Accordingly the factor \( 1/d \) is termed as frequency variation factor (FVF). The rate of variation of isolator frequency with sliding displacement is directly proportional to FVF.

The properties of VFPI have been shown in Fig. 1 for two values of FVF but having same initial frequency. Properties of FPS with frequency same as initial frequency of VFPI is also shown for comparison. It is observed that the VFPI has two distinguishing characteristics that makes it more effective than the other friction type isolators: (1) the isolator frequency decreases with increase in sliding displacement so as to provide frequency separation between the structure and the excitation (Fig. 1b) and (2) the isolator restoring force has softening mechanism for large sliding displacements which limits the maximum force that is transmitted to the structure during high levels of excitation thereby providing fail-safe mechanism (Fig. 1c) (Pranesh and Sinha, 2000a, 2002). The rate of change of frequency can be controlled by choosing suitable geometrical parameters. From Eq. (3) it is seen that the vertical displacement is bounded by the value \( b \) which is attained only at infinity. A comparison of sliding surface profile of VFPI and FPS is shown in Fig. 1a, in which it can be clearly seen that the vertical displacement in VFPI is smaller than that in FPS.

3. Mathematical formulation

The mathematical formulation has been given below for an \( N \)-storey shear building isolated by VFPI, although the formulation can be easily extended for general three-dimensional structures. Due to the action of frictional forces at the sliding surface, the motion consists of two phases namely, non-sliding (stick) phase and sliding (slip) phase. The equations of motion are different in the two phases and the overall behaviour consisting of a random series of sliding and non-sliding phases is highly non-linear. Depending on the phase of motion, the corresponding equations govern the response of structure and equipment.

3.1. Non-sliding phase

In this phase the structure behaves as a conventional fixed-base structure. Due to frictional resistance there is no relative movement between the base mass and the slider. The equations of motion governing this phase are
\[
\mathbf{M}_t \ddot{\mathbf{x}}_b + \mathbf{C}_b \dot{\mathbf{x}}_b + \mathbf{K}_b \mathbf{x}_b = -\mathbf{M}_t \dot{\mathbf{x}}_b^g
\]  
(6)

and
\[
\dot{x}_b = \ddot{x}_b = 0 \quad \text{and} \quad \dot{\mathbf{x}}_b = \text{constant}
\]  
(7)

with
\[
\left\{ \sum_{i=1}^{N} m_i (\dddot{x}_i + \dddot{x}_b) + m_b \dddot{x}_b \right\} + m_b \dot{\mathbf{x}}_b^g < m_b \mu g
\]  
(8)

In the above equations, \( \mathbf{M}_t \), \( \mathbf{C}_b \) and \( \mathbf{K}_b \) are the \( N \times N \) mass, damping and stiffness matrices of the structure (excluding base mass), respectively, \( \mathbf{x}_b \) the vector of relative displacements of the structure with respect to the base mass, \( \mathbf{x}_b \) the sliding displacement of isolator, \( r_b \) the force influence vector containing unit values, \( m_i \) the mass of the \( i \)th floor, \( m_b \) the base mass and \( m_t \) the total mass of the structure (Fig. 2). The coefficient of friction is given by \( \mu \) and over-dots indicate derivative with respect to time. The left-hand side of Eq. (8) is the absolute value of sum of total inertia
force and restoring force at the isolator level and the right-hand side is the friction force that must be overcome for sliding motion to take place. Eq. (6) can be readily solved by the standard modal analysis procedures (Chopra, 2001).

3.2. Sliding phase

The structure starts sliding when the forces on the system exceed the static friction force leading to motion of the base mass relative to the ground. The
structure now has one additional degree-of-freedom. The equations of motion are given by

\[ \ddot{x} + \ddot{x}_g - \dot{r} \mu_f (\dot{x}_b) = - \ddot{r} \mu_f x_b - \mu_t \omega^2 x_b \mid \sum_{i=1}^{N} m_i (\ddot{x}_i + \ddot{x}_g + \ddot{x}_b) + m_b (\ddot{x}_b + \ddot{x}_g) \mid + m_t \omega^2 x_b \mid \sum_{i=1}^{N} m_i (\ddot{x}_i + \ddot{x}_b) + m_b (\ddot{x}_b + \ddot{x}_g) \mid + m_t \omega^2 x_b \mid \]  

(11)

The signum function, \( \text{sgn}(\dot{x}_b) \), assumes a value of +1 for positive sliding velocity and −1 for negative sliding velocity, and is given by

\[ \text{sgn}(\dot{x}_b) = \frac{\sum_{i=1}^{N} m_i (\ddot{x}_i + \ddot{x}_g + \ddot{x}_b) + m_b (\ddot{x}_b + \ddot{x}_g) + m_t \omega^2 x_b}{\sum_{i=1}^{N} m_i (\ddot{x}_i + \ddot{x}_b) + m_b (\ddot{x}_b + \ddot{x}_g) + m_t \omega^2 x_b} \]  

(10)

The value of signum function remains constant in a given sliding phase. The end of a sliding phase is governed by the condition that the sliding velocity of the base is equal to zero, i.e.,

\[ \dot{x}_b = 0 \]  

(12)

As soon as Eq. (12) is satisfied, Eqs. (6) and (7) corresponding to the non-sliding phase are used to evaluate the response quantities and check the validity of inequality in Eq. (8). This decides whether, during the next time step, the structure continues in the sliding phase or enters a non-sliding phase.

3.3. Solution procedure

Eq. (9) can be directly solved by standard numerical integration techniques in order to obtain the time history of responses. However, this approach is computationally intensive for large size problems, and does not provide any further insight into the structural behaviour. Better understanding of the structural behaviour can be obtained by solving the equations of motion in frequency domain. In this formulation, due to non-classical damping, complex modal analysis has been used to uncouple the equations of motion. The modal properties of the fixed-base structure are assumed to remain unchanged in the sliding phase to reduce the size of the complex eigenvalue problem. The analysis uses the non-linear state vector approach (Singh and Suarez, 1992). The uncoupled equations are solved by step-by-step integration procedures. The close-form final expressions for the evaluation of modal response in both non-sliding and sliding phase are analogous to those for SDOF systems presented in Pranesh and Sinha (2000a).

The equations are linear in non-sliding phase and may be linear or non-linear in sliding phase depending on the nature of restoring force. The solution in any phase depends on the response at the end
of previous phase. Further, a particular phase of response may last for a very short duration. As a result, the transient component of response strongly influences the total response in any step. The change in phase must be evaluated very precisely for accurate determination the structure response. Due to highly non-linear behaviour of the system, and the strong influence of initial conditions at each time-step on non-linear behaviour of the system, and the strong determination the structure response. Due to highly non-linear phase must be evaluated very precisely for accurate evaluation. The total response in any step. The change in the transient component of response strongly influences the overall performance of the isolator. VFPI has been used herein to represent the overall performance in a more unified manner and can be used to decide the overall response to the acceleration of its supporting floor in a similar manner as response of primary structure to ground excitations. The floor accelerations at the support points are strongly influenced by their dynamic properties relative to those of the primary structure, such as frequency ratio (ratio of frequency of secondary system to the fundamental frequency of the primary system), mass ratio (ratio of mass of the secondary system to the mass of the primary system) and damping in the secondary system (Igusa and Der Kiureghian, 1985). The secondary systems exhibit highly amplified responses when the frequency ratio is close to unity. Isolating the primary system can change the dominant frequency of excitation at the base of the secondary system as well as reduces excitation amplitude.

The energy balance equation of a base isolated system is given by (Pranesh, 2000)

\[
\frac{1}{2} \dot{x}_0^2 M_0 x_0 + \frac{1}{2} m_b \dot{y}_0^2 + m_b g y + \frac{1}{2} k_b x_b + \int \left( k_b^2 \delta_b x_b \right) dt + \int m_b \mu g \frac{\delta_b}{|\delta_b|} dx_b \\
= \int \left[ \frac{1}{2} m_r \dot{r}_r^2 \right] dt + \int m_b dx_b (13)
\]

where the subscript ‘i’ of response quantities indicates their absolute values. The equation can be simply written as

\[
E_k + E_p + E_i + E_d + E_{\mu} = E_i (14)
\]

where \(E_k\), \(E_p\) and \(E_i\) are kinetic energy, potential energy due to vertical displacement of the structure during sliding, and strain energy, respectively. These components represent the conservative energy in the system. \(E_d\) and \(E_{\mu}\) are the non-conservative energies due to structural damping and sliding friction, respectively. The absolute input energy given by right-hand side terms has been represented by \(E_i\).

4. Response of isolated structure–equipment systems

Many structures support sub-structures or secondary systems and equipments whose safety and functional integrity during earthquake ground motions is essential. A low-mass secondary structure responds to the acceleration of its supporting floor in a similar manner as response of primary structure to ground excitations. The floor accelerations at the support points of the secondary system are typically narrow-banded due to filtering effect of the structure. As a result, the response characteristics of secondary systems are strongly influenced by their dynamic properties relative to those of the primary structure, such as frequency ratio (ratio of frequency of secondary system to the fundamental frequency of the primary system), mass ratio (ratio of mass of the secondary system to the mass of the primary system) and damping in the secondary system (Igusa and Der Kiureghian, 1985). The secondary systems exhibit highly amplified response when the frequency ratio is close to unity. Isolating the primary system can change the dominant frequency of excitation at the base of the secondary system as well as reduces excitation amplitude.
In the present investigations the response of a light equipment mounted on top of an example shear structure has been considered. The structure consists of a five-storey shear building (excluding base mass), 5 m² in plan (Fig. 2). The story stiffness has been taken as 112,600 kN/m, and the floor mass has been chosen such that the fundamental frequency of the fixed-base structure is close to the predominant frequency of El Centro 1940 ground motion (approximately 2.0 Hz). For this system, the different storeys, including the base have equal mass of 60,080 kg. The investigations have been presented for El Centro ground motions with two different scaling or intensity factors: (1) intensity factor of 1.0 representing the original recorded ground motion, and (2) intensity factor of 2.0, i.e. accelerations intensified by a factor of 2.0 representing high intensity earthquake. The natural frequencies and effective modal mass of the fixed-base structure and the structure isolated by VFPI are shown in Table 1. It should, however, be kept in mind that frequencies of a structure isolated by VFPI change continuously with isolator displacement. The frequencies shown in Table 1 indicate an upper bound that is obtained when the isolator sliding displacement is zero.

A light equipment, that has a mass of 1% of the total mass of the structure (including the base mass), is placed on the top storey of the example structure. The equipment has been modelled by a mass with linear spring and a damper attached to the top floor of the structure. The structural and equipment damping are both assumed to be equal to 5% of critical damping to eliminate the influence of non-classical damping from this example. The parameters of VFPI are chosen as $b = 0.01 \text{ m}$, $d = 0.10 \text{ m}$ and FVF of 10 per m so that it has initial time period of 2.0 s and the frequency reduces sharply with sliding displacement (as shown in Fig. 1). The coefficient of friction has been taken as 0.02.

4.1. Time-history response

The response of structure-equipment system has been obtained by solving Eq. (6) or Eq. (9) depending on the phase of motion. The equipment stiffness is chosen so that the equipment natural frequency is 3.85 Hz, which tunes with the second natural frequency of the isolated structure (first frequency is the isolator frequency) and represents the most severe case of structure-equipment tuning. Typical time-history plot of absolute acceleration and displacement of the equipment relative to the support are given in Fig. 3a. It can be observed from the time-history responses that there is considerable reduction in peak response of the equipment in comparison with both the equipment on fixed-base structure and structure isolated by FPS. It is to be noted that at around $t = 5.5 \text{ s}$, the response of equipment on structure isolated by PF system is greater than that on a structure isolated by VFPI and FPS isolators. This is possibly due to the rapidly varying stick-slip motions in PF system inducing high frequency response in the structure, which in turn affects the response of equipment. Such motions are minimised in case of VFPI and FPS isolators. However, as expected, the peak equipment response is largest in case of structure isolated by FPS.

The typical time-history for recoverable energy of equipment (kinetic energy + strain energy) (Pranesh, 2000) is shown in Fig. 3b. The energy result gives additional information, which cannot be obtained from the response time-history. From this plot it is observed

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Isolator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-base</td>
<td>Frequency (Hz)</td>
<td>–</td>
<td>1.96</td>
<td>5.72</td>
<td>9.02</td>
<td>11.59</td>
</tr>
<tr>
<td></td>
<td>Effective modal mass (%)</td>
<td>–</td>
<td>87.95</td>
<td>87.72</td>
<td>2.42</td>
<td>0.75</td>
</tr>
<tr>
<td>Isolated</td>
<td>Frequency (Hz)</td>
<td>0.49</td>
<td>3.64</td>
<td>6.92</td>
<td>9.76</td>
<td>11.93</td>
</tr>
<tr>
<td></td>
<td>Effective modal mass (%)</td>
<td>99.93</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
that the maximum recoverable energy in equipment is considerably reduced when structure is isolated by VFPI in comparison to that isolated by FPS. However, as expected, it is higher than that of a PF system. As the absolute kinetic energy has been considered in the mathematical formulation, the effect of rigid body movements is also implicitly included. For instance the sliding surface of FPS being circular, it develops higher restoring force with sliding displacement resulting in higher sliding velocities which in turn increases the kinetic energy of the equipment. Also the extent of vertical movement of the equipment during sliding is higher in case of FPS than that in case of VFPI due to the geometry of FPS which increases the potential energy component. This effective increase in recoverable energy is manifested through the various peaks in the recoverable energy time-history for FPS isolated structure. These peaks are drastically reduced in case of VFPI and PF systems indicating more stable response of the equipment.
4.2 Floor response spectra of equipment

The maximum response of single-degree-of-freedom equipment can be conveniently evaluated in terms of its floor response spectra. The floor response spectra also enables one to evaluate the effectiveness of various isolation systems for secondary systems or equipment with different properties.

The displacement and acceleration floor response spectra for equipment on the example structure have been shown in Fig. 4. The effects of structure–equipment interaction, including the effect of equipment on structure, are fully considered in the analysis. The displacement spectra are normalised with respect to the peak displacement of equipment (equal to 0.28 m) mounted on fixed-base structure. In case of a fixed-base structure the equipment acceleration response is maximum when the equipment frequency tunes with fundamental frequency of the structure (approximately 1.85 Hz). However for base-isolated structures, the first natural frequency is the isolator frequency, which is much lower than that of both the fixed-base structure and the equipment. Consequently, the possibility of equipment frequency tuning with the isolator frequency is very unlikely and has not been included. The equipment response shows a peak when the equipment frequency is close to the second frequency of the isolated structure. This can be clearly observed from the acceleration spectra (Fig. 4b) wherein the peak in FPS occurs at 3.85 Hz, which is close to the second isolated frequency (see Table 1). It is to be noted that the amplification of equipment response due to tuning is almost non-existent in case of structure isolated with VFPI. This is due to variation in frequency of VFPI with sliding displacement, which constantly changes fundamental frequency of the isolated structure. It is further observed that response of equipment mounted on structure with VFPI performs better for the entire range of equipment frequencies. The VFPI is effective even for flexible equipment whereas a conventional FPS shows higher response.

The maximum equipment acceleration when its natural frequency is tuned to the natural frequencies of the isolated structure is shown in Table 2. It has been found that for all frequencies of equipment, the use of VFPI for isolating the structure is more effective in reducing the response than the use of FPS. This is due to the almost constant fundamental period of structure

Table 2

<table>
<thead>
<tr>
<th>Equipment frequency (Hz)</th>
<th>3.75</th>
<th>7.14</th>
<th>10.00</th>
<th>12.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure isolated by FPS</td>
<td>0.635</td>
<td>0.510</td>
<td>0.368</td>
<td>0.306</td>
</tr>
<tr>
<td>Structure isolated by VFPI</td>
<td>0.440</td>
<td>0.353</td>
<td>0.328</td>
<td>0.262</td>
</tr>
</tbody>
</table>
Intensity Factor = 1.0

When isolated by FPS so that the structure behaves like a narrow banded filter. On the other hand the fundamental period of a structure isolated by VFPI continuously changes with sliding displacement, making it behave as a wide band filter and results in better vibration control of the secondary system.

The floor-response spectra of recoverable energy for medium and high intensity excitations are shown in Fig. 5. For both excitations there is a substantial reduction in the peak equipment response for VFPI-isolated structure than that for FPS-isolated structure. It is interesting to note that the recoverable energy in equipment on FPS-isolated structure increases with intensity of excitation, whereas the energy in equipment isolated with VFPI and PF systems is relatively independent of the excitation intensity. It is also seen that the variation in equipment response with its frequency is very small when the structure is isolated using VFPI. This shows that the performance of secondary system or equipment is relatively independent of the frequency content and amplitude of excitation when structure is isolated by VFPI. Use of VFPI for structure isolation therefore is very effective for passive vibration control of secondary systems mounted on primary structures.

5. Conclusions

The mathematical formulation for response and energy of a structure–equipment system isolated by the variable frequency pendulum isolator has been presented in this paper. The VFPI provides the ability to vary isolation time period with sliding displacement thereby eliminating the possibility of tuning between the structure and isolator or between the isolator and ground motions. The response of equipment mounted on structures isolated using VFPI, FPS and PF system have been evaluated to evaluate the effectiveness of VFPI.

Based on this investigation following conclusions can be drawn:

1. The VFPI is a robust isolation device and is very effective in controlling the response of structure–equipment placed system when compared to other friction isolation systems such as FPS and PF systems.

2. The VFPI is effective in vibration control of equipment with wide range of structural properties.

3. The behaviour of structure–equipment system isolated by VFPI is relatively independent of the frequency content and amplitude of base excitation.

References


